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SRA MATHEMATICS LEARNING SYSTEM TEXT 6

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SRA MATHEMATICS LEARNING SYSTEM TEXT

M. Vere DeVault
Professor of Curriculum and Instruction
University of Wisconsin

Herbert J. Greenberg
Chairman, Department of Mathematics
University of Denver

Helen Frehmeyer
Classroom Teacher and
Educational Consultant

Stanley J. Bezuszka
Director
Boston College Mathematics Institute

CANADIAN CONSULTANT
William Bober
Supervisor of Mathematics
Edmonton Catholic School District

SRA
SCIENCE RESEARCH ASSOCIATES (CANADA) LIMITED

Toronto, Montreal
Chicago, Palo Alto, Sydney
Henley-on-Thames, Paris, Stuttgart

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1

NUMBER RELATIONSHIPS AND OPERATIONS WITH + AND −

You have been adding and subtracting whole numbers for a long time. Do you know how good you really are? Your goal is to find out for yourself how many addition and subtraction skills you have. You'll be surprised how much you really know.

Sam delivers newspapers in the city. One day his dad asked him how many papers were sold each weekday. Sam called the newspaper office. He was given the following figures.

Newspapers Sold Each Weekday

	To Homes	To Newsstands
City	11 300	7690
Suburb	10 120	3800

1. About how many newspapers were sold in all?
2. Sales in the city total about 19 000. How many are sold outside the city?
3. It was estimated that there were 140 000 persons who might buy the paper. How many don't buy this newspaper?

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1. People who get their papers at home usually pay the newsboy once a month. When Sam goes collecting each month, he also gets tipped. He thinks that he gets more tips on weekends than on weekdays.

Monday's Tips		Saturday's Tips		
24¢ he rounds to	20¢	57¢ he rounds to	60¢	Did he get tipped more
38¢ he rounds to	40¢	89¢ he rounds to	90¢	on Saturday or
92¢ he rounds to	90¢	31¢ he rounds to	30¢	on Monday?
	?		?	

2. Use Sam's rounding method to estimate the tips for the other days of the week.

Tues.	Wed.	Thurs.	Fri.	Sun.
59¢	57¢	62¢	89¢	65¢
12¢	31¢	79¢	43¢	55¢
100¢		16¢	90¢	41¢
				28¢

3. Add the tips for each day of the week.

4. What day did he collect the most money in tips? How much more than on Monday? than on Thursday? than on Sunday?

5. Did Sam get tipped more on Friday, Saturday, and Sunday than he did on Monday through Thursday?

2556887

You have to round a number before you estimate.
But how do you round?

1. Round 27 to nearest *ten*.

THINK 27 is between 20 and 30.
27 is closer to 30 than to 20.
So round to 30.

2. Round 34 to nearest *ten*.

THINK 34 is between 30 and 40.
34 is closer to 30 than to 40.
So round to ?.

3. Round 55 to nearest *ten*.

THINK 55 is between 50 and 60.
55 is halfway.
So round to 60.

4. Round 380 to nearest *hundred*.

THINK 380 is between 300 and 400.
380 is closer to 400 than to 300.
So round to ?.

5. Round 145 to nearest *hundred*.

THINK 145 is between 100 and 200.
145 is closer to 100 than to 200.
So round to 100.

6. Round 550 to nearest *hundred*.

THINK 550 is between 500 and 600.
550 is halfway.
So round to ?.

1. Sam was rounding numbers to the nearest ten. This number line shows multiples of 10. It will help you understand why he rounded up or down.

a 24 was rounded to 20 because 24 is closer to 20 than to 30.

b 38 was rounded to 40 because 38 is closer to 40 than to 30.

c 92 was rounded to 90 because 92 is closer to 90 than to 100.

d Is 59 closer to 50 or 60?

e Is 12 closer to 10 or 20?

f Is 31 closer to 30 or 40?

g Is 16 closer to 10 or 20?

h Is 65 closer to 60 or 70? STOP! What was the agreement? Halfway numbers should be rounded up to the next ten.

i Why doesn't 20 or 50 or 90 need to be rounded?

2. Rounding is a skill needed to estimate answers. Estimates help you make sure your answer is reasonable.

a

58	rounds to	60
+20	- - - - - - - - -	+20
?		80

is a reasonable answer. Will the answer be more than 80 or less than 80? Why? What is the answer?

b

53	rounds to	50
+40	- - - - - - - - -	+40
?		90

is a reasonable answer. Will the answer be more than 90 or less than 90? Why? What is the answer?

c $34 + 90 = \underline{?}$

Round and then estimate.
Will your answer be more than your estimate?
Will it be less?

1. Think before you answer.

a Is 27 + 20 closer to 40 or 50?

b Is 62 + 50 closer to 110 or 120?

c Is 60 + 22 closer to 80 or 90?

d Is 40 + 77 closer to 110 or 120?

e Is 46 + 30 closer to 70 or 80?

f Is 86 + 30 closer to 110 or 120?

g Is 90 + 17 closer to 100 or 110?

h Is 65 + 70 closer to 130 or 140?

Remember?
That symbol > means "is greater than"

2. Complete each of the following.
Name the multiple of 10 that is nearest to the correct answer.

a $42 + 10 > \underline{?}$

b $25 + 20 > \underline{?}$

c $82 + 20 > \underline{?}$

d $59 + 30 < \underline{?}$

e $60 + 18 < \underline{?}$

f $49 + 70 < \underline{?}$

---

3. Both numbers will need rounding on these.
But it is not a hard job.

a Is 21 + 57 closer to 70 or 80?

b Is 33 + 61 closer to 90 or 100?

c Is 28 + 33 closer to 60 or 70?

d Is 48 + 69 closer to 110 or 120?

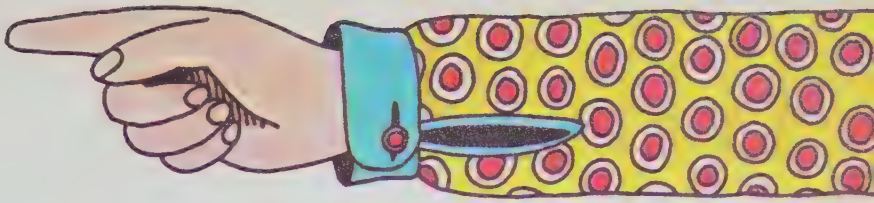
e Is 44 + 26 closer to 60 or 70?

f Is 35 + 45 closer to 80 or 90?

Estimate the answers for each.

	a	**b**	**c**	**d**	**e**	**f**	**g**
4.	35 + 40	60 + 27	70 + 18	57 + 30	81 + 50	92 + 63	49 + 74
5.	57 + 41	82 + 28	46 + 31	84 + 25	72 + 17	69 + 52	95 + 59

Are estimates
wild guesses
or can they predict
a reasonable answer?



1. Estimate the answers first. Then compute the exact answers.

a	b	c	d	e
48	38	72	23	44
+27	+35	+59	+36	+89

How close to the exact answer was your estimate?

2. Copy and complete. Give the estimate to the nearest ten. Then compute.

	Estimate	Exact
46 + 37	**a**	**b**
49 + 38	**c**	**d**
53 + 44	**e**	**f**

	Estimate	Exact
56 + 47	**g**	**h**
62 + 53	**i**	**j**
66 + 57	**k**	**l**

PROGRESS CHECK

Give the estimate to the nearest ten. Then compute.

	Estimate	Exact
13 + 26	(1.)	(2.)
19 + 37	(3.)	(4.)
21 + 42	(5.)	(6.)

	Estimate	Exact
27 + 46	(7.)	(8.)
32 + 52	(9.)	(10.)
36 + 57	(11.)	(12.)

Four students were given the problem $39 + 57 = \underline{?}$ as homework. Their answers were all different. Estimate the answer to see which is probably correct.

Ralph

$$\begin{array}{r} 39 \\ +57 \\ \hline 816 \end{array}$$

Olga

$$\begin{array}{r} 39 \\ +57 \\ \hline 86 \end{array}$$

Steven

$$\begin{array}{r} {}^{1}39 \\ +57 \\ \hline 186 \end{array}$$

Jan

$$\begin{array}{r} 39 \\ +57 \\ \hline 96 \end{array}$$

1. What was Ralph's mistake? What must he remember to do?
2. What help does Olga need?
3. Where did Steven make his mistake?
4. Jan's answer was correct. She computed the answer in two steps.

Problem	Thought Step 1	Thought Step 2
$\begin{array}{r} 39 \\ +57 \end{array}$ →	tens | ones $\begin{array}{r} {}^{1} \\ 3 \mid 9 \\ +\ 5 \mid 7 \\ \hline 6 \end{array}$	→ tens | ones $\begin{array}{r} {}^{1} \\ 3 \mid 9 \\ +\ 5 \mid 7 \\ \hline 9 \mid 6 \end{array}$

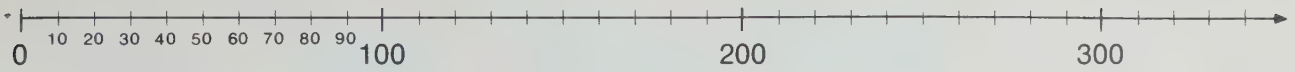
Step 1. Jan knew that $9 + 7 = 16$. She knew that 16 was 1 ten and 6 ones. She wrote the 6 in the ones column and the 1 at the top of the tens column.

Step 2. She added 1 ten, 3 tens, and 5 tens.

Copy these problems. If you estimate your answers before you compute, you will not make the kinds of errors that Ralph and Steven did. Try to think as Jan did.

	a	b	c	d	e	f	g
5.	$\begin{array}{r} 29 \\ +38 \end{array}$	$\begin{array}{r} 17 \\ +67 \end{array}$	$\begin{array}{r} 75 \\ +15 \end{array}$	$\begin{array}{r} 53 \\ +29 \end{array}$	$\begin{array}{r} 46 \\ +36 \end{array}$	$\begin{array}{r} 68 \\ +26 \end{array}$	$\begin{array}{r} 82 \\ +15 \end{array}$
6.	$\begin{array}{r} 82 \\ +19 \end{array}$	$\begin{array}{r} 75 \\ +36 \end{array}$	$\begin{array}{r} 84 \\ +77 \end{array}$	$\begin{array}{r} 33 \\ +99 \end{array}$	$\begin{array}{r} 56 \\ +34 \end{array}$	$\begin{array}{r} 69 \\ +48 \end{array}$	$\begin{array}{r} 97 \\ +55 \end{array}$

We can round hundreds, too.
Use the number line to help you answer the question.



1. Is 110 closer to 100 or 200?
Is 220 closer to 200 or 300?
Estimate the sum of 110 + 220.

110	rounds to	100
+ 220	rounds to	+ 200
		? estimate

Will the sum be > or < the estimate?

2. Is 240 closer to 200 or 300?
Is 170 closer to 100 or 200?
Estimate the sum of 240 and 170.

240	rounds to	200
+ 170	rounds to	+ 200
		? estimate

Will the sum be > or < the estimate?

3. Answer these.

a Is 260 + 200 closer to 400 or 500?

b Is 310 + 420 closer to 700 or 800?

c Is 420 + 260 closer to 600 or 700?

d Is 495 + 160 closer to 600 or 700?

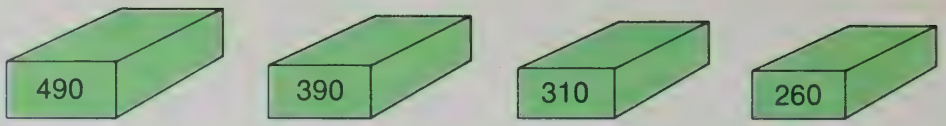
e Is 332 + 521 closer to 800 or 900?

f Is 167 + 221 closer to 300 or 400?

4. Estimate the sums.

a 500 + 400 **b** 750 + 150 **c** 220 + 740 **d** 180 + 190 **e** 290 + 570

1. Jane promised she would get about 600 advertising flyers delivered to the store. She went to pick them up at the printer's and found they had been put into boxes. The quantity was on the label of each box. Which boxes should she take to the store?



a 490 + 390 ⓘ 600

b 490 + 310 ⓘ 600

c 490 + 260 ⓘ 600

d 390 + 310 ⓘ 600

e 390 + 260 ⓘ 600

f 310 + 260 ⓘ 600

g Did she take more than or less than 600? Did she have a good second choice about which boxes she could take? What other choices did she have?

h If she had wanted exactly 600, what would she have had to do?

i If she had taken all the boxes, about how many hundred would she have?

Round to the nearest hundred.
Then estimate the answers.

2.

a	**b**	**c**
420	640	160
+330	+190	+480

d	**e**	**f**
280	461	722
+260	+942	+275

3.

a	**b**	**c**
620	430	670
+290	+540	+720

d	**e**	**f**
560	870	891
+790	+240	+511

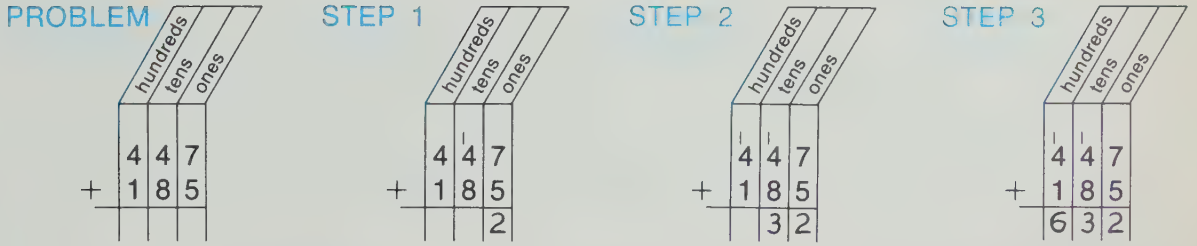
4. Find the exact answers for either problem 2 or problem 3.

Several mistakes were made on one of the class's homework problems. The problem was 447 + 185 = ?
What is your estimate of the answer?

Cecile	Rozanne	Simone	Babs
447	447	447	447
+185	+185	+185	+185
51372	1532	522	632

1. What was Cecile's mistake?
2. What should she have done?
3. Where did Rozanne go wrong?
4. Who was right—Simone or Babs?

She worked the problem in three steps.



STEP 1 7 + 5 = 12. Rename 12 as 1 ten and 2 ones.
Write the 2 in the ones column and
the 1 at the top of the tens column.

STEP 2 Add the tens. 1 ten + 4 tens + 8 tens = 13 tens.
Rename 13 tens as 1 hundred and 3 tens.
Write the 3 in the tens column, and
the 1 at the top of the hundreds column.

STEP 3 Add the 1 hundred, the 4 hundreds, and the 1 hundred.

Copy and compute. Try to think as Babs did.

5.	6.	7.	8.	9.	10.
148	289	639	298	628	262
+364	+516	+148	+736	+492	+879

Time out to check your skills in addition

DO THESE THREE PROBLEMS

a

$$\begin{array}{r} 436 \\ +218 \\ \hline \end{array}$$

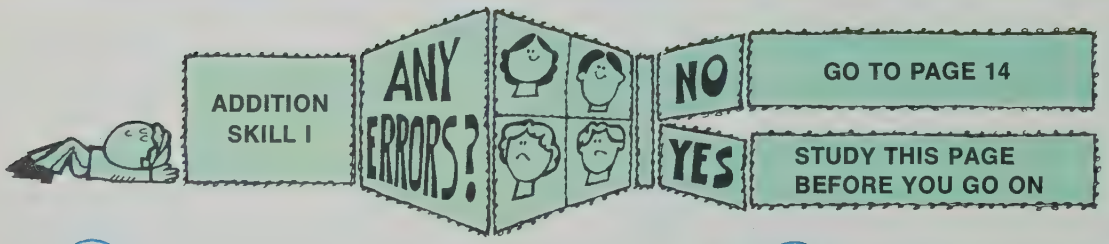
b

$$\begin{array}{r} 34 \\ 17 \\ +25 \\ \hline \end{array}$$

c

$$\begin{array}{r} 7 \\ 6 \\ 5 \\ +3 \\ \hline \end{array}$$

Check your answers with the key on the next page.



1

$$\begin{array}{r} 436 \\ +218 \\ \hline 4 \end{array}$$

6 + 8 is 14. Write 4 and carry 1.

$$\begin{array}{r} 436 \\ +218 \\ \hline 54 \end{array}$$

$$\begin{array}{r} 436 \\ +218 \\ \hline 654 \end{array}$$

2

$$\begin{array}{r} 34 \\ 17 \\ +25 \\ \hline 6 \end{array}$$

4 + 7 is 11. 11 + 5 is 16. Write 6 and carry 1.

$$\begin{array}{r} 34 \\ 17 \\ +25 \\ \hline 76 \end{array}$$

Now for the tens—
1 + 3 is 4.
4 + 1 is 5.
5 + 2 is 7.

3

$$\begin{array}{r} 7 \\ 6 \\ 5 \\ +3 \\ \hline 21 \end{array}$$

Add two numbers at a time.
7 + 6 is 13.
13 + 5 is 18.
18 + 3 is 21. Write 21.

4

$$\begin{array}{r} 21 \\ 57 \\ 43 \\ +68 \\ \hline \blacksquare 9 \end{array}$$

Add the ones column first.
1 + 7 is 8.
8 + 3 is 11.
11 + 8 is 19. Write 9 in the ones column.
Remember to add that 1 ten as you add the tens column.

PROGRESS CHECK

ADD.
(Remember to estimate to prevent careless mistakes.)

1.
$$\begin{array}{r} 3 \\ 4 \\ +7 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 9 \\ 2 \\ 2 \\ +4 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 3 \\ 6 \\ 2 \\ +5 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 4 \\ 7 \\ 5 \\ +9 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 21 \\ 42 \\ +19 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 13 \\ 28 \\ +40 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 59 \\ 17 \\ +18 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 24 \\ 15 \\ +27 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 40 \\ 13 \\ 19 \\ +17 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 24 \\ 19 \\ 18 \\ +15 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 13 \\ 24 \\ 15 \\ +28 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 36 \\ 15 \\ 19 \\ +24 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 32 \\ +39 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 57 \\ +18 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 73 \\ +18 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 27 \\ +46 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 353 \\ +218 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 427 \\ +133 \\ \hline \end{array}$$

19.
$$\begin{array}{r} 269 \\ +458 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 732 \\ +189 \\ \hline \end{array}$$

Go on to the next page if you have time.

a 654 b 76 c 21

Do these two addition problems.

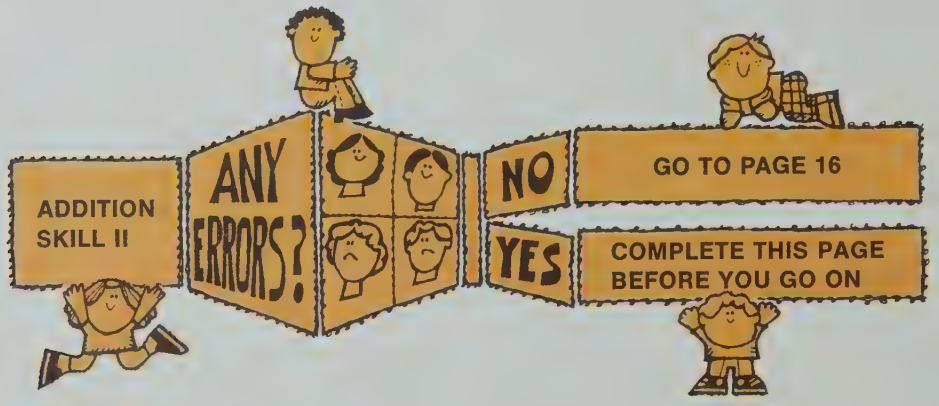
a

$$\begin{array}{r} 3684 \\ +\,2498 \\ \hline \end{array}$$

b

$$\begin{array}{r} 478 \\ 251 \\ +\,639 \\ \hline \end{array}$$

Check your answers with the key on the next page.



1.

$$\begin{array}{r} {\scriptstyle 1} \\ 3684 \\ +\,2498 \\ \hline 2 \end{array}$$

4 + 8 is 12.

1 2

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 3684 \\ +\,2498 \\ \hline 82 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1\,1} \\ 3684 \\ +\,2498 \\ \hline 182 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1\,1} \\ 3684 \\ +\,2498 \\ \hline 6182 \end{array}$$

2.

$$\begin{array}{r} {\scriptstyle 1} \\ 478 \\ 251 \\ +\,639 \\ \hline 8 \end{array}$$

8 + 1 is 9.
9 + 9 is 18.

1 8

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 478 \\ 251 \\ +\,639 \\ \hline 68 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,1} \\ 478 \\ 251 \\ +\,639 \\ \hline 1368 \end{array}$$

PROGRESS CHECK

Add. (Estimates help prevent mistakes.)

(1.) 378 + 224	(2.) 483 + 134	(3.) 271 + 548	(4.) 333 + 267
(5.) 4173 + 3394	(6.) 6752 + 1586	(7.) 4520 + 2899	(8.) 1537 + 3416
(9.) 5487 + 2596	(10.) 3278 + 4956	(11.) 2107 + 3886	(12.) 4548 + 3667
(13.) 369 254 + 103	(14.) 273 224 + 318	(15.) 194 630 + 128	(16.) 239 182 + 457
(17.) 303 247 + 198	(18.) 172 449 + 259	(19.) 250 348 + 365	(20.) 231 249 + 270

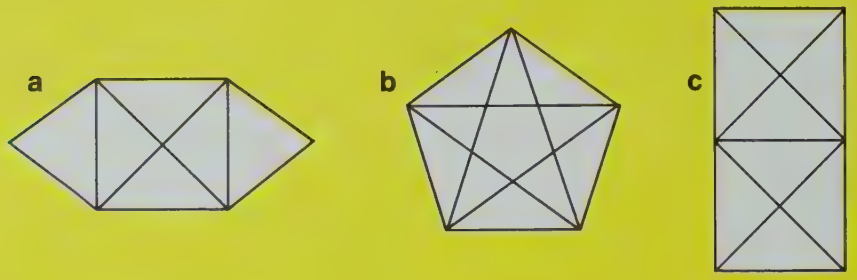
Go on to the next page if you have time.

a 6182 b 1368

Just for fun

1. Arrange the digits 1 through 9 in such a way that when added the answer is 100. Use each digit only once.

2. Put a sheet of paper over the three shapes. Find out which you can trace without lifting your pencil and without going over any part more than once.



3. Most codes use letters or numbers to stand for other letters or numbers. This code is based on addition.

ALPHABET	A	B	C	D	E	F	. . .	Z
CODE	2	3	?	8	13	?	. . .	317 811

Here's a clue: A + C = 7.
Now decode this supersecret dispatch.

TOP SECRET

10 946 – 13 – 5 – 6765 – 13 – 17 711 *

610 – 89 – 10 946 – 10 946 – 89 – 1597 – 987 * 2 *

10 946 – 28 657 – 5 – 5 – 13 – 10 946 – 10 946!

1. Sam delivers newspapers to 37 families in one building. There are 72 families living in the building. About how many families do not get papers from Sam?

$$\begin{array}{rcl} 72 & \text{rounds to} & 70 \\ -37 & \text{rounds to} & -40 \\ \hline & & ? \end{array}$$

2. Sam delivers papers to several apartment buildings. Look at the table below. How many families in each building do not get papers from Sam?

	Bldg. A	Bldg. B	Bldg. C	Bldg. D	Bldg. E
Families in all	64	57	89	72	47
Families who get papers from Sam	43	19	32	49	29

3. Sam delivers papers to 172 families in buildings A, B, C, D, and E. The total number of families living in those buildings is 329. About how many families do not get papers from Sam?

$$\begin{array}{rcl} 329 & \text{rounds to} & 300 \\ -172 & \text{rounds to} & -200 \end{array}$$

4. Round to hundreds and estimate the difference.

a	b	c	d	e	f	g
614	490	569	719	964	423	870
-290	-379	-110	-180	-275	-331	-442

5. Copy and complete. Name the multiple of 10 that is nearest the exact answer.

a $47 - 30 > n$ **b** $82 - 30 < n$ **c** $66 - 20 < n$ **d** $79 - 40 > n$

e $99 - 40 < n$ **f** $27 - 10 > n$ **g** $84 - 60 < n$ **h** $68 - 40 < n$

1 You will have to round only one number to answer these.

a Is 47 − 20 closer to 30 or 20?

b Is 92 − 20 closer to 70 or 60?

c Is 70 − 46 closer to 30 or 20?

d Is 60 − 19 closer to 50 or 40?

e Is 80 − 57 closer to 30 or 20?

f Is 50 − 36 closer to 20 or 10?

g Is 140 − 77 closer to 70 or 60?

h Is 140 − 54 closer to 90 or 80?

2 You will round both numbers to answer these.

a Is 54 − 32 closer to 30 or 20?

b Is 36 − 16 closer to 20 or 30?

c Is 73 − 48 closer to 30 or 20?

d Is 40 − 16 closer to 20 or 30?

e Is 97 − 42 closer to 50 or 60?

f Is 71 − 46 closer to 30 or 20?

g Is 122 − 78 closer to 50 or 40?

h Is 157 − 89 closer to 60 or 70?

Estimate the answer for each.

	a	**b**	**c**	**d**	**e**	**f**
3	53 − 30	47 − 30	28 − 10	79 − 20	75 − 30	27 − 14
4	79 − 37	64 − 21	35 − 13	77 − 64	70 − 14	30 − 21
5	80 − 48	40 − 27	90 − 33	44 − 26	75 − 17	47 − 14

Find the exact answers to the exercises in any row above.
How close were your estimates to the exact answers?

Jerry sells papers on the corner. One day he sold 42. The next day he sold only 28. He subtracted to find the difference. He made some mistakes before he finished.

First try

42	He estimated	42	rounded to	40
−28	the answer to	−28	rounded to	−30
24	check his work.			10

The difference between 24 and his estimate of 10 was large enough to convince him that he had probably made a mistake.

Second try

Step 1

	tens	ones
	4	2
−	2	8

Step 2

	tens	ones
	3	1
	~~4~~	2
−	2	8
	1	4

Copy and subtract to find the exact answer. Rename as Jerry did.

1.	2.	3.	4.	5.	6.
42	65	74	43	81	60
−27	−46	−39	−25	−38	−23

7. Write four subtraction problems that do not require renaming.

8. Write four addition problems that do not require renaming.

9. Write four subtraction problems that do require renaming.

10. Write four addition problems that do require renaming.

11. What is the relationship between the sizes of the two numbers in the ones place that makes renaming necessary in subtraction? in addition?

The next set of numbers to be subtracted will have three digits. If there are more digits, are there more patterns? fewer patterns? Will the patterns that signal renaming be the same or different? Find out.

$$\begin{array}{rl} 151 & \text{rounded to } 150 \\ -143 & \text{rounded to } -140 \end{array}$$

—How "good" is this estimate?
—Is it within ten greater or less than the accurate answer?
—Would this estimate help prevent a silly mistake?

1 Subtract.

a $700 - 300 = \underline{?}$ **b** $360 - 140 = \underline{?}$ **c** $620 - 480 = \underline{?}$

d $720 - 270 = \underline{?}$ **e** $530 - 420 = \underline{?}$ **f** $700 - 410 = \underline{?}$

2 Estimate the answers.

a	437	**b**	892	**c**	659	**d**	718	**e**	435
	−217		−226		−382		−109		−213
f	364	**g**	736	**h**	274	**i**	465	**j**	657
	−192		−525		−181		−177		−384

3 Find exact answers to any four exercises.
How "good" were your estimates?

4 Was renaming sometimes needed in the ones place?
Was renaming sometimes needed in the tens place?
Was renaming sometimes needed in the tens place AND the ones place?

1. Find the answers to these next problems. Your completed work should serve as a reference for the questions on this page.

a
$$\begin{array}{r} 579 \\ -232 \\ \hline ? \end{array}$$
Did you rename ones, tens, or hundreds?

b
$$\begin{array}{r} 347 \\ +232 \\ \hline ? \end{array}$$

c
$$\begin{array}{r} 682 \\ -154 \\ \hline ? \end{array}$$
Did you rename ones, tens, or hundreds?

d
$$\begin{array}{r} 528 \\ +154 \\ \hline ? \end{array}$$

e
$$\begin{array}{r} 575 \\ -379 \\ \hline ? \end{array}$$
Did you rename ones, tens, or hundreds?

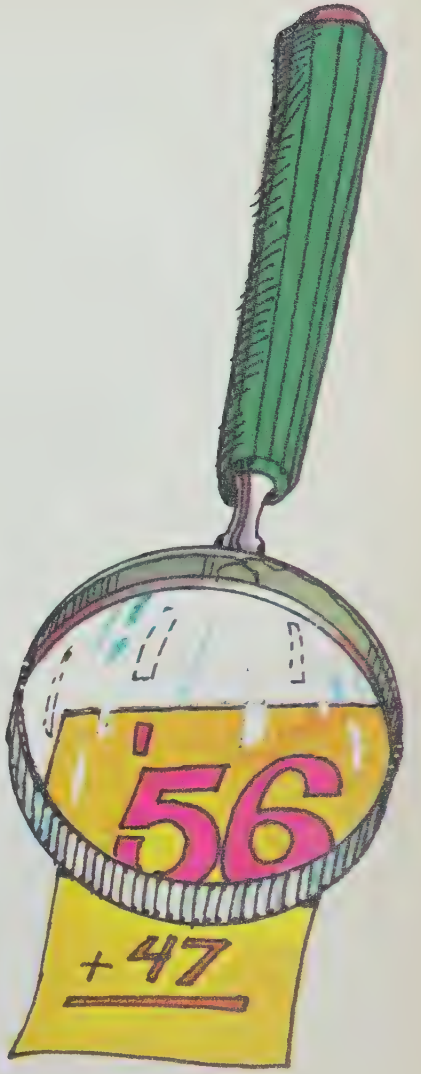
f
$$\begin{array}{r} 196 \\ +379 \\ \hline ? \end{array}$$

g
$$\begin{array}{r} 600 \\ -436 \\ \hline ? \end{array}$$
Did you rename ones, tens, or hundreds?

h
$$\begin{array}{r} 164 \\ +436 \\ \hline ? \end{array}$$

2. You may have to rename pairs of numbers no matter what their value. Would this be true if you had two 4-digit numbers to add or subtract? How about renaming with two 5-digit numbers? With two 6-digit numbers?

3. Let's use the magnifying-glass routine to look just at the renaming. If you add any two 2-digit numbers, what is the largest number you will rename? Prove your answer.



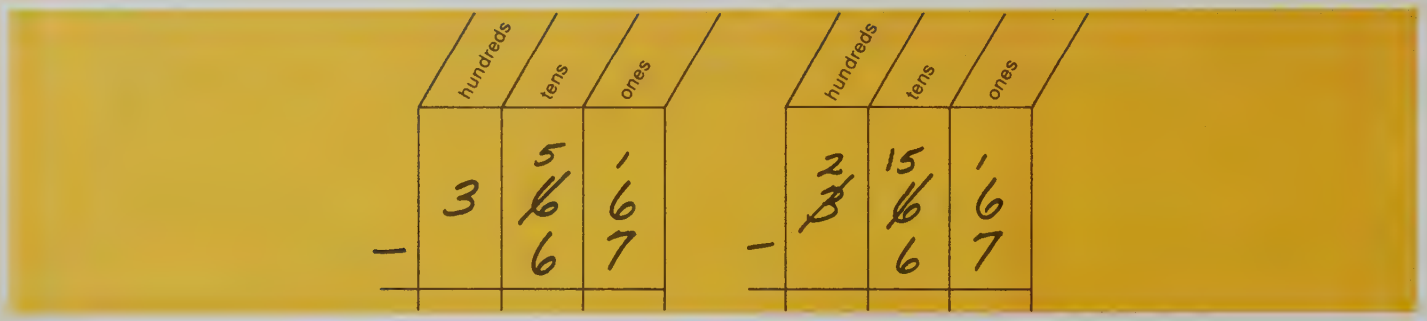
1. What single-digit numbers subtracted from 50 make renaming necessary?

50	50	50	50	50	50	50	50	50	50
− 0	− 1	− 2	− 3	− 4	− 5	− 6	− 7	− 8	− 9

2. What single-digit numbers subtracted from 51 make renaming necessary?

51	51	51	51	51	51	51	51	51	51
− 0	− 1	− 2	− 3	− 4	− 5	− 6	− 7	− 8	− 9

3. What single-digit numbers subtracted from 52 make renaming necessary? from 53? from 54? from 55?



4. In the example above, you must rename twice. How many tens must be renamed as ones? How many hundreds must be renamed as tens? Make up two more subtraction problems in which you have to rename twice. Then find the answers.

Subtract.

5.

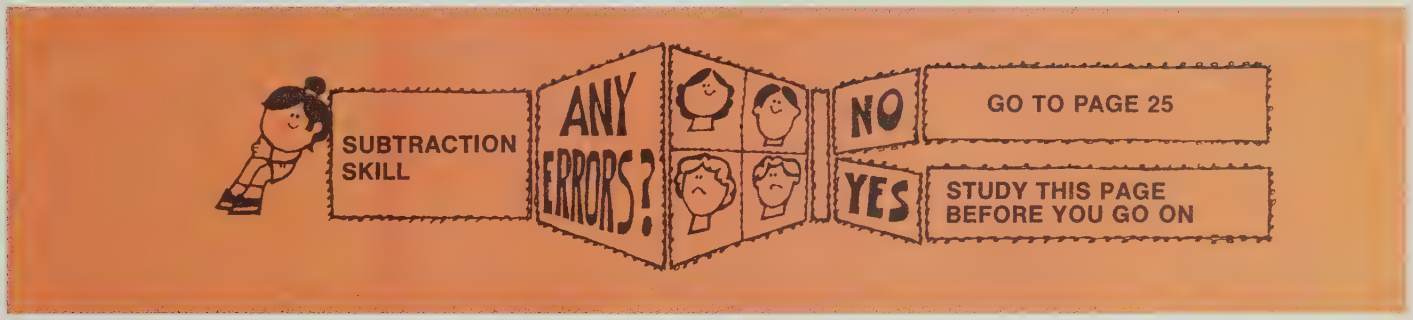
a	b	c	d	e
968	454	867	900	5280
−756	−129	−768	−737	− 591

Time out to check your skills in subtraction.

Do these two problems.

a	b
755	631
−297	− 48

Check your answers with the key on the next page.



1.

$$\begin{array}{r} {\scriptstyle 4\ 1} \\ 7\not{5}5 \\ -297 \\ \hline 8 \end{array}$$

15 − 7 is 8.
But 4 − 9 cannot be done here.

Rename 7 hundreds becomes 6.
4 tens becomes 14.

$$\begin{array}{r} {\scriptstyle 6\ 14\ 1} \\ 7\not{5}5 \\ -297 \\ \hline 8 \end{array}$$

$$\begin{array}{r} {\scriptstyle 6\ 14\ 1} \\ 7\not{5}5 \\ -297 \\ \hline 58 \end{array}$$

Now work on tens.
14 − 9 is 5.
Write 5.

$$\begin{array}{r} {\scriptstyle 6\ 14\ 1} \\ 7\not{5}5 \\ -297 \\ \hline 458 \end{array}$$

Finish with hundreds.
6 − 2 is 4.
Write 4.

2.

$$\begin{array}{r} {\scriptstyle 5\ 12\ 1} \\ 6\not{3}1 \\ -\ 48 \\ \hline 83 \end{array}$$

Compute ones, then tens.
11 − 8 is 3. Write 3.
12 − 4 is 8. Write 8.

$$\begin{array}{r} {\scriptstyle 5\ 12\ 1} \\ 6\not{3}1 \\ -\ 48 \\ \hline 583 \end{array}$$

And finish the hundreds.

Write 5.

PROGRESS CHECK

Subtract. (Remember to estimate to prevent careless mistakes.)

1. 647 − 159	2. 871 − 485	3. 722 − 555	4. 983 − 485
5. 426 − 239	6. 873 − 186	7. 945 − 554	8. 754 − 259
9. 723 − 338	10. 590 − 287	11. 438 − 189	12. 923 − 356
13. 563 − 87	14. 294 − 89	15. 314 − 28	16. 733 − 84
17. 512 − 172	18. 443 − 244	19. 638 − 79	20. 922 − 156
21. 745 − 176	22. 828 − 98	23. 652 − 367	24. 771 − 498

Go on to the next page if you have time.

a 458 b 583

Just for fun

1. **a** Pick any three different digits. Write them in order.
b Reverse the order of the digits.
c Subtract the smaller number from the larger number.
d If there are only two digits in the answer, put a zero in front of them to form a 3-digit number.
e Reverse the order of the digits in your answer.
f Add these two numbers (the numbers in steps **c** and **e**).
g Your answer will be 1089.

Try 3-digit numbers and see if you can get an answer other than 1089.

Try 4-digit numbers. Is the answer 10 890?

2. Here's another code just for fun. This code is based on a pattern of subtraction.

ALPHABET	A	B	C	D	E	F	G	H	I	J	. . .
CODE	260	250	?	?	?	?	?	?	?	?	. . .

Here's a clue: A − C = 20.
Now decode this spy message.

80 − 220 − 130 − 230 * 70 − 190 − 220 *
110 − 260 − 240 − 160 − 260 − 200 − 220 *
70 − 120 * 20 − 120 − 60 − 90 *
240 − 120 − 130 − 70 − 260 − 240 − 70

Computation Sidetrack

Shortcuts

**Some people are very good at mental arithmetic.
They use all sorts of shortcuts.
Here's one. Maybe you'll want to use this idea.**

Think about subtracting 9. Another name for 9 is $10 - 1$.
It's easier to subtract 10 and then add 1 to the answer.

$47 - 9 = \underline{?}$ **Think** $47 - 10 = 37$
$37 + 1 = 38$
So $47 - 9 = 38$

$56 - 9 = \underline{?}$ **Think** $56 - 10 = 46$
$46 + 1 = 47$
So $56 - 9 = 47$

It works for subtracting 19 too.

$63 - 19 = \underline{?}$ **Think** $63 - 20 = 43$
$43 + 1 = 44$
so $63 - 19 = 44$

$76 - 19 = \underline{?}$ **Think** $76 - 20 = 56$
$56 + 1 = 57$
So $76 - 19 = 57$

People who subtract nines this way usually add nines in much the same way.

$47 + 9 = \underline{?}$ **Think** $47 + 10 = 57$
$57 - 1 = 56$
So $47 + 9 = 56$

$83 + 9 = \underline{?}$ **Think** $83 + 10 = 93$
$93 - 1 = 92$
So $83 + 9 = 92$

Would this work for adding 19? 29? 39? 59?

Could the idea be used for subtracting or adding 8?
What changes would have to be made in the "Think" steps?

CHECKOUT

1. Round each number to the nearest ten.

a 41 **b** 56 **c** 79

d 35 **e** 64 **f** 16

2. Round each number to the nearest hundred.

a 125 **b** 156 **c** 179

d 935 **e** 550 **f** 779

3. Estimate. Watch out! You will check out both addition and subtraction.

a Is $570 + 230$ closer to 700 or 800?

b Is $310 - 170$ closer to 200 or 100?

c Is $460 - 215$ closer to 200 or 300?

d Is $265 + 492$ closer to 700 or 800?

4. Compute. You will check out both addition and subtraction.

a $\begin{array}{r} 590 \\ +230 \\ \hline \end{array}$ **b** $\begin{array}{r} 460 \\ -310 \\ \hline \end{array}$ **c** $\begin{array}{r} 525 \\ +387 \\ \hline \end{array}$

d $\begin{array}{r} 635 \\ -595 \\ \hline \end{array}$ **e** $\begin{array}{r} 407 \\ +649 \\ \hline \end{array}$ **f** $\begin{array}{r} 423 \\ -367 \\ \hline \end{array}$

2

NUMBER RELATIONSHIPS AND OPERATIONS WITH × AND ÷

You know what you know about addition and subtraction. What about your skills in multiplication and division? Your goal is to check out these skills. It won't hurt a bit!

SALE PRICES NOW IS THE TIME TO BUY

Onions	1 kg	2 kg	3 kg	4 kg	10 kg
Price	$0.44	?	?	?	?

A The amount you pay depends on how much you buy. If you buy 2 kg, will this math sentence fit the situation? Let m stand for the money you pay. $m = 2 \times \$0.44$

B What sentence would fit if you bought 3 kg? 4 kg?

1. Go back and complete the chart.

The onions had to be packaged before the sale. There were 16 onions in each kilogram bag.

The number of bags depends on the number of onions. You have 48 onions. Let b stand for the number of bags. Will this sentence fit the situation? $b = 48 \div 16$

Number of onions	16	48	80	128	176	320
Number of bags	1	3	?	?	?	?

2. Copy and complete the chart.

Here is a warm-up

Have you ever sent a message to a friend in code? Codes are used in serious business too. There are some people who earn their living breaking codes and decoding messages. The alphabet is frequently coded into number patterns. The message depends on finding the number patterns and breaking the code.

1. Here's an easy code. What is the number pattern? Complete the code for the whole alphabet.

ALPHABET	A	B	C	D	E	F	G	H	I	J	K	L	...
CODE I	1	3	5	7	9	?	?	?	?	?	?	?	...

2. Try this one. What is the number pattern? Finish the code for the whole alphabet.

ALPHABET	A	B	C	D	E	F	G	H	I	J	K	L	...
CODE II	1	2	4	7	11	?	?	?	?	?	?	?	...

3. Here's another. What is the number pattern? Finish the code for the whole alphabet.

ALPHABET	A	B	C	D	E	F	G	H	I	J	K	L	...
CODE III	52	50	48	?	?	?	?	?	?	?	?	?	...

4. This one is a real challenge. (Hint: Two operations are used.) Try to complete this code for the whole alphabet.

ALPHABET	A	B	C	D	E	F	G	H	I	J	K	L	...
CODE IV	2	4	3	6	5	10	9	?	?	?	?	?	...

There are four messages on this page. Each message uses one of the codes on the page before. Each message has the word *the* in it. The code for *the* is underlined. Decoding this one word will help you decide which code to use to decode the whole message.

1. 301 — 106 — 211 * 79 — 211 — 172 — 191 *
22 — 11 — 191 * 191 — 29 — 11 *
121 — 1 — 121 — 11 — 154 — 172

2. 39 — 15 — 9 * 25 — 1 — 27 * 45 — 17 — 23 — 23 *
3 — 9 * 39 — 15 — 9 — 35 — 9

3. 40 — 24 * 14 — 24 * 14 — 38 — 44 *
38 — 24 — 12 — 16 — 44 * 52 — 14 *
26 — 36 — 26 — 44

4. 514 — 5 — 258 — 129 — 514 — 1026 * 1026 — 129 *
1026 — 18 — 5 * 4 — 129 — 513 — 513 * 129 — 130 *
1026 — 1025 — 5 — 513 — 6 — 2 — 4097

1

Jack has a good deal. Every time he puts some money into his bank, his father puts in 25¢. He showed this on a chart like this.

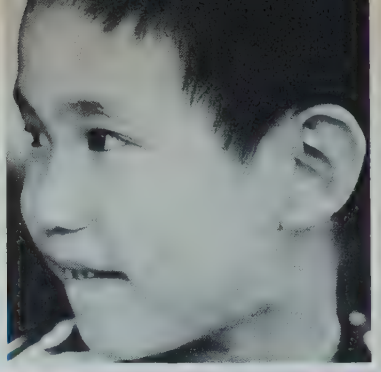
IF I SAVE	DAD ADDS
10¢	25¢
25¢	25¢
75¢	25¢
1.30	25¢
ANY AMOUNT	25¢

The amount Jack's dad adds depends only on Jack saving some money. Jack saves 10¢. His dad adds 25¢.
Total saved is ?.

2

Complete this chart.

Jack saves	His dad adds $0.25	Total saved
$0.10	$0.25	a
$0.25	$0.25	b
$0.75	$0.25	c
$1.30	$0.25	d
e	$0.25	$4.75



3

Jay has a good deal too. Every time he puts some money in his bank, his father puts in the same amount. Jay made this chart.

IF I SAVE	DAD ADDS
10¢	10¢
25¢	25¢
75¢	75¢
1.30	1.30
ANY AMOUNT	MATCHING AMOUNT

The amount Jay's dad adds depends on how much money Jay saves.

Who has the best deal?
Jack or Jay?

Look at the relationship between the two sets of numbers.
The number in column B depends on the number in column A.
But how?

A	B
2	5
3	6
5	8
7	?
9	?

Think

What is the relationship between A and B?
What has been added to 2 to get 5?
What has been added to 3 to get 6?
Does that same number added to 5 give 8?
Add that same number and get the second.
Add that same number and get the second.

Would this sentence tell what's happening?
The first number plus another number equals the second number.

$a + \blacksquare = b$ $\quad 2 + 3 = 5$

Your job is to find the relationship between A and B.
Then complete the table. (Hint: Think addition.)

A	B
10	20
15	25
20	30
21	?
35	?

A	B
3	104
6	107
8	109
12	?
27	?

A	B
21	155
50	184
79	213
82	?
98	?

A	B
135	235
147	247
168	268
205	?
494	?

A	B
20	177
34	191
86	243
159	?
268	?

A	B
1	1000
13	1012
67	1066
101	?
599	?

1

Mr. Tanzini runs a bakery. He bakes nut bread once a week. He delivers 23 loaves every week to a local restaurant. He sells the rest. How many does he have left to sell? It depends on how many loaves he bakes.

Complete the chart.

	Loaves baked	Loaves delivered	Loaves to sell
Week 1	46	23	23
Week 2	52	23	**a**
Week 3	95	23	**b**
Week 4	47	23	**c**
Week 5	61	23	**d**
Week 6	100	23	**e**

Would this sentence tell what has happened?

Number of loaves baked minus those delivered equals number to sell.

$b - d = s$ $\qquad$ $46 - 23 = 23$

2

Jan's mother lets her babysit to earn spending money. But she has to put $0.45 in her bank each time she babysits. The money she can spend depends on how much she earned.

Complete the chart.

	Money earned	Money in bank	Money to spend
Week 1	$1.20	$0.45	$0.75
Week 2	$2.50	$0.45	**a**
Week 3	$1.75	$0.45	**b**
Week 4	$3.25	$0.45	**c**
Week 5	$5.00	$0.45	**d**
Week 6	$3.10	$0.45	**e**

f Write a sentence that tells what has happened.

Look for the relationship between these two sets of numbers.

A	B
5	1
7	3
10	6
12	?
25	?

Think

What is the relationship between A and B?
Is the same relationship true?
Is the same relationship true again?
Then you know the missing number.
You know this one too.

Complete each table.
Ask yourself: What is the relationship between A and B?

1

A	B
15	5
37	27
58	48
110	?
201	?
300	?

2

A	B
72	51
85	64
91	70
94	?
103	?
200	?

3

A	B
204	79
356	231
400	275
562	?
604	?
875	?

4

A	B
235	200
236	201
240	205
500	?
614	?
1000	?

5

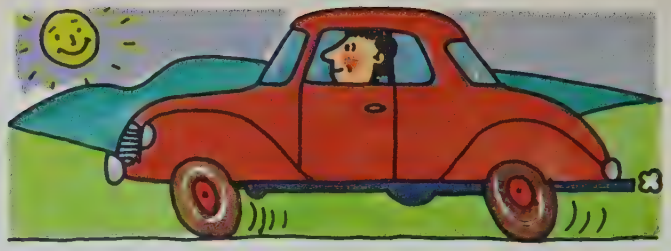
A	B
109	0
198	89
229	120
625	?
700	?
1108	?

6

A	B
999	0
1000	1
1999	1000
3009	?
4909	?
5000	?

1.
Mr. Simons is a salesman. He rents the cars he drives. The Davis Rent-A-Car Company charges $0.20/km . How much does Mr. Simons pay? That depends on how many kilometres he drives.

Mr. Simons usually travels the same roads. Here is a chart that shows 5 days of driving. Complete the chart.



He drives	40 km	60 km	120 km	200 km	500 km
He pays	$8.00	$12.00	?	?	?

a Is it true that the more he drives, the more he pays?
The cost depends on cost per kilometre times the number of kilometres.
Will this math sentence fit the story if he drives 400 kilometres?

$$c = \$0.20 \times 400$$

What does c stand for?
How much would he pay if he drove 400 km?

b Someone said the amount was $800. Is that answer reasonable? What did that person forget?

2.
Mr. Simons tried another car-rental company for a while. They charged $14 a day plus $0.12/km .

He prepared another chart.
You complete it.

He drives	40 km	60 km	120 km	200 km	500 km
He pays	$14 plus $4.80	$14 plus ?	?	?	?

3.
He heard of still another agency that charged $20 a day with no charge for distance. Would he save money here?

Look for the relationship between these two sets of numbers.

A	B
1	5
5	25
6	30
8	?
10	?

Think

What is the relationship between A and B?
Is the same relationship true?
Is it true again?
Then you know the missing number.
You know this one too.

Complete each table.
Ask yourself: What is the relationship between A and B?

A	B
1	10
10	100
20	200
70	?
100	?
1000	?

A	B
2	18
6	54
8	72
9	?
12	?
15	?

A	B
1	12
10	120
12	144
18	?
36	?
73	?

A	B
1	23
2	46
3	69
7	?
15	?
19	?

A	B
1	65
2	130
5	325
12	?
156	?
279	?

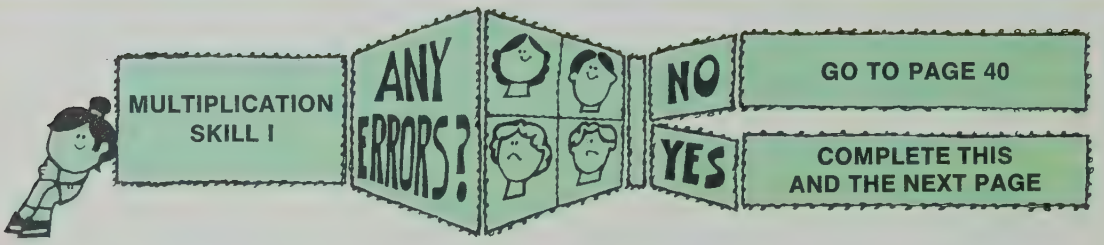
A	B
2	74
4	148
10	370
56	?
123	?
370	?

Time out to check your skills in multiplication.

DO THESE THREE PROBLEMS

a	b	c
53 × 40	31 × 23	49 × 65

Check your answers with the key on the next page.



1.

53 × 40	53 × 40	4 × 53 is 212.
0	2120	Write 212.
		Can you figure out what happened in this shortcut?

2.

31 × 23		31 × 23
93	3 × 31 is 93. Write 93.	93
620	20 × 31 is 620. Write 620.	620
	93 + 620 is 713. Write 713.	713

3.

49 × 65		49 × 65
245	5 × 49 is 245. Write 245.	245
2940	60 × 49 is 2940. Write 2940.	2940
	245 + 2940 is 3185. Write 3185.	3185

PROGRESS CHECK

1. 38×40	2. 46×20	3. 72×60	4. 29×40
5. 21×33	6. 42×12	7. 23×31	8. 22×42
9. 63×11	10. 12×14	11. 33×23	12. 22×22
13. 34×23	14. 28×34	15. 19×43	16. 46×12
17. 78×23	18. 49×45	19. 61×38	20. 39×67
21. 64×54	22. 75×48	23. 89×78	24. 96×87

Check your answers on page 53.

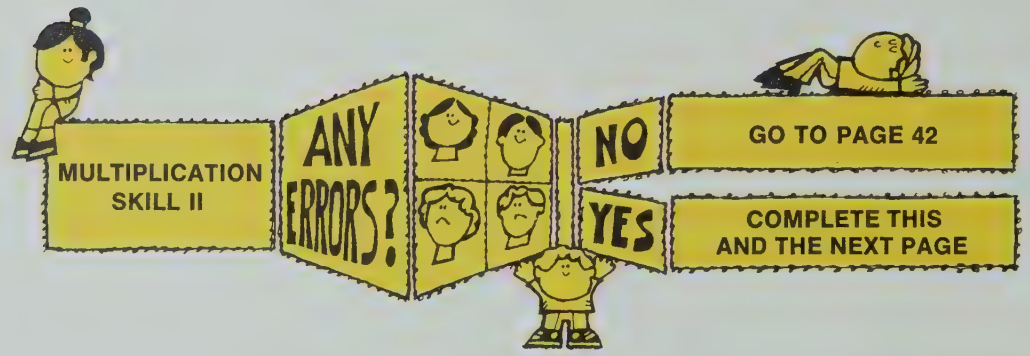
Go to the next page. Do the first two problems.

a 2120 b 713 c 3185

Do these two multiplication problems.

a	b
376 × 43	654 × 300

Check your answers with the key on the next page.



1.

$$\begin{array}{r} 376 \\ \times\ 43 \\ \hline 1128 \end{array}$$

3×376 is 1128.
Write 1128.

$$\begin{array}{r} 376 \\ \times\ 43 \\ \hline 1128 \\ 15040 \end{array}$$

40×376 is 15 040.
Write 15 040.

$$\begin{array}{r} 376 \\ \times\ 43 \\ \hline 1128 \\ 15040 \\ \hline 16168 \end{array}$$

$1128 + 15\,040$ is 16 168.
Write 16 168.

2.

$$\begin{array}{r} 654 \\ \times 300 \\ \hline 00 \end{array}$$

$$\begin{array}{r} 654 \\ \times 300 \\ \hline 196200 \end{array}$$

3×654 is 1962.
So 300×654 is 196 200.
Write 196 200.

PROGRESS CHECK

1. $\begin{array}{r} 443 \\ \times\ 24 \\ \hline \end{array}$	2. $\begin{array}{r} 517 \\ \times\ 53 \\ \hline \end{array}$	3. $\begin{array}{r} 383 \\ \times\ 45 \\ \hline \end{array}$	4. $\begin{array}{r} 245 \\ \times\ 37 \\ \hline \end{array}$
5. $\begin{array}{r} 729 \\ \times\ 25 \\ \hline \end{array}$	6. $\begin{array}{r} 638 \\ \times\ 36 \\ \hline \end{array}$	7. $\begin{array}{r} 218 \\ \times\ 49 \\ \hline \end{array}$	8. $\begin{array}{r} 935 \\ \times\ 68 \\ \hline \end{array}$
9. $\begin{array}{r} 369 \\ \times\ 89 \\ \hline \end{array}$	10. $\begin{array}{r} 726 \\ \times\ 94 \\ \hline \end{array}$	11. $\begin{array}{r} 338 \\ \times\ 26 \\ \hline \end{array}$	12. $\begin{array}{r} 474 \\ \times\ 83 \\ \hline \end{array}$
13. $\begin{array}{r} 573 \\ \times\ 89 \\ \hline \end{array}$	14. $\begin{array}{r} 694 \\ \times\ 78 \\ \hline \end{array}$	15. $\begin{array}{r} 783 \\ \times\ 86 \\ \hline \end{array}$	16. $\begin{array}{r} 967 \\ \times\ 29 \\ \hline \end{array}$
17. $\begin{array}{r} 658 \\ \times\ 86 \\ \hline \end{array}$	18. $\begin{array}{r} 378 \\ \times\ 77 \\ \hline \end{array}$	19. $\begin{array}{r} 947 \\ \times\ 59 \\ \hline \end{array}$	20. $\begin{array}{r} 775 \\ \times\ 84 \\ \hline \end{array}$
21. $\begin{array}{r} 793 \\ \times 400 \\ \hline \end{array}$	22. $\begin{array}{r} 429 \\ \times 300 \\ \hline \end{array}$	23. $\begin{array}{r} 382 \\ \times 600 \\ \hline \end{array}$	24. $\begin{array}{r} 166 \\ \times 700 \\ \hline \end{array}$

Check your answers on page 53.
Go on to the next page.

Just for fun

1. Suppose you were offered a job and were promised 1 cent for the first day's work, double that, or 2 cents, for the second day, 4 for the third, and so on, getting double the preceding day's pay for every day in the month. Would you take the job? Now figure out how much you would have made had you taken the job for the month of March.

2. Multiply these numbers.

   $24 \times 4 \times 10 \times 5 \times 0 \times 1 = ?$

3. Are you ready for another code? The message depends on what each symbol stands for. This code is based on a simple pattern of multiplication.

ALPHABET	A	B	C	D	E	F	G	H	I	J	K	. . .
CODE	2	?	8	?	?	?	?	?	?	?	?	. . .

Here's a clue: $A \times B = 8$.
Now decode the intercepted diplomatic letter.

Office of the Ambassador

65 536 – 4096 – 32 – 2 – 524 288 – 32 *

524 288 – 32 – 16 384 – 16 *

524 288 – 32 – 8 – 262 144 – 32 – 1 048 576 *

64 – 512 – 4096 – 32

1

The supermarket is doing a test. The manager thinks that items sell faster if they are on a certain shelf. The manager experiments in this way: There are 4 shelves. She puts the same number of items that are alike on each shelf. The number of items on each shelf depends on how many she has in all.

Complete this chart.

	Total number	Number of shelves	Number on each shelf
Packages of hard rolls	44	4	11
Hamburger buns	76	4	a
Hotdog buns	64	4	b
Loaves of white bread	96	4	c
Loaves of rye bread	48	4	d
Loaves of wheat bread	60	4	e

Does this sentence tell what's happening?
The total divided by the number of shelves equals the number on each shelf. $t \div s = n$ $44 \div 4 = 11$

2

The club made artificial flowers to sell. They put them in boxes that held 12. How many boxes could they sell? That depends on the number they made in all.

Complete this chart.

	Total number made	Number in each box	Number of boxes
January	12	12	1
February	60	12	a
March	36	12	b
April	144	12	c
May	96	12	d

e Write a sentence that tells what's happening.

Look for the relationship between these two sets of numbers.

A	B
100	10
150	15
170	17
200	?
350	?

Think

What is the relationship between A and B?
Is the same relationship true?
Is the same relationship true again?
Then you know the missing number.
You know this one too.

There is a relationship between A and B in each of these tables. Find out what it is and complete the table.

A	B
25	1
50	2
150	6
275	?
325	?
500	?

A	B
39	3
91	7
273	21
728	?
1053	?
1300	?

A	B
92	2
138	3
276	6
460	?
598	?
1242	?

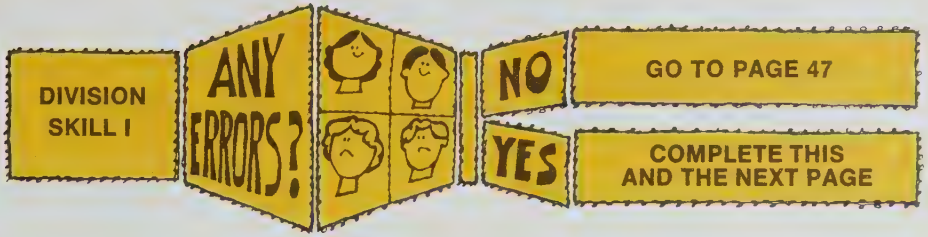
A	B
22	2
66	6
121	11
473	?
957	?
1100	?

A	B
140	4
245	7
280	8
?	10
420	?
945	?

A	B
57	1
114	2
171	3
684	?
1083	?
1425	?

Do these three division problems. **a** $2\overline{)68}$ **b** $3\overline{)639}$ **c** $4\overline{)288}$

Check your answers with the key on the next page.



1. $2\overline{)68}$

$$\begin{array}{r} 30 \\ 2\overline{)68} \\ \underline{60} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

30 × 2 is 60. Write 60. Then subtract.

How many 2s in 68?
20? 20 × 2 = 40
30? 30 × 2 = 60
40? 40 × 2 = 80 too many

$$\begin{array}{r} ? \\ \hline 4 \\ 30 \\ 2\overline{)68} \\ \underline{60} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

How many 2s in 8? 4 × 2 = 8

O.K. Finish it. Add the parts of your answer on top.

2. $3\overline{)639}$

$$\begin{array}{r} ? \\ \hline 3 \\ 10 \\ 200 \\ 3\overline{)639} \\ \underline{600} \\ 39 \\ \underline{30} \\ 9 \\ \underline{9} \\ 0 \end{array}$$

How many 3s in 639?
200 × 3
How many 3s in 39?
10 × 3
How many 3s in 9?
3 × 3

3. $4\overline{)288}$

$$\begin{array}{r} ? \\ \hline 2 \\ 70 \\ 4\overline{)288} \\ \underline{280} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

How many 4s in 288?
70 × 4
How many 4s in 8?
2 × 4

PROGRESS CHECK

1. $3\overline{)93}$	2. $4\overline{)48}$	3. $6\overline{)66}$	4. $2\overline{)86}$
5. $3\overline{)36}$	6. $4\overline{)88}$	7. $5\overline{)55}$	8. $2\overline{)28}$
9. $3\overline{)39}$	10. $4\overline{)44}$	11. $3\overline{)96}$	12. $2\overline{)64}$
13. $2\overline{)468}$	14. $4\overline{)844}$	15. $3\overline{)690}$	16. $3\overline{)366}$
17. $2\overline{)682}$	18. $2\overline{)448}$	19. $4\overline{)804}$	20. $2\overline{)264}$
21. $3\overline{)963}$	22. $2\overline{)822}$	23. $3\overline{)936}$	24. $5\overline{)550}$
25. $3\overline{)189}$	26. $5\overline{)255}$	27. $4\overline{)328}$	28. $4\overline{)244}$
29. $7\overline{)497}$	30. $8\overline{)560}$	31. $5\overline{)305}$	32. $3\overline{)276}$
33. $6\overline{)426}$	34. $3\overline{)249}$	35. $7\overline{)567}$	36. $9\overline{)819}$

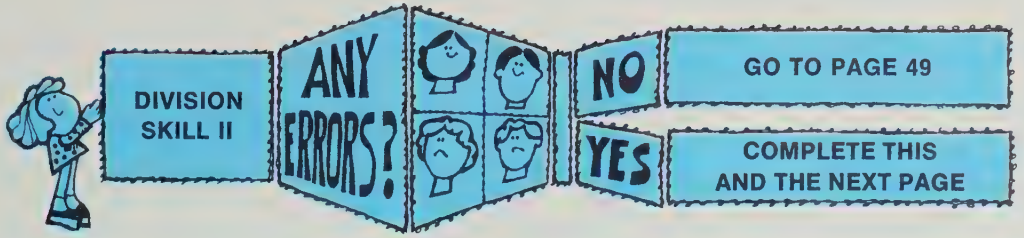
Check your answers on page 53.
Go to the next page. Do the first two problems.

a 94 b 213 c 72

DO THESE TWO DIVISION PROBLEMS.

a $34\overline{)788}$ **b** $49\overline{)8925}$

Check your answers with the key on the next page.



It helps to round your divisor when you have a two-digit divisor.

1.

```
   ?
   3
  20
34)788   About how many 30s in 788?   10 × 30 = 300?
 680     20 × 34                      20 × 30 = 600?   Good estimate
 108     How many 30s in 108?         30 × 30 = 900?   for 34 too.
 102     3 × 34
   6     This is a remainder. It goes with your answer.
```

2.

```
   ?
   2
  80
 100
49)8925   About how many 50s in 8925?   Good estimate for 49 too.
4900       100 × 49
4025       About how many 50s in 4025?
3920       80 × 49
 105       About how many 50s in 105?
  98       2 × 49
   7       This is a remainder. It goes with your answer.
```

PROGRESS CHECK

1. $21\overline{)476}$	2. $39\overline{)726}$	3. $55\overline{)941}$	4. $28\overline{)733}$	5. $42\overline{)594}$	6. $53\overline{)837}$
7. $18\overline{)448}$	8. $71\overline{)975}$	9. $45\overline{)789}$	10. $31\overline{)4876}$	11. $25\overline{)7412}$	12. $66\overline{)7432}$
13. $43\overline{)5893}$	14. $27\overline{)7194}$	15. $38\overline{)5858}$	16. $72\overline{)8561}$	17. $34\overline{)7734}$	18. $29\overline{)9569}$
19. $24\overline{)1798}$	20. $48\overline{)2229}$	21. $33\overline{)2054}$	22. $79\overline{)5142}$	23. $93\overline{)4621}$	24. $49\overline{)4297}$
25. $61\overline{)1378}$	26. $55\overline{)5039}$	27. $16\overline{)1248}$	28. $42\overline{)597}$	29. $31\overline{)7830}$	30. $72\overline{)689}$
31. $53\overline{)2625}$	32. $58\overline{)944}$	33. $49\overline{)3047}$	34. $13\overline{)1971}$	35. $35\overline{)718}$	36. $52\overline{)9282}$

Check your answers on page 53. Go on to the next page.

a. 23 R6 b. 182 R7

Think about putting some of the facts in each story into a table. It might help you find the answers.

1 The sign read: BUY 2 AND GET 1 MORE FREE. How many free ones would you get if you bought 4? 8? 12? How many free ones would you get if you bought 15?

2 Only 5 would fit into a box. How many boxes would you need to pack 15? 50? 60? 75? 88?

3 One jar of coffee makes 32 cups of coffee. How many cups can be made from 5 jars? 10 jars? 12 jars? How many jars will be needed to make 640 cups of coffee?

4 In some restaurants a service charge is added to the bill instead of the customer leaving a tip. In one restaurant the service charge is 15¢ for every dollar spent. How much would the service charge be if your bill is $2.00? $5.00? $10.00? $100.00?

5 He gets a discount on everything he buys. If he buys something that costs $1.00, he pays $0.90. If he buys something that costs $2.00, he pays $1.80. How much does he pay for something that costs $3.00? $5.00? $10.00?

Just for fun

Several years ago a mail-order catalogue used in the late 1800s was reprinted. It was great fun to look at the old catalogue and see how much things cost in those days.

Here are the prices for men's socks that appeared in the catalogue.

The cheapest cotton socks cost \$0.08 a pair.
The most expensive cotton socks cost \$0.45 a pair.
The most expensive silk socks cost \$0.98 a pair.

The amount your great-grandfather would have paid for socks from this catalogue would depend on what things?

Did you remember the socks would be sent through the mail? That is part of the cost too. At that time postage was 2 cents a pair.

amount paid = number of pairs × cost per pair + postage

This is one math sentence that could be used if grandpa bought 3 pairs of cheap socks: $\$ = 3 \times \$0.08 + 3 \times \$0.02$

Goodness! Where did all those numbers come from?

This is another math sentence that could be used for the same purchase: $\$ = 3 \times (\$0.08 + \$0.02)$

The parentheses mean that you complete that part of the problem first. This sentence says that each pair really cost \$0.10. Can that be true? Will the answer be the same no matter which math sentence you use? Prove it.

1. Compute the cost for grandpa's socks if he bought—
   a 4 pairs of each kind **b** 6 pairs of each kind
   c 12 pairs of expensive cotton socks and 6 pairs of silk socks

1 How much do socks cost now? Pretend you're going to buy socks for some man you know. You will go to the store so that you don't have to worry about postage. How much will 2 pairs cost? 3 pairs?

2 That same catalogue had prices for ladies' stockings, too.
The cheapest cotton stockings cost $0.05 a pair.
The most expensive cotton stockings cost $0.50 a pair.
The most expensive silk stockings cost $2.47 a pair.
The postage rate was $0.03 a pair.

Compute the cost for grandma's stockings if she had bought—

a 2 pairs of each kind

b 6 pairs of each kind of cotton stockings

c 12 pairs of the cheapest cotton stockings and 2 pairs of silk stockings

3 Could grandma have bought nylon stockings at the turn of the century? Why? What do you think was the difference between the 5-cent and the 50-cent stockings besides the price? Try to find out what the price range of stockings is today.

4 Postage rates are much different now. Try to find out the cost of sending a letter by first-class mail now. What is the cost of airmail? How are postage rates determined?

Could grandma have sent a letter in the late 1800s by airmail? Why?

5 Why do you think mail-order catalogues were popular in the 1800s?

CHECKOUT

1. Compute. Watch the operation signs.

a 9785 − 8342	**b** 7592 − 4335	**c** 5641 − 1269	**d** 6430 − 3657
e 6324 + 2643	**f** 4605 + 3258	**g** 7348 + 1587	**h** 5976 + 6349

2. Multiply.

a 64 × 30	**b** 57 × 46	**c** 314 × 34	**d** 624 × 87

3. Divide.

a $9\overline{)729}$	**b** $6\overline{)427}$	**c** $32\overline{)452}$	**d** $23\overline{)5451}$

4. Here's Jay's homework paper. Did he get all of them right? If there are errors, show how the problem should be completed.

a) 21 R24
1
20
45 | 1961
900
61
45
24

b) 702
× 50
35100

c) 15
~~16~~30
− 783
853

ANSWERS PAGE 39

1.	2120	**7.**	713	**13.**	782	**19.**	2318
2.	920	**8.**	924	**14.**	952	**20.**	2613
3.	4320	**9.**	693	**15.**	817	**21.**	3456
4.	1160	**10.**	168	**16.**	552	**22.**	3600
5.	693	**11.**	759	**17.**	1794	**23.**	6942
6.	504	**12.**	484	**18.**	2205	**24.**	8352

ANSWERS PAGE 41

1.	10 632	**7.**	10 682	**13.**	50 997	**19.**	55 873
2.	27 401	**8.**	63 580	**14.**	54 132	**20.**	65 100
3.	17 235	**9.**	32 841	**15.**	67 338	**21.**	317 200
4.	9065	**10.**	68 244	**16.**	28 043	**22.**	128 700
5.	18 225	**11.**	8788	**17.**	56 588	**23.**	229 200
6.	22 968	**12.**	39 342	**18.**	29 106	**24.**	116 200

ANSWERS PAGE 46

1.	31	**10.**	11	**19.**	201	**28.**	61
2.	12	**11.**	32	**20.**	132	**29.**	71
3.	11	**12.**	32	**21.**	321	**30.**	70
4.	43	**13.**	234	**22.**	411	**31.**	61
5.	12	**14.**	211	**23.**	312	**32.**	92
6.	22	**15.**	230	**24.**	110	**33.**	71
7.	11	**16.**	122	**25.**	63	**34.**	83
8.	14	**17.**	341	**26.**	51	**35.**	81
9.	13	**18.**	224	**27.**	82	**36.**	91

ANSWERS PAGE 48

1.	22 R14	**10.**	157 R9	**19.**	74 R22	**28.**	14 R9
2.	18 R24	**11.**	296 R12	**20.**	46 R21	**29.**	252 R18
3.	17 R6	**12.**	112 R40	**21.**	62 R8	**30.**	9 R41
4.	26 R5	**13.**	137 R2	**22.**	65 R7	**31.**	49 R28
5.	14 R6	**14.**	266 R12	**23.**	49 R64	**32.**	16 R16
6.	15 R42	**15.**	154 R6	**24.**	87 R34	**33.**	62 R9
7.	24 R16	**16.**	118 R65	**25.**	22 R36	**34.**	151 R8
8.	13 R52	**17.**	227 R16	**26.**	91 R34	**35.**	20 R18
9.	17 R24	**18.**	329 R28	**27.**	78	**36.**	178 R26

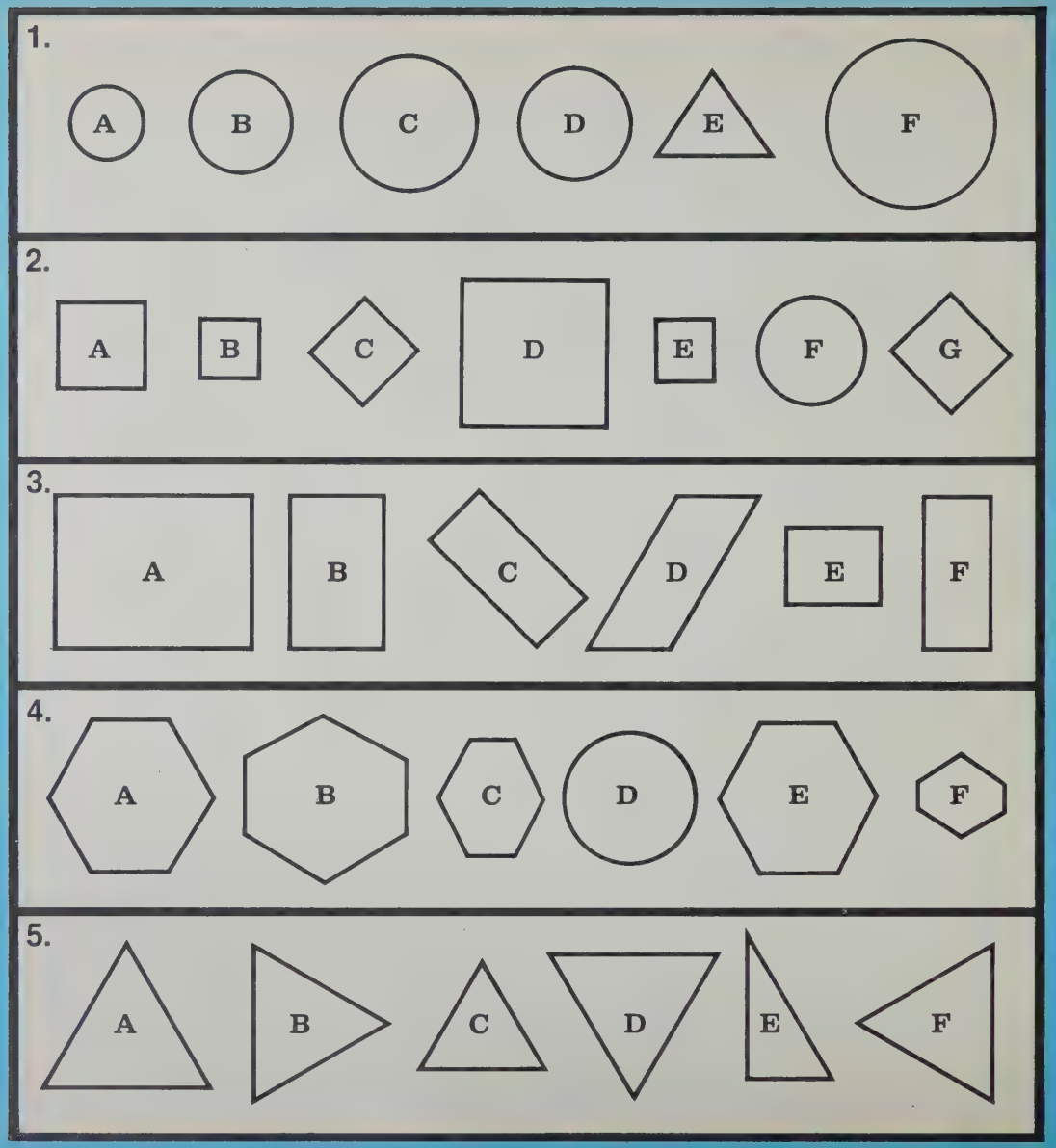
3
GEOMETRY
CONGRUENCE

Two containers.
Each holds the same amount.
Both have the same label on them.
Both come from the same shop.
And the shop buys them from
the same dairy. The dairy buys
the containers from the same factory.
The factory uses the same machine
to put them together and the same
machine to fold and cut them.
They are cut out of the same sheet
of cardboard.

Are the containers probably
THE SAME SIZE AND SHAPE?

What things do we depend upon being the same size and shape? Find out. That's your goal. Find out how, when, and why things are the same size and shape.

Look at each row.
Name the figure that
does not belong.
Why doesn't it belong?



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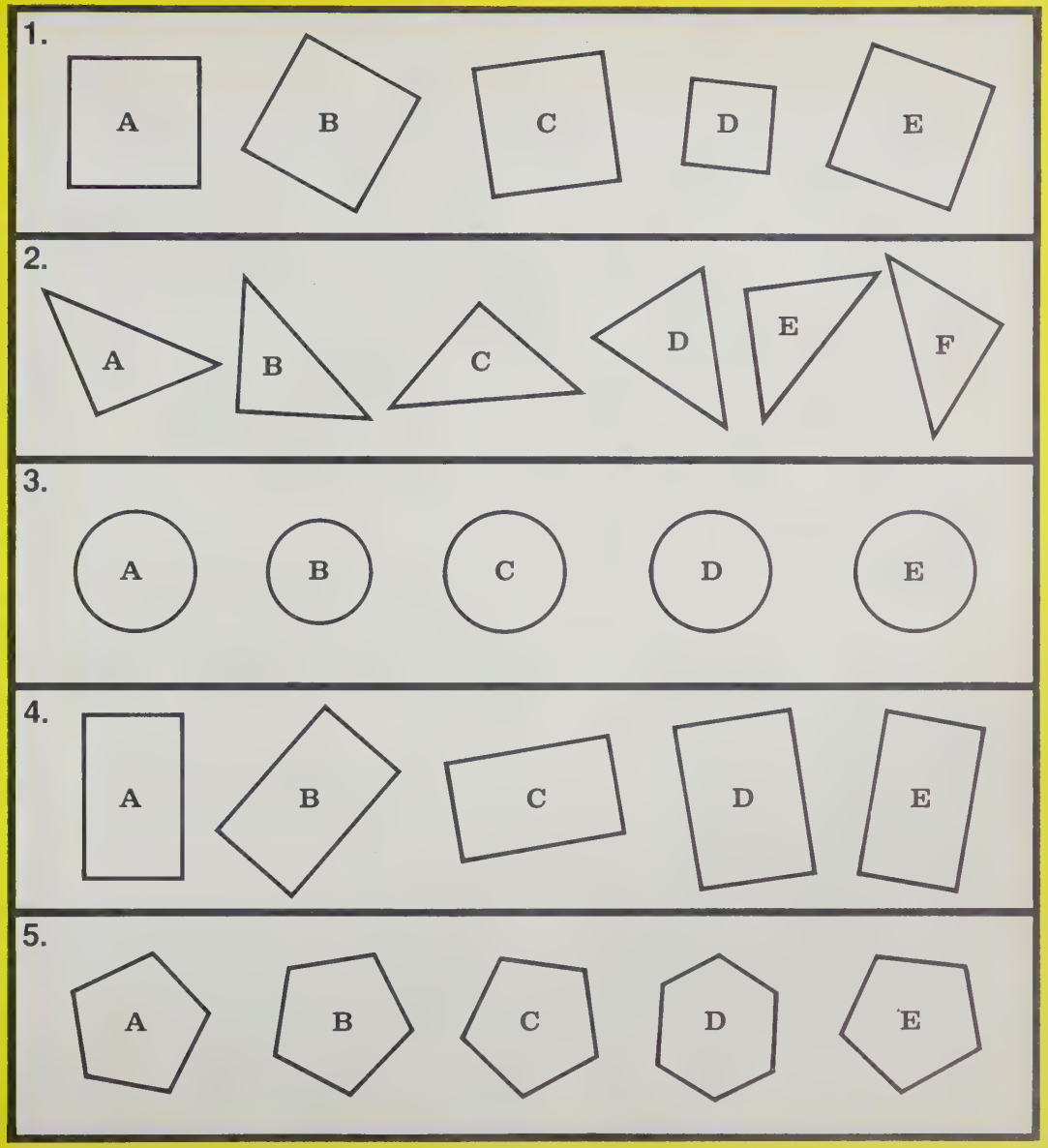

Look at each row.
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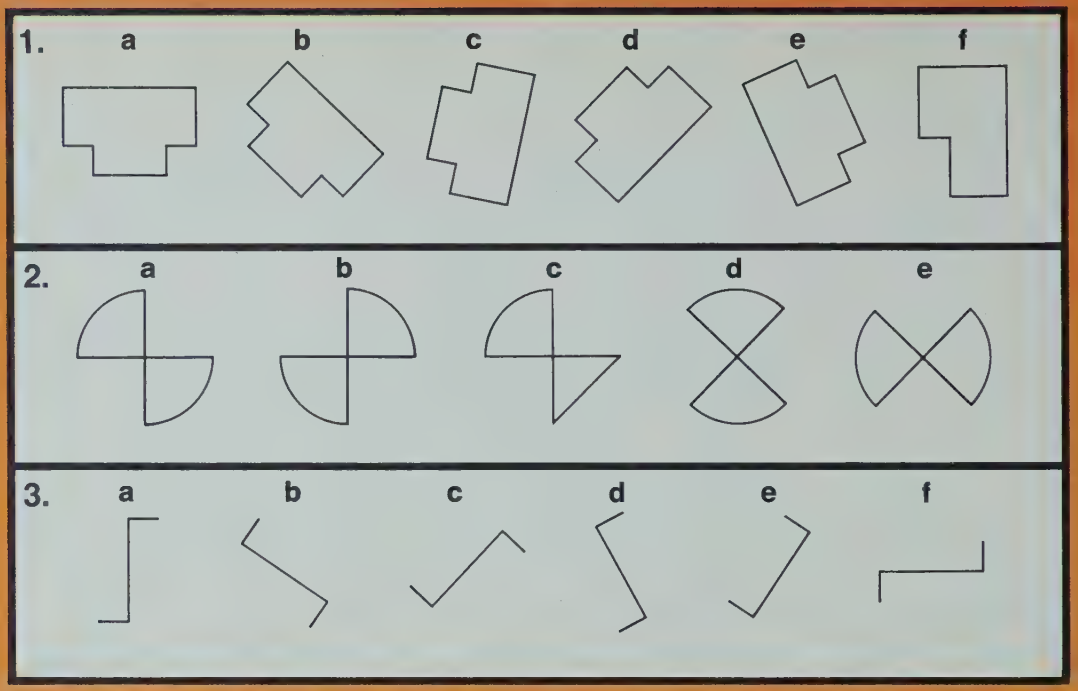
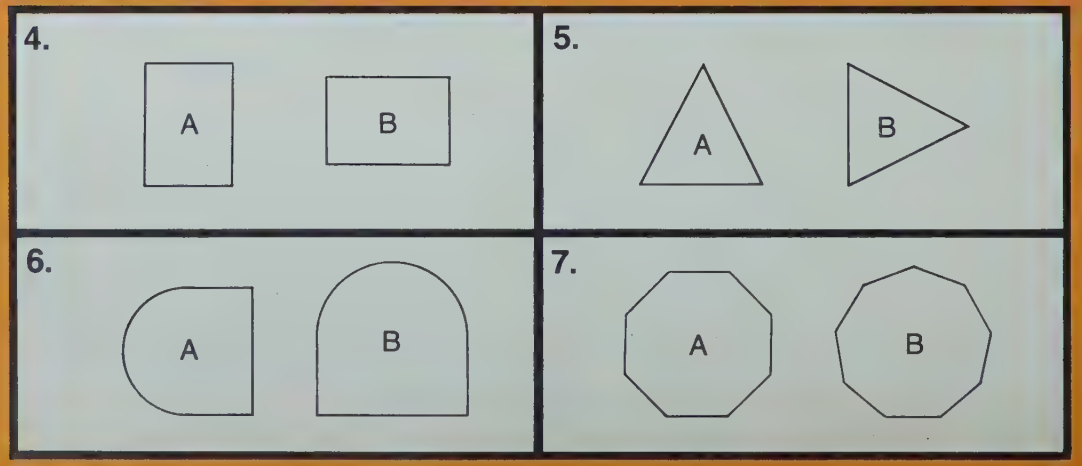


Look at each pair of figures. Is figure A the same size and shape as figure B?

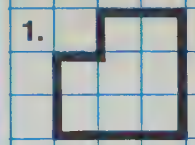


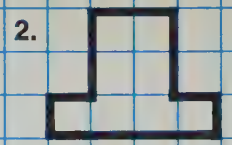
1.
Three of these rabbits are the same shape and the same size. Find them. Would it help to do some tracing?

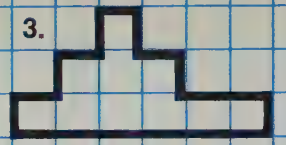
2.
Here is a pattern of a car. Trace it.
Find the other two pictures of a car that have been drawn from the same pattern.

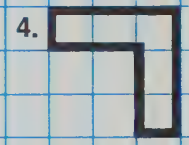
3.
If a picture is drawn from a pattern, does it have the same size and shape as the pattern?

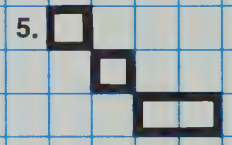
Get a sheet of graph paper. Make a copy of each of these figures.

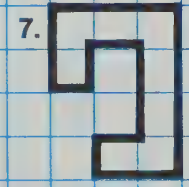


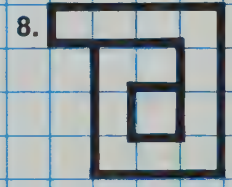


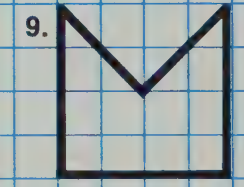




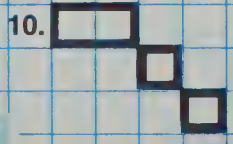


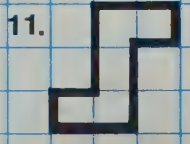


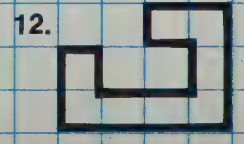




Are any of these figures the same size and shape as those you made?

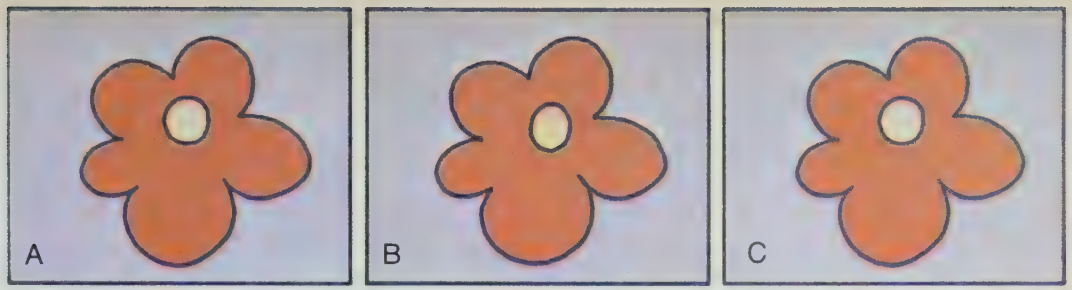




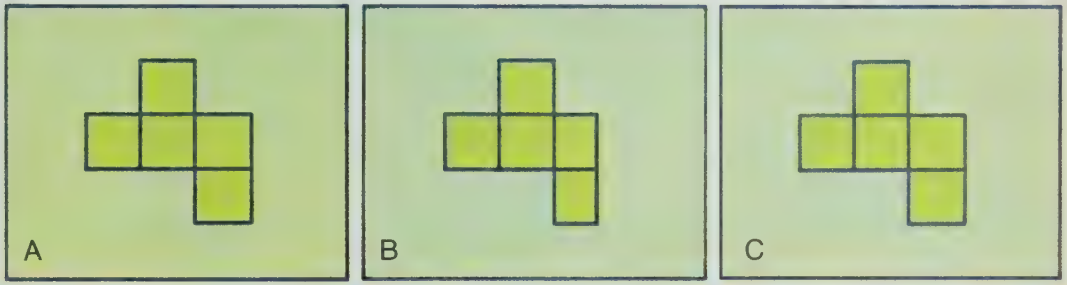


Trace to make sure.

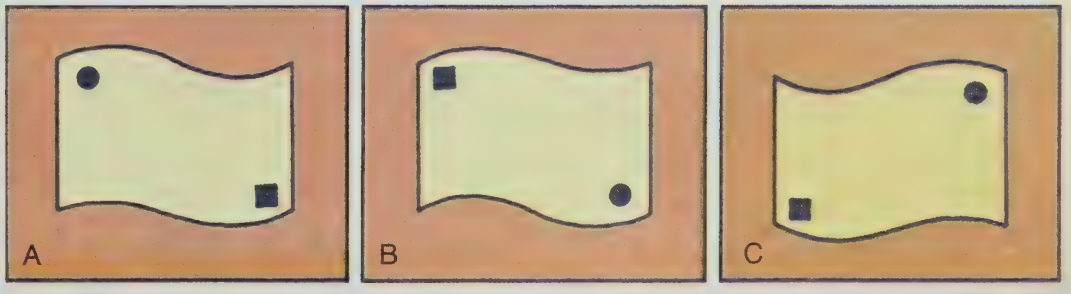
1.
Trace A. Which figure is the same as A?



2.
Trace A. Which figure is the same as A?

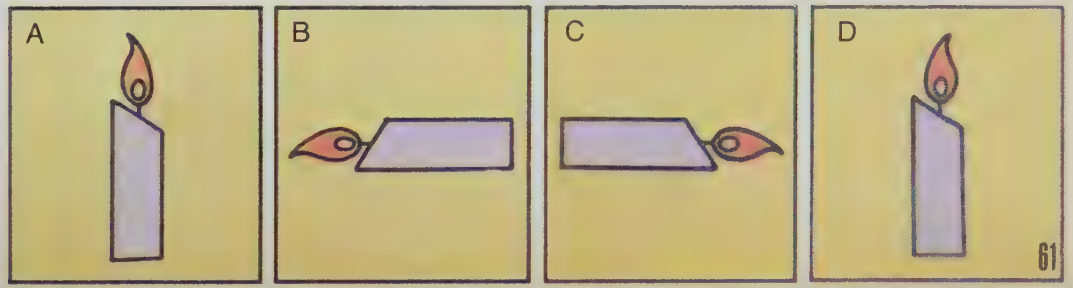


3.
Trace A. Which figure is the same as A?

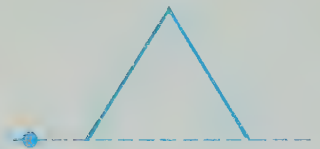


When a tracing of a figure matches another figure, you can say the two figures are congruent. That means they are the same size and shape even though they may be in a different position.

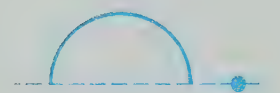
4.
Which of these figures are congruent?



Each of these drawings has a broken line on one side and a dot on that line. Use tracing paper to trace the drawing, the line, and the dot.

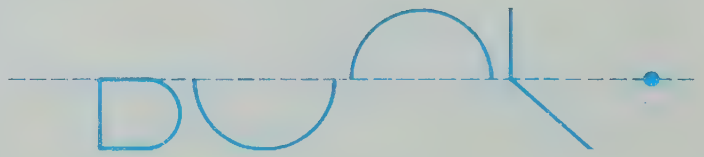


Now flip your tracing paper so that the line you drew matches the line in the drawing. Make the dot you drew also match the dot on the picture. The original picture looked like a triangle. What does the new image look like?



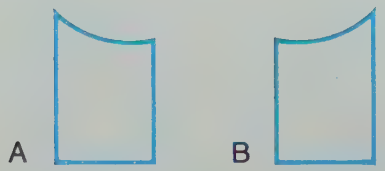
Try it again. Trace. Flip. Match the line. Match the dot. The original picture looked like a half-circle. What does the new image look like?

Look carefully at this one. Trace every mark shown in the figure. Then flip your tracing. Match the line and the dot.

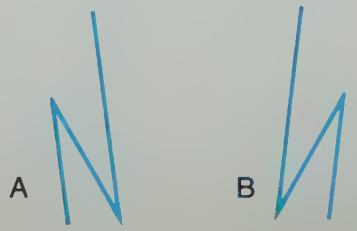


Now try these. Can B be a flip of A? Trace and make sure.

1.



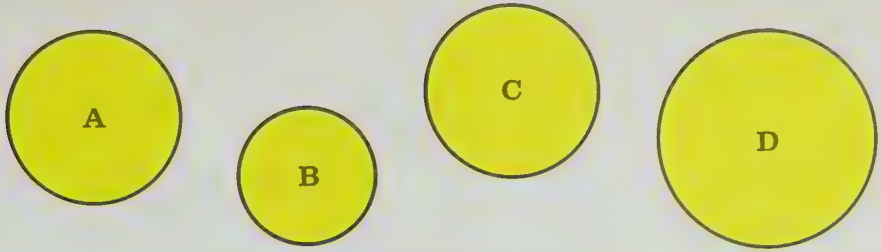
2.



Do you see a flip of you when you look in a mirror?

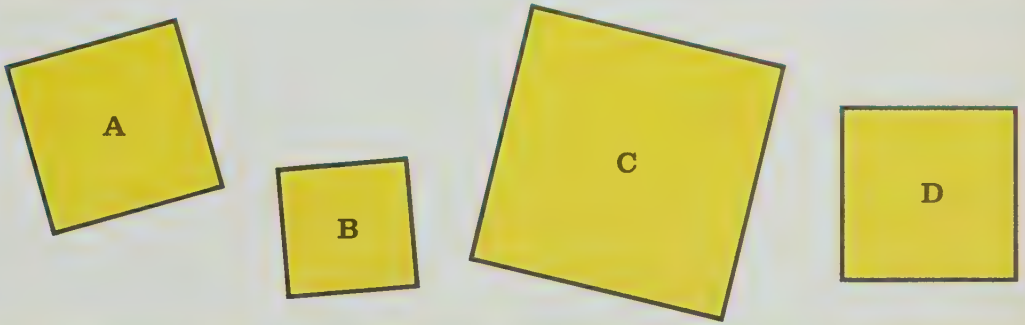
1

Are circles always the same shape? Are they always the same size? Which of these circles are congruent?



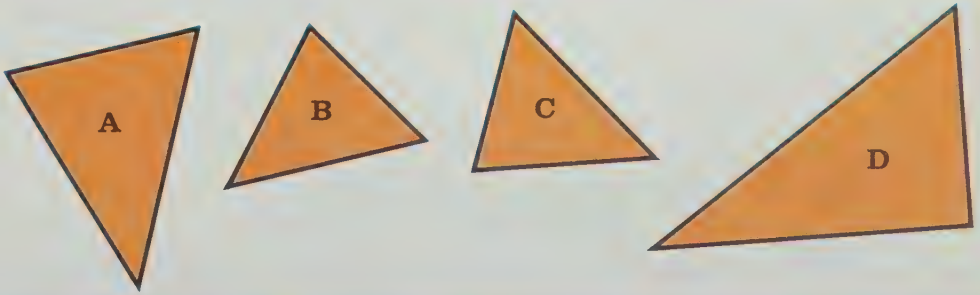
2

Are squares always the same shape? Are they always the same size? Which of these are congruent?



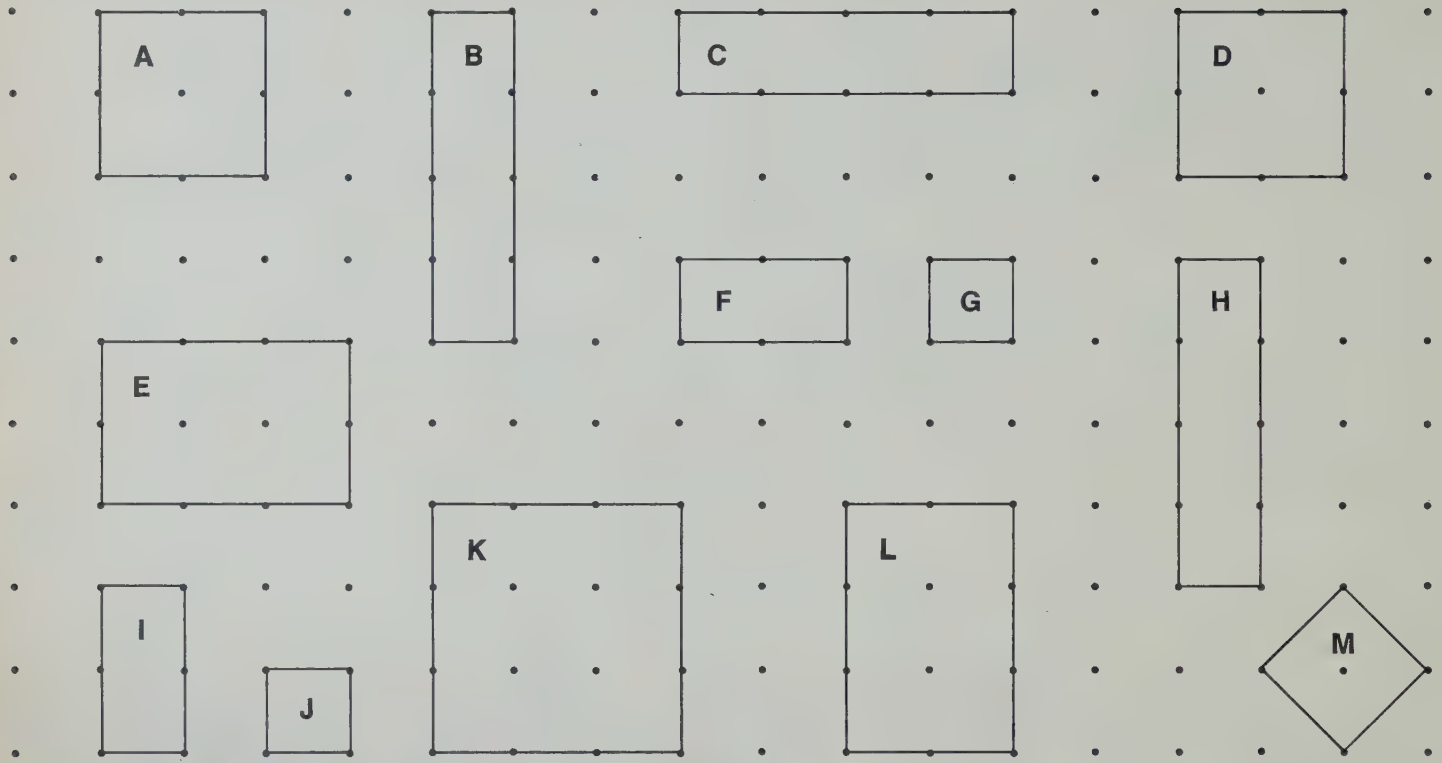
3

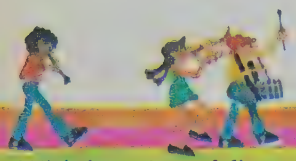
Are triangles always the same shape? Are they always the same size? Which of these triangles are congruent?



4 Does it make any difference in the looks of a circle or a square if either of the shapes is flipped?

Are rectangles always the same shape?
Are they always the same size?
Which of these rectangles are congruent?

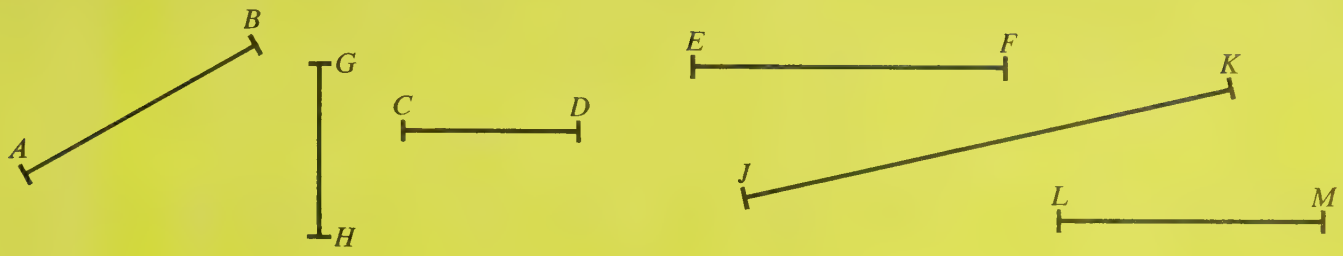




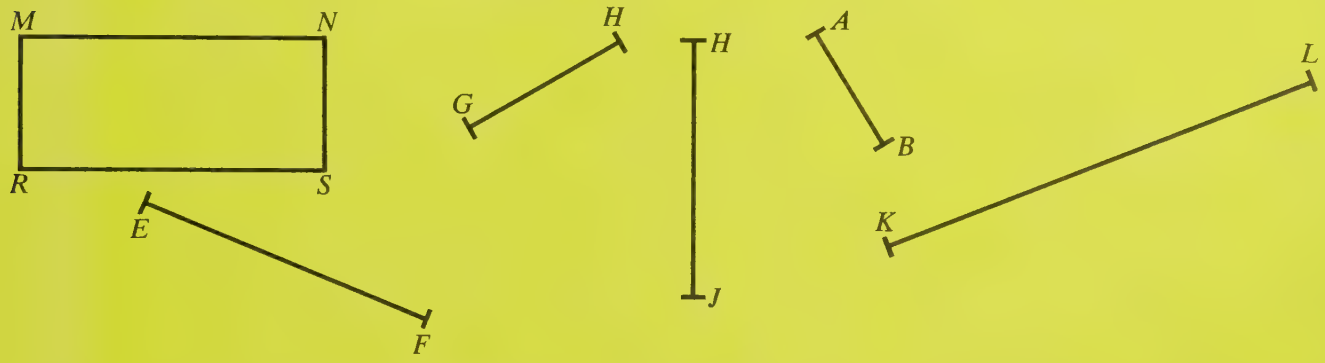
PROGRESS CHECK

Which pairs of figures are congruent?

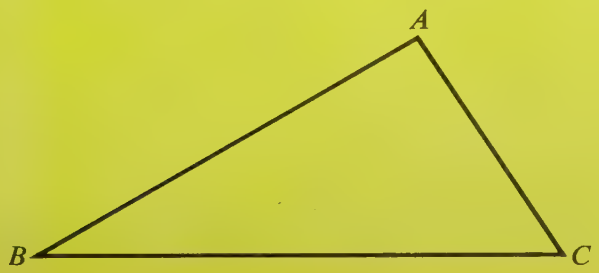
1. Which line segments are congruent?

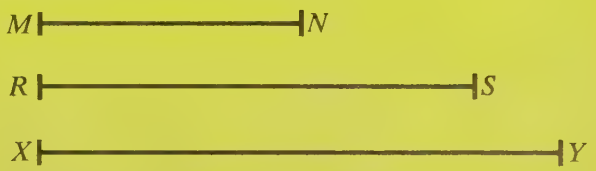


2. Which two sides of the rectangle are congruent?
Is there another line segment that is congruent to these sides?



3. Are any sides of the triangle congruent? What is the greatest number of congruent sides a triangle could have?





Are any of these line segments congruent to any side of the triangle?

Look at each row. Name the figure that does not belong. Why doesn't it belong? Be careful—these are tricky. Trace to make sure.

1.

a

b

c

d

2.

a

b

c

d

3.

a

b

c

d

4.

a

b

c

d

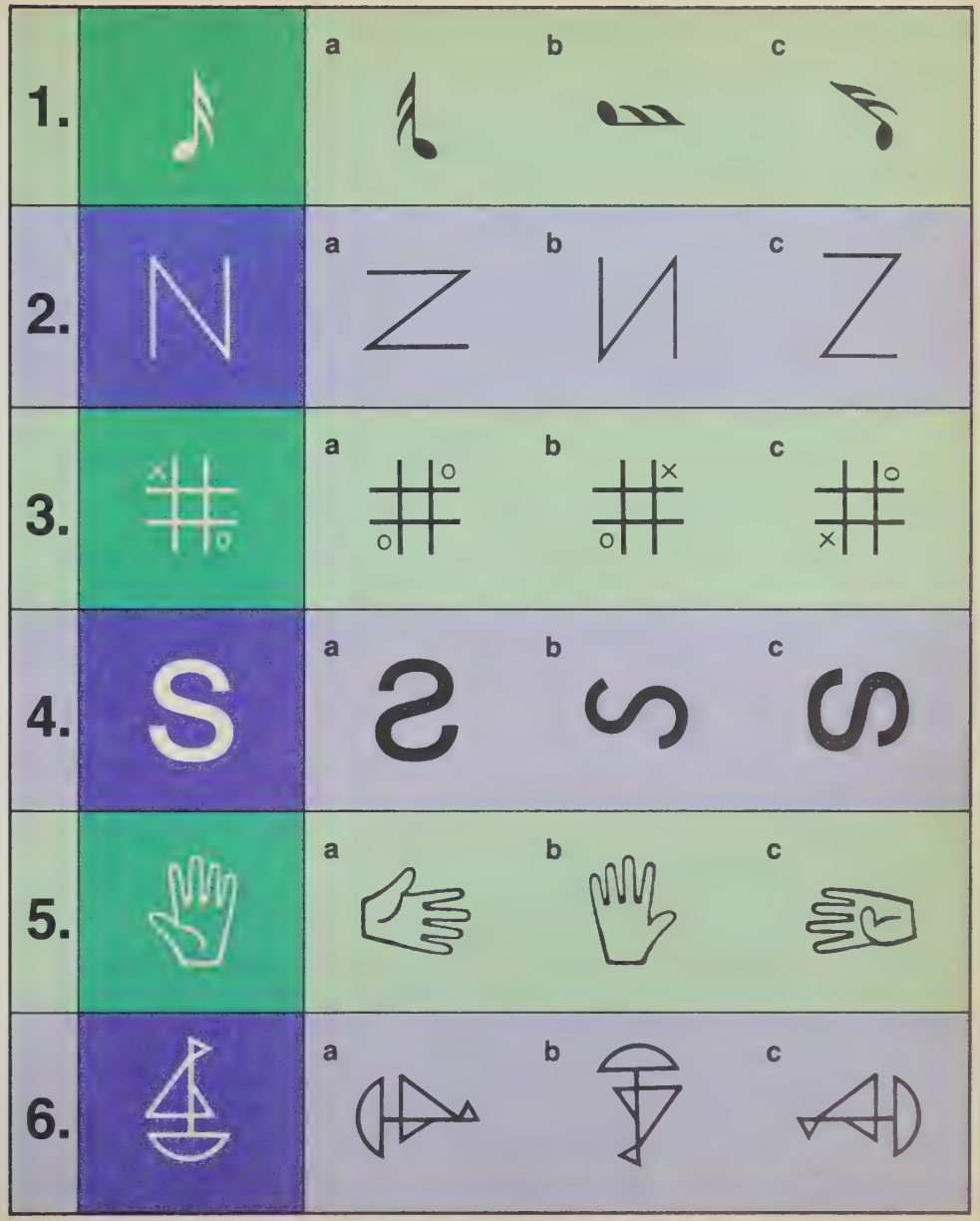
5.

Look at **4c**. Are you sure that it is the same as **a** and **b**? Does your tracing match exactly? What do you have to do to make it the same?

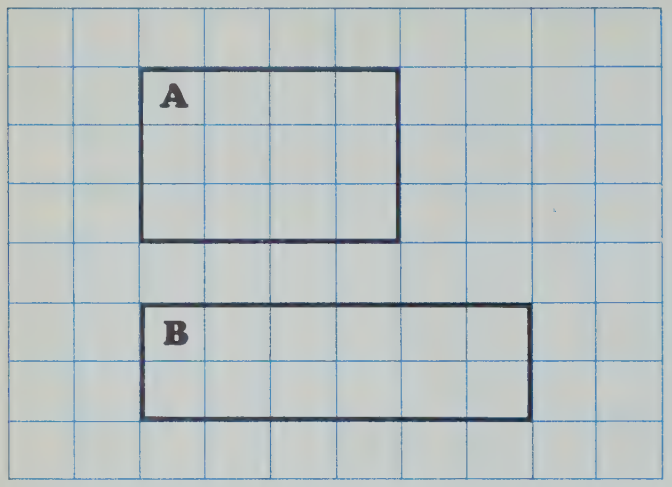
6.

Pretend you have on a new pair of shoes. Is the sole of the right shoe exactly the same size and shape as the sole of the left?

Look at the pattern. Then look at the tracing. One tracing was turned. One tracing was flipped. And one got messed up and just doesn't belong. What happened to each tracing?

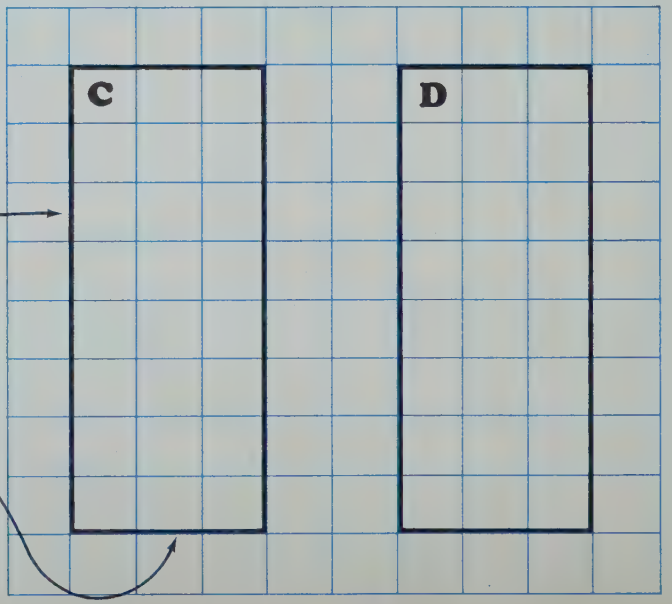


Area is the number of square units covered by a plane figure.



Time out to look at the area of a rectangle.

1. Does the distance around a rectangle tell you anything about the number of squares covered by the rectangle?
   Check rectangles A and B to make sure.
2. How many squares are covered by rectangle A?
   The area of A is ? square units.
3. What is the area of rectangle B?
4. Are rectangles A and B congruent?
   How do you know?



5. Are rectangles C and D congruent?
   How do you know?
6. Does rectangle C have the same area as rectangle D?
7. How many squares here?
   This is the *height* of rectangle C.

   How many squares here?
   This is the *base* of rectangle C.
8. Is there a way to find the number of squares covered other than by adding?

THINK Then answer.

1. If two rectangles cover the same number of squares, are they congruent?
2. Can a rectangle and a square cover the same number of squares?
3. If a rectangle and a square cover the same number of squares, does that mean that they are congruent?
4. Can a rectangle and a square be congruent?
5. Can any sides of a square be congruent? If so, which ones?
6. Can any sides of a rectangle be congruent? If so, which ones?
7. Can any sides of a triangle be congruent? If so, which ones?
8. Would any two congruent figures cover the same number of squares?

Congruent shapes are equal in all ways. They are exact copies of one another. They are the same size *and* the same shape. They must match exactly.

Can solids be congruent?

1

Think about cans of fruit juice that are the same brand. Are the cans the same size? Are they the same shape? Do they have to be?

2

Think about the bases of light bulbs for a lamp. Are they the same size? Are they the same shape? What would happen if they weren't the same size and shape?

3

Think about a pair of gloves. Are the two gloves the same size? Are they the same shape?

4

Think about a coin—a cent, for example. Are all new cents the same size? Are they the same shape? What would happen if they weren't the same size and shape?

5

Think about floor tiles. If you were going to put them down on a floor, would you depend on their being the same size and shape? What would happen if they weren't?

6

Think about film for a camera. Does it all need to be the same size and shape? Why?

7

Are machine-made items more likely to be congruent than handmade items? Why?

What would happen if—

1. Pages of a book were not congruent?

2. All coins were congruent?

3. The top of a shoe box were congruent to the bottom of the box?

4. Each plate in a set of dishes were a different size and shape?

5. The size and shape of every soap bar were different?

6. All doors were different in size and shape?

7. Every litre container were different in size and shape?

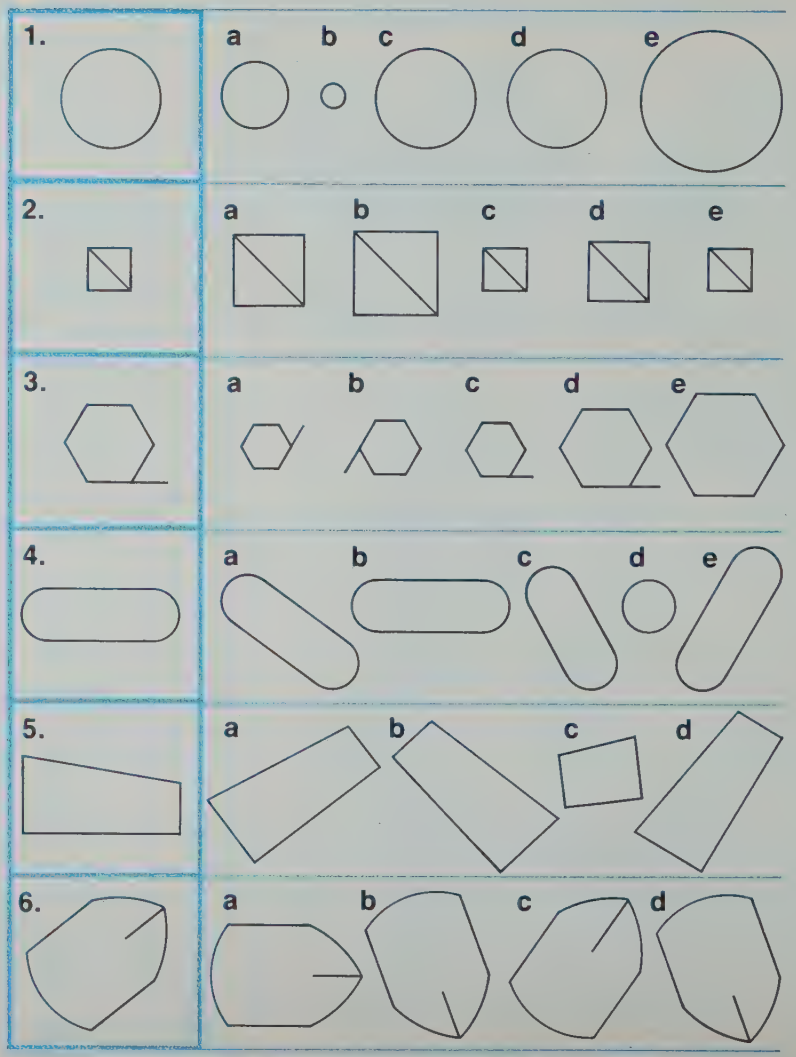
8. All theatre seats were different in size and shape?

9. The shape of the front wheels of a car were different from the shape of the back wheels?

10. All people were the same in size and shape?

CHECKOUT

In each row, which figures are congruent with the one in the box?



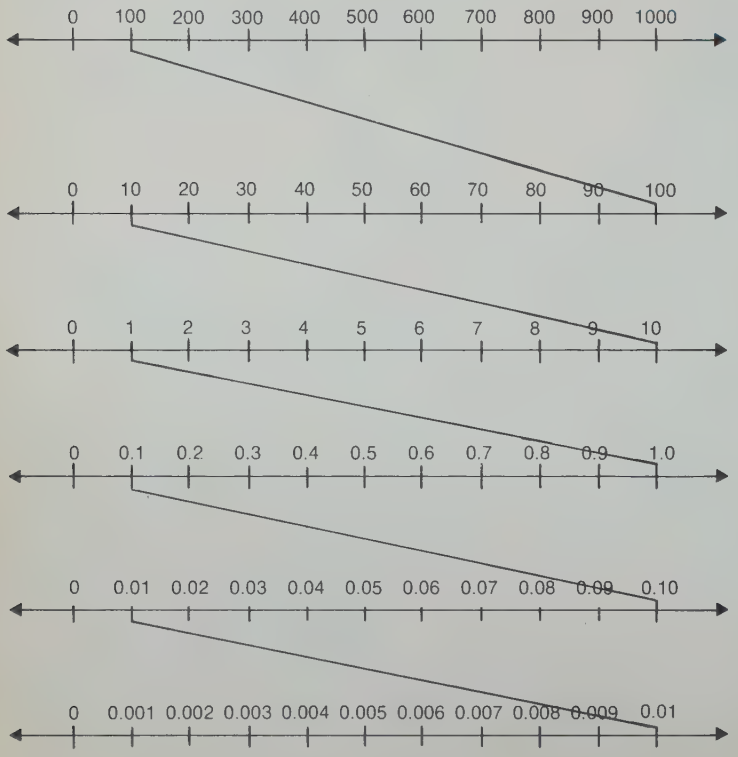
4 DECIMALS

If you moan when you see a fraction like $\frac{1}{10}$, you'll like these pages.

YOUR GOAL

is to learn about another way to deal with parts less than 1.
And you'll learn another way to write big numbers, too.

One of the ten parts shows one hundred.



Take part of the number line between 0 and 100. Enlarge it. One of the ten parts now shows ? .

Take a part between 0 and 10. Enlarge it. One of the ten parts now shows ? .

Take a part between 0 and 1. Enlarge it. One of the ten parts now shows ? .

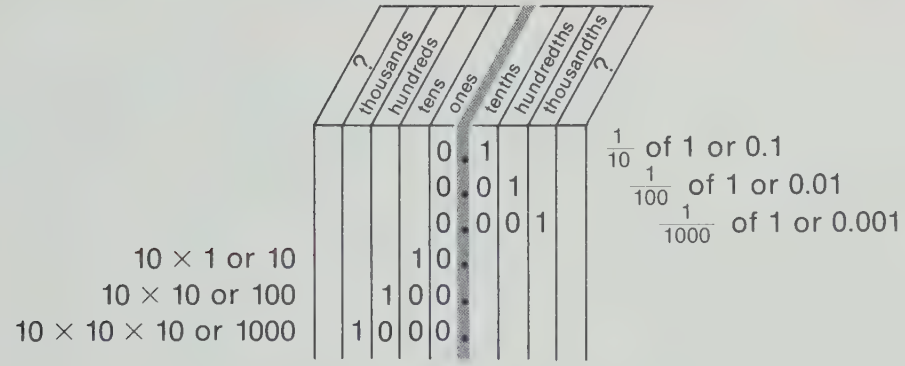
Take a part between 0 and 0.1. Enlarge it. One of the ten parts now shows ? .

Take a part between 0 and 0.01. Enlarge it. One of the ten parts now shows ? .

Could you enlarge the part between 0 and 0.001?

1 Our number system has only ten digits. The value of a digit depends on its place in relation to other digits and its relation to a decimal point.

← larger numbers ← • → smaller fractional parts →



2 List all the decimal fractions you can find below. On your list, put a ✓ by the smallest number. Put an X by the largest number.

Paper box and bag industries. These industries include manufacturers of folding cartons and set-up boxes, manufacturers of corrugated boxes and manufacturers of paper bags. Their total shipments in 1970 amounted, respectively, to $182.8 million, $281.8 million and $191.8 million, compared with $174.6 million, $274.1 million and $184.4 million, respectively, in 1969.

...rise from 5.3 to 5.4 rooms. Not all provinces followed this pattern, however: there was a decline in the average number of rooms per dwelling in all provinces east of Ontario. Throughout the nation as a whole, Prince Edward Island had the highest average in 1971 at 6.1 rooms per dwelling, and the Yukon Territory and Northwest Territories ...

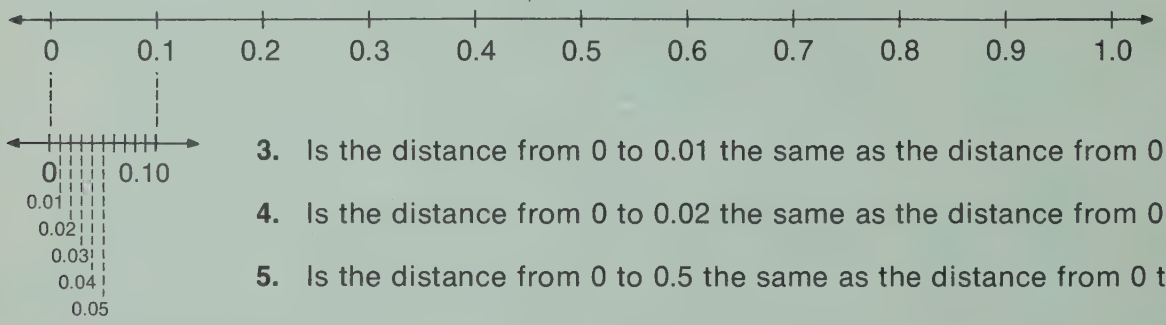
Total person nights in 1971 numbered 30.0 million, an increase over the previous year. The average stay decreased from 26 nights in 1970 to 25.1 in 1971, and the total of nights stayed has approximately doubled since 1968. The average expenditure in 197 was $341 compared with $352 a year earlier and does not include international transportation fares, which have been excluded for the calculations; the average expenditure has changed only marginally since 1968, when it was $350.

Industry	*1961-72*	*1970-72*
Agriculture	1.9	2.3
Forestry	2.8	-1.1
Fishing/trapping	0.3	-8.9
Mines	6.0	4.6
Manufacturing	6.0	5.9
Non-durables	4.9	4.8
Durables	7.3	7.0
Construction	4.9	5.1
Trade	5.5	6.8
Wholesale	6.4	6.4
Retail	4.9	7.1

THINK

Imagine one number line marked with tenths and another
marked with hundredths and another
marked with thousandths put on top of one another.

1. Would the distance from 0 to 0.5 be the same as the distance from 0 to 0.50?
2. Would the distance from 0 to 0.50 be the same as the distance from 0 to 0.500?



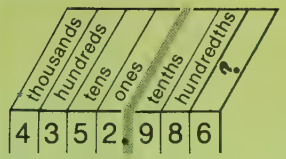
3. Is the distance from 0 to 0.01 the same as the distance from 0 to 0.1?
4. Is the distance from 0 to 0.02 the same as the distance from 0 to 0.2?
5. Is the distance from 0 to 0.5 the same as the distance from 0 to 0.05?

LOOK OUT!

6. Is the distance from 0 to 0.1 the same as the distance from 0 to 1.0?
7. Is the distance from 0 to 0.01 the same as the distance from 0 to 1.0?
*8. Copy each pair of numbers. Replace ? with $>$, $<$, or $=$ to make each sentence true.

a 0.1 ? 0.4 **b** 1.0 ? 0.9 **c** 0.10 ? 1.0 **d** 1.0 ? 1.1

e 0.03 ? 3.0 **f** 0.20 ? 0.02 **g** 0.1 ? 0.10 **h** 1.5 ? 1.50



A What is the place value of the 5?
The number it represents is written 50.

B What is the place value of the 9?
The number it represents can be written 0.9 or $\frac{9}{10}$.

C What is the place value of the 4?
The number it represents is written 4000.

D What is the place value of the 6?
6 thousandths can be written 0.006 or $\frac{6}{1000}$.

Express each of these fractions as a decimal.

	a	b	c	d	e
1.	$\frac{2}{10}$	$\frac{2}{100}$	$\frac{2}{1000}$	$\frac{20}{100}$	$\frac{20}{1000}$
2.	$\frac{5}{10}$	$\frac{5}{100}$	$\frac{5}{1000}$	$\frac{55}{100}$	$\frac{55}{1000}$

Express each of these decimals as a fraction.

	a	b	c	d	e
3.	0.651	0.908	0.550	0.938	0.025
4.	0.428	0.230	0.55	0.026	0.005
5.	0.337	0.066	0.055	0.410	0.600

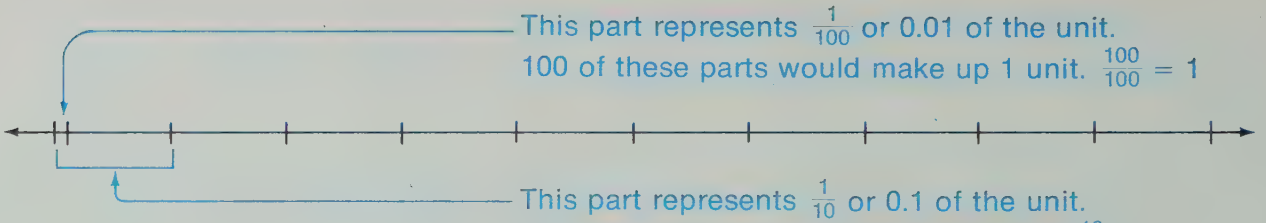
Tell how many tenths, hundredths, or thousandths.

	a	b
6.	0.4 = ? tenths	0.873 = ? thousandths
7.	0.429 = ? thousandths	0.56 = ? hundredths
8.	0.9 = ? tenths	0.385 = ? thousandths
9.	0.50 = ? hundredths	0.27 = ? hundredths

Tell what place value the last digit in each number represents.

	a	b	c	d	e
10.	0.399	0.21	0.3	0.95	0.8
11.	0.160	0.20	0.766	0.001	10.1

Suppose that the length of the whole segment drawn on this page represents 1 unit.


This part represents $\frac{1}{100}$ or 0.01 of the unit.
100 of these parts would make up 1 unit. $\frac{100}{100} = 1$
This part represents $\frac{1}{10}$ or 0.1 of the unit.
10 of these parts would make up 1 unit. $\frac{10}{10} = 1$

1 can be written as 1 or 1.0 or 1.00 or $\frac{1}{1}$ or $\frac{10}{10}$ or $\frac{100}{100}$.

10 means 1 ten ---------- 0.1 means 1 tenth
100 means 1 hundred ------- 0.01 means 1 hundredth

Write each of these fractions as a decimal.

	a	b	c	d	e
1.	$\frac{4}{10}$	$\frac{30}{100}$	$\frac{11}{100}$	$\frac{20}{100}$	$\frac{1}{10}$
2.	$\frac{53}{100}$	$\frac{8}{100}$	$\frac{7}{100}$	$\frac{70}{100}$	$\frac{84}{100}$
3.	$\frac{60}{100}$	$\frac{1}{100}$	$\frac{50}{100}$	$\frac{23}{100}$	$\frac{6}{100}$

Write each of these decimals as a fraction.

	a	b	c	d	e
4.	0.4	0.28	0.60	0.40	0.5
5.	0.59	0.08	0.8	0.2	0.31
6.	0.09	0.44	0.9	0.84	0.90
7.	0.04	0.1	0.01	0.10	0.11

You can *explain* decimals to someone by reading them this way:

a 0.8 is "8 tenths"

b 0.65 is "65 hundredths"

c 0.437 is "437 thousandths"

1. How do you know that **a** is tenths, **b** is hundredths, and **c** is thousandths?

When you *read* decimals aloud, you name each digit one at a time.

14.2 is "fourteen point two or fourteen decimal two".

8.53 is "eight point five three or eight decimal five three".

11.664 is "eleven point six six four or eleven decimal six six four".

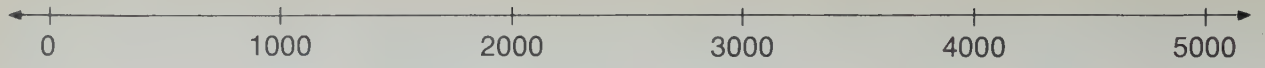
It is not correct to read a decimal fraction like a whole number. 11.664 should not be read as "eleven point six hundred sixty-four".

2. Read each of these decimals aloud.

a 0.68	**b** 0.9	**c** 0.47	**d** 0.297	**e** 0.2
f 0.707	**g** 0.5	**h** 0.099	**i** 0.38	**j** 0.164

3. Read each of these numbers aloud.

a 6.45	**b** 10.6	**c** 9.304	**d** 2.497
e 10.051	**f** 6.33	**g** 22.3	**h** 4.002



1. The numbers go on and on.

a Name a number between 0 and 100.

b Name a number between 0 and 10.

c Name a number between 0 and 1.

d Name a number between 0 and 0.1.

e Name a number between 0 and 0.01.

And more and more numbers can go between any two numbers.

2. A fraction can be written as a decimal.

Which fraction is the largest — 0.55, $\frac{99}{100}$, $\frac{9}{10}$, or 0.95?

3. Canadian and Australian coins represent fractional parts of one dollar.

a 10 cents is what fractional part of one dollar?

b One cent is what fractional part of one dollar?

c How many cents in ten dollars?
in twenty dollars?
in thirty dollars?
in fifty dollars?

4. There are all sorts of ways to write a number.

a Which of these fractions does not name the number 1?

$\frac{10}{10}$ 1.0 $\frac{1.0}{10}$ $\frac{1}{1}$ 1.00 $\frac{100}{100}$ $\frac{1000}{1000}$

b Write two more names for 1.

Gene boasted to Alec, Gill, and Pamela, "I can throw a ball so far you could measure the distance in kilometres." Of course he had to prove it. When he threw the ball, it went a full 67 m . Everybody laughed. But Gene said, "See! That's zero point zero six seven kilometres!"
Was he right?

Remember: 1000 m = 1 km
1 m = 0.001 km

1. Write each of these metre distances as decimal fractions of a kilometre.

   a 3 m **b** 44 m **c** 102 m **d** 999 m **e** 500 m

2. Write each of these decimal fractions of a kilometre as metres.

   a 0.750 km **b** 0.112 km **c** 0.050 km **d** 0.425 km **e** 0.550 km

3. Look at these distances:

   0.475 km 0.089 km 0.809 km 0.800 km 0.500 km

   a Which distance is longest?

   b Which is shortest?

   c Order the distances from longest to shortest.

   d Rewrite the distances as metres.

   e Order the distances in metres from longest to shortest.

   f Compare the answers you got in **e** to the answers you got in **c**.

1. Write $>$, $<$, or $=$ for each $\underline{?}$.

a	$0.435 \underline{?} 0.433$	**b**	$0.025 \underline{?} 0.032$	**c**	$0.63 \underline{?} 0.629$
d	$0.077 \underline{?} 0.66$	**e**	$0.5 \underline{?} 0.501$	**f**	$0.71 \underline{?} 0.74$
g	$0.3 \underline{?} 0.077$	**h**	$0.08 \underline{?} 0.009$	**i**	$0.05 \underline{?} 0.4$
j	$0.34 \underline{?} 0.340$	**k**	$0.3 \underline{?} 0.009$	**l**	$0.98 \underline{?} 0.981$

2. Write each set of decimals in order from least to greatest.

a	0.62	0.619	0.089	0.09	0.6	0.621
b	0.3	0.92	0.093	0.34	0.339	0.4
c	2.3	0.24	0.241	2.31	23.1	2.031

3. Use a dollar sign and a decimal point.
(Add the zero, if required, before the decimal point.)
Write 1 cent. Write 10 cents. Write 100 cents.
Write 1 dollar. Write 10 dollars. Write 100 dollars.

4. Think money.
Does $0.01 = 0.10 = 1.00$? Then does $\frac{1}{100} = \frac{10}{100} = \frac{100}{100}$?
Does $0.1 = 0.10 = 0.100$? Then does $\frac{1}{10} = \frac{10}{100} = \frac{100}{1000}$?

5. Think money again.
Does $0.05 = 0.50 = 5.00$? Then does $\frac{5}{100} = \frac{50}{100} = \frac{500}{100}$?
Does $0.5 = 0.50 = 0.500$? Then does $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$?

PROGRESS CHECK

Complete.

1. 0.516 = 5 tenths 1 hundredth 6 thousandths
2. 0.492 = ? tenths ? hundredths ? thousandths
3. 0.704 = ? tenths ? hundredths ? thousandths
4. 3.295 = ? ones ? tenths ? hundredths ? thousandths
5. 0.349 = ?
6. 0.650 = ?
7. 14.227 = ?
8. 5.008 = ?

Answer the questions.

9. Name two decimals between 0.210 and 0.220.
10. Name two decimals between 0.480 and 0.490.
11. Name two decimals between 0.350 and 0.360.
12. Name two decimals between 0.35 and 0.36.
13. Name two decimals between 0.09 and 0.12.
14. Name two decimals between 0.036 and 0.046.

Write each as a decimal.

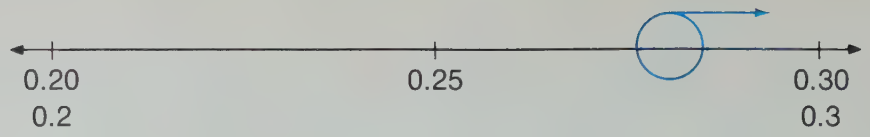
15. $\frac{3}{10}$
16. $\frac{20}{100}$
17. $\frac{13}{100}$
18. $\frac{13}{1000}$
19. $\frac{3}{100}$
20. $\frac{100}{1000}$
21. $\frac{10}{1000}$
22. $\frac{1}{1000}$

Write each as a fraction.

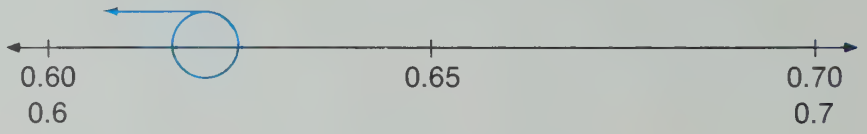
23. 0.13
24. 0.6
25. 0.07
26. 0.149
27. 0.40
28. 0.040
29. 0.065
30. 0.106

1. More about rounding.

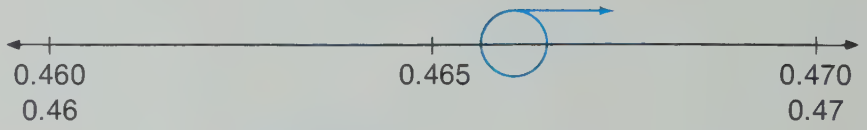
a Round 0.28 to the nearest *tenth*.
0.28 is between 0.20 and 0.30.
0.28 is closer to 0.30.
0.30 = 0.3, so round 0.28 to 0.3.



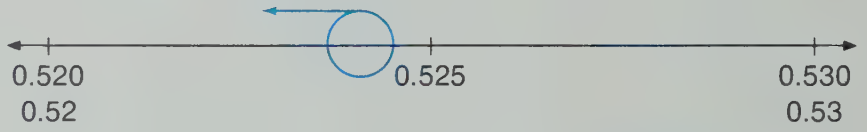
b Round 0.62 to the nearest *tenth*.
0.62 rounds to ?.



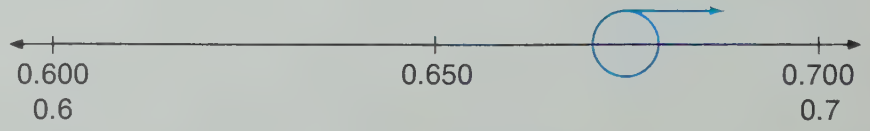
c Round 0.466 to the nearest *hundredth*. 0.466 rounds to ? .



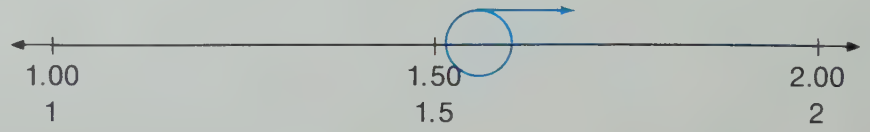
d Round 0.524 to the nearest *hundredth*. 0.524 rounds to ? .



e Round 0.674 to the nearest *tenth*.
0.674 rounds to ? .



f Round 1.56 to the nearest *whole number*. 1.56 rounds to ? .



2. Round these to tenths.

a	0.75	**b**	0.56
c	0.44	**d**	0.567
e	0.405	**f**	0.095

3. Round these to hundredths.

a	0.463	**b**	0.628
c	0.456	**d**	0.609
e	0.249	**f**	0.048

4. Round these to a whole number.

a	2.7	**b**	6.45
c	10.56	**d**	4.09
e	5.5	**f**	3.49

Here's another way to round.

Round 0.76 to the nearest *tenth*. You want one decimal place.
0.76 — look at this digit. Five or more? Round to 0.8.

Round 0.845 to the nearest *tenth*. You want one decimal place.
0.845 — look at these digits. Fifty or more? Round to 0.8.

Round 1.56 to the nearest *tenth*. You want one decimal place.
1.56 – look at this digit. Five or more? Round to 1.6.

Here are more examples.

Round 0.578 to the nearest *hundredth*. You want two decimal places.
0.578 — look at this digit. Five or more? Round to 0.58.

Round 1.056 to the nearest *tenth*. You want one decimal place.
1.056 – look at these digits. Fifty or more? Round to 1.1.

What will happen now?

Round 1.056 to the nearest whole number or unit. You don't want any decimal place.
1.056 – look at these digits. Five hundred or more? Round to 1.

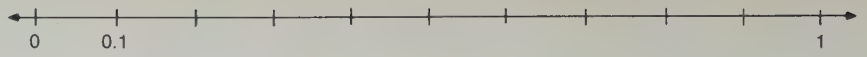
You try these. Think about them in any way you want. BUT round each of these numbers.

1. Round to the nearest tenth.
a 0.34 **b** 0.278 **c** 1.346 **d** 2.45

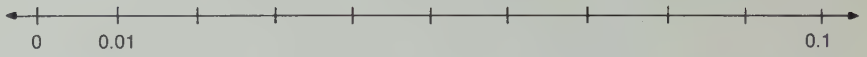
2. Round to the nearest hundredth.
a 0.868 **b** 0.323 **c** 3.455 **d** 1.184

3. Round to the nearest whole number.
a 2.7 **b** 1.642 **c** 3.584 **d** 1.032

1. **a** Is 0.6 closer to 0 or 1?
b Is 0.3 closer to 0 or 1?
c Is 0.5 closer to 0 or 1?
Agree that you will round 0.5 up to the nearest unit.



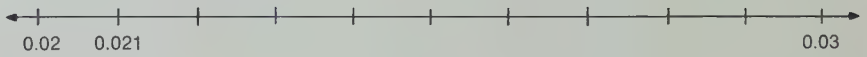
2. **a** Is 0.04 closer to 0 or 0.1?
b Is 0.09 closer to 0 or 0.1?
c Is 0.05 closer to 0 or 0.1?



3. **a** Is 0.007 closer to 0 or 0.01?
b Is 0.001 closer to 0 or 0.01?
c Is 0.005 closer to 0 or 0.01?

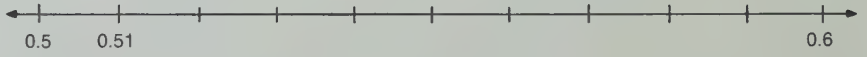
WATCH OUT!

4. **a** Is 0.026 closer to 0.02 or 0.03?
b Is 0.023 closer to 0.02 or 0.03?
c Is 0.025 closer to 0.02 or 0.03?



KEEP YOUR EYES OPEN.

5. **a** Is 0.59 closer to 0.5 or 0.6?
b Is 0.50 closer to 0.5 or 0.6?
c Is 0.55 closer to 0.5 or 0.6?



4.876 rounded to the nearest hundredth is 4.88.
4.876 rounded to the nearest tenth is 4.9.
4.876 rounded to the nearest whole number is 5.

6. Round each of these numbers to the nearest hundredth, then to the nearest tenth, and finally to the nearest whole number.

a 14.651 **b** 20.268 **c** 14.449 **d** 9.945 **e** 9.894 **f** 8.493 **g** 6.065 **h** 99.445

PROGRESS CHECK

Round to the nearest tenth.

(1.) 0.32 (2.) 0.48 (3.) 0.55

Round to the nearest hundredth.

(4.) 0.446 (5.) 0.359 (6.) 0.273

Round to the nearest whole number.

(7.) 5.56 (8.) 3.09 (9.) 11.603

Which is the smaller number?

(10.) 0.36 or 0.43 (11.) 0.127 or 0.163 (12.) 0.447 or 0.53

(13.) 0.23 or 0.139 (14.) 0.462 or 0.07 (15.) 0.287 or 0.37

Write each set of decimals from smallest to largest.

(16.) 1.0 0.1 0.01 1.1 0.001

(17.) 4.6 46.0 0.46 460.0 0.046

(18.) 3.7 0.96 1.74 0.99 0.704

(19.) 0.201 0.02 0.021 0.210 0.200

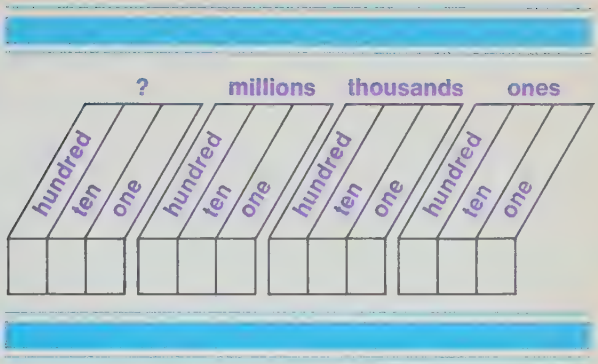
Round to the nearest whole number and then find the sum.

(20.) $8.2 + 4.8$ (21.) $5.15 + 3.25$ (22.) $10.6 + 4.35$

All the planets move around the sun in their own paths. The paths are not circular. These noncircular paths are usually called elliptical orbits. The orbits are curved, but they are longer than they are wide. The distance of a planet from the sun is different from time to time. The range of distances is given below.

Distances from the Sun in Kilometres

Planet	Maximum distance	Minimum distance
Mercury	69 802 000	45 991 000
Venus	108 920 000	107 450 000
Earth	152 068 000	147 060 000
Mars	249 164 000	207 416 000
Jupiter	815 803 000	740 503 000
Saturn	1 506 268 000	1 347 087 000
Uranus	3 005 550 000	2 735 068 000
Neptune	4 536 014 000	4 459 056 000
Pluto	7 406 000 000	4 443 600 000



1. Mercury's maximum distance from the sun is read "sixty-nine million, eight hundred two thousand kilometres." How would you read the maximum distance of Venus? of Earth? Mars? Jupiter?

2. The maximum distance of Saturn is read "one *billion*, five hundred six million, two hundred sixty-eight thousand kilometres."

a How would you read the maximum distance of Uranus? of Neptune? of Pluto?

b How would you read each of the minimum distances?

3. How many billions are there in 63 521 740 609?
How many millions? How many thousands? How many ones?

4. How many billions are there in 935 004 078 333?
How many millions? How many thousands? How many ones?

5. How many billions are there in 5 672 000 984?
How many millions? How many thousands? How many ones?

6. Find the missing numbers.

a 43 764 271 909 = ? billions ? millions ? thousands ? ones

b 833 000 891 754 = ? billions ? millions ? thousands ? ones

c 1 934 244 700 = ? billions ? millions ? thousands ? ones

Why do all the distances to faraway places such as the planets usually end in three or more zeros?
Tell to which place each distance has been rounded on page 88.
Can you always tell? Why?

1 Write a numeral for each.

a one hundred million

b six hundred billion

c one billion

d fifty-eight billion

e ten million

f nine billion, two hundred thousand

g one hundred thousand

h thirty billion, five hundred thousand

i ten billion

j ten billion, four hundred thousand

2 Do some research.

a How many people live in your town?

b What is the population of the largest city in your state or province?

c What is the population of the largest city in the world?

d What is the population of your country?

e What is the population of the largest country in the world?

f What does it mean to have someone or something insured?

g What amount of life insurance could a person have?

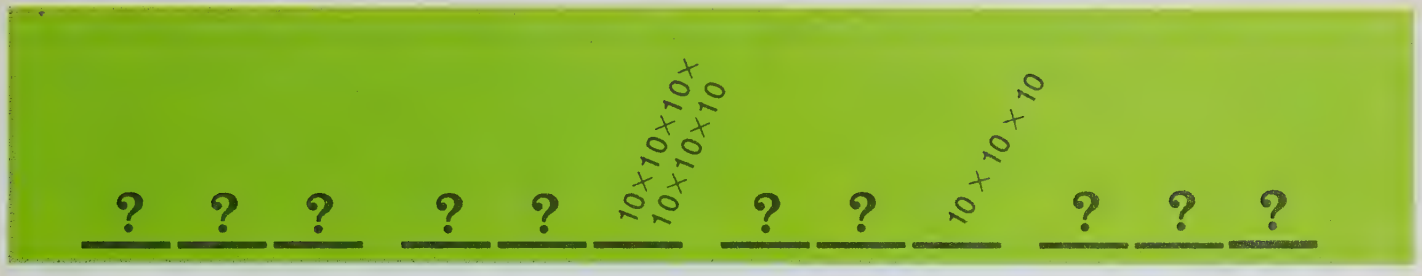
h How much could you save in a bank and have the amount insured?

3 What is the difference between a millionaire and a billionaire? How much difference might there be?

$1000 = 10 \times 10 \times 10$

$1\,000\,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

A What other numbers can be written as a product of tens?



A number is a *power* of 10 if it can be written as a product with *only* 10s as factors.

B Which of the following are powers of 10?

2100	49	100 000 000 000
100	100 000	36
101 000	50 000 000	10 000 000
1 000 000 000	10 000	10 000 000 000
37 000 000 000	100 000 000	11 000
25	100 100	125

C Write all the powers of 10 as products. Use only 10s as factors.

Can you find any powers of 5 in the list above?
any powers of 6? any other powers?

Find the products.

1. 10×10 **2.** $10 \times 10 \times 10$ **3.** $10 \times 10 \times 10 \times 10$

Write the numbers as products of 10.

4. 100 **5.** 1000 **6.** 100 000

$10 \times 10 \times 10 \times 10 \times 10$ is another way of writing 100 000

It shows that 100 000 can be written as a product using 10 as a factor 5 times. Can you think up a shorter way of writing $10 \times 10 \times 10 \times 10 \times 10$, or 100 000?

EXPLAIN YOUR WAY

Products of 10 can be written in shortened form by using *exponents*. For example:

$$10 \times 10 = 10^2$$

The small, raised 2 is the exponent. It tells how many times 10 is used as a factor in the product.

Consider these exponents

$$10^4 = 10 \times 10 \times 10 \times 10$$
$$10^3 = 10 \times 10 \times 10$$
$$10^2 = 10 \times 10$$
$$10^1 = \underline{?}$$

What does the exponent 1 mean?

We can think of 10 as a power of 10, even though 10 is a factor only once.

10^1 is the first power of 10,
10^2 is the second power of 10,
10^3 is the third power of 10, and so on.

10^1 is read "ten to the first power."
10^2 is read "ten to the second power," or sometimes "ten squared."
10^3 is read "ten to the third power," or sometimes "ten cubed."
10^4 is read "ten to the fourth power."

How would you read 10^5? 10^6?

10 000 is a standard numeral.
10 000 can be written in factor form, $10 \times 10 \times 10 \times 10$.
And $10 \times 10 \times 10 \times 10$ can be written in exponent form, 10^4.
Similarly, 10^4 can be written in factor form or as a standard numeral.

1. Write each standard numeral in factor form.

a 1000 **b** 1 000 000 **c** 100 000 000

d 10 000 000 000 **e** 100 000 **f** 1 000 000 000

2. Write each factor in exponent form.

a $10 \times 10 \times 10 \times 10 \times 10$ **b** $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

c 10×10 **d** $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

e $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

3. Write each standard numeral in exponent form.

a 1000 **b** 10 000 000 **c** 100 000 000

d 10 000 000 000 **e** 100 000 **f** 100 000 000 000

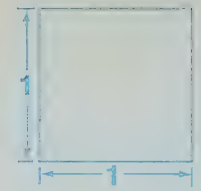
Can you find a shortcut?

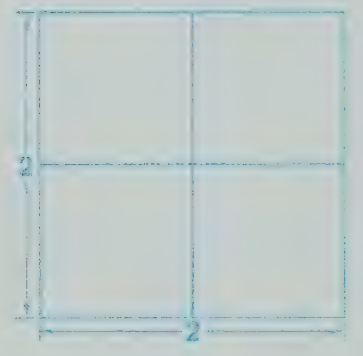
4. Write each numeral in factor form.

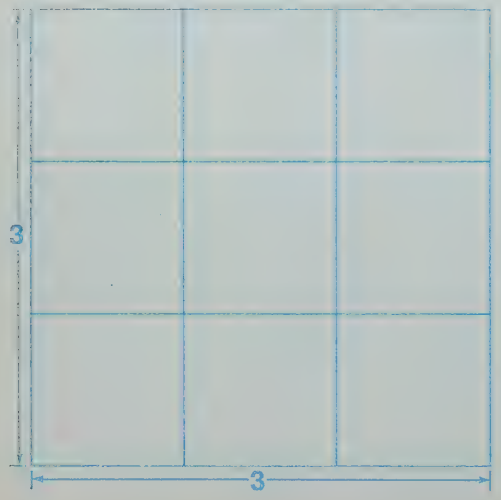
a 10^3 **b** 10^6 **c** 10^5 **d** 10^{11} **e** 10^9 **f** 10^{10}

5. Write each numeral in standard-numeral form.

a 10^3 **b** 10^6 **c** 10^7 **d** 10^8 **e** 10^5 **f** 10^{11}







The Bigshot Construction Company puts up buildings with square floor plans. There are always exactly as many rooms going east-west as there are going north-south. Some typical floor plans are here.

Number of rooms expressed as a product	Number of rooms expressed with an exponent	Number of rooms expressed with a standard numeral
1×1	1^2	1
2×2	2^2	4
3×3	3^2	9
?	4^2	?
6×6	?	?

1. Draw the floor plan for a building with 7 rooms on a side. Express the number of rooms as a product, with an exponent, and with a standard numeral.

2. The following numbers are called *square numbers*. Can you think of a reason for this name? Write the standard numeral for each of these square numbers.

a 9^2 **b** 15^2 **c** 10^2

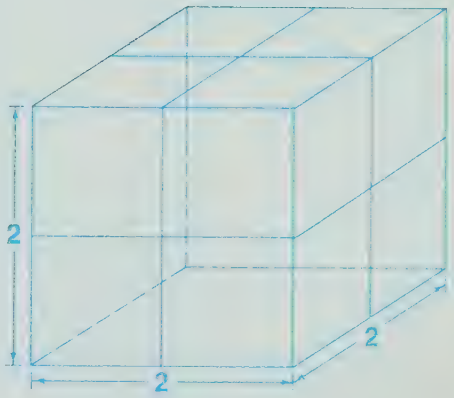
d 13^2 **e** 17^2 **f** 25^2

3. Which of these numbers are square numbers?

64 56 121 45 110 36

The buildings the Bigshot Construction Company puts up always have as many floors as there are rooms along one side of a floor. The buildings are cubes. There are exactly as many rooms going up-down as there are going east-west or north-south.

	Number of rooms on each floor	Number of floors	Total number of rooms expressed		
			as a product	with an exponent	in standard notation
1.	1×1	1	$1 \times 1 \times 1$	1^3	1
2.	2×2	2	**a**	2^3	**b**
3.	3×3	**a**	**b**	3^3	**c**
4.	4×4	4	**a**	**b**	64
5.	6×6	6	$6 \times 6 \times 6$	6^3	**a**



6. Draw a sketch of a building with 9 rooms on each floor. How many rooms are in the building all together? Express the total number of rooms as a product, with an exponent, and with a standard numeral.

7. The following numbers are called *cubic numbers*. Can you think of a reason for this name? Write the standard numeral for each of these cubic numbers.

a 5^3 **b** 8^3 **c** 7^3 **d** 10^3 **e** 12^3 **f** 15^3

PROGRESS CHECK

Write each product, using an exponent.

1. $10 \times 10 \times 10 = ?$
2. $7 \times 7 \times 7 \times 7 = ?$
3. $6 \times 6 \times 6 \times 6 \times 6 = ?$
4. $2 \times 2 \times 2 \times 2 = ?$
5. $3 \times 3 \times 3 \times 3 \times 3 \times 3 = ?$
6. $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = ?$
7. $12 \times 12 \times 12 = ?$
8. $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = ?$
9. $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = ?$
10. $8 \times 8 \times 8 \times 8 \times 8 = ?$

Write each number as the product of equal factors.

11. 3^5 12. 6^2 13. 7^5 14. 8^4
15. 5^3 16. 3^4 17. 4^8 18. 2^{10}

Write each number in standard-numeral form.

19. 2^4 20. 4^2 21. 4^3 22. 3^4 23. 4^4

*24. Copy and complete the table.

Standard numeral	Product of equal factors	Written with an exponent
36	6 x 6	6^2
a	b	5^3
c	4 x 4 x 4	d
e	f	3^4
100	10 x 10	g
10 000	h	i
j	k	10^8

Look at another way of naming numbers.
Tell how many full hands (all 5 fingers up) and how many fingers extra.

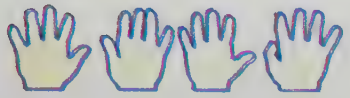
1. ? hands ? fingers

2. ? hands ? fingers

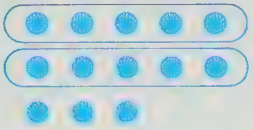
3. ? hands ? fingers

4. ? hands ? fingers

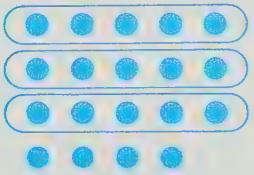
5. ? hands ? fingers

6.  ? hands ? fingers

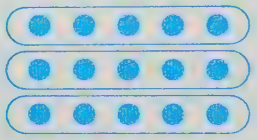
Tell how many fives and how many ones.

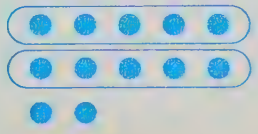
7.  ? fives ? ones

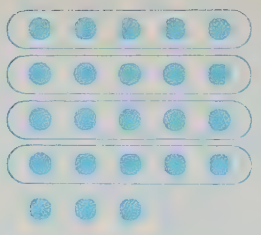
8. ? fives ? ones

9.  ? fives ? ones

10. ? fives ? ones

11.  ? fives ? ones

12.  ? fives ? ones



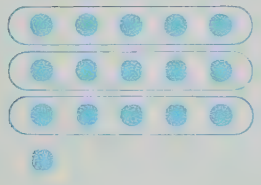
We can describe the number of dots like this: 43_{five}

The first digit (4) tells how many fives. The second digit tells how many ones. The "five" says that the dots are grouped in fives.

43_{five} is called a *base-five numeral.*

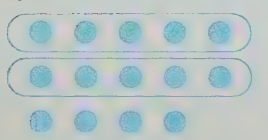
1. Write a base-five numeral to tell how many dots.

a

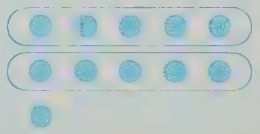


b

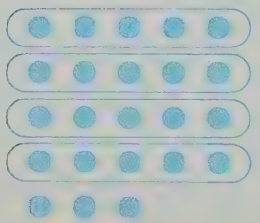
c



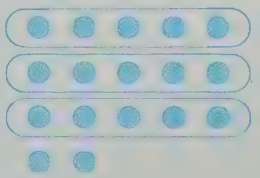
d



e



f



2. Write a base-five numeral to tell how many for each set of fingers and each set of dots shown on the page before.

3. Draw dots to show how many.

a 24_{five} **b** 31_{five} **c** 33_{five}

d 20_{five} **e** 14_{five} **f** 1_{five}

4. Which digits do base-five numerals use? How many different digits is this?

5. The numerals we usually use are called *base-ten numerals*. Which digits do base-ten numerals use? How many different digits is this?

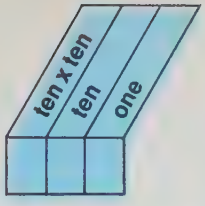
6. Draw dots and then write a base-five numeral to tell how many.

a 8_{ten} **b** 13_{ten} **c** 23_{ten}

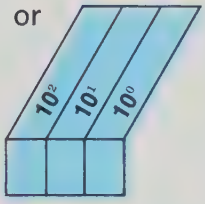
d 18_{ten} **e** 9_{ten} **f** 10_{ten}

7. How many is 10_{ten}? How many is 10_{five}?

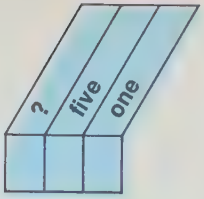
Our base-ten system is called the *decimal system*. In the decimal system a place value is written with a 10 and an exponent or as a product of tens.



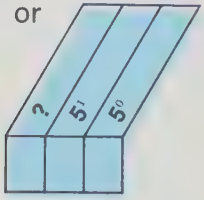
or



What do you think the next place value is in base five?



or



The base-five system can be compared to a money system with only pennies (ones), nickels (fives), and quarters (five times five). Whenever possible, five pennies must be exchanged for a nickel, and five nickels must be exchanged for a quarter. Use this system to complete the following table.

	Dollars and cents	Quarters	Nickels	Pennies
1.	$0.58	a	b	c
2.	0.44	a	b	c
3.	0.91	a	b	c
4.	1.22	a	b	c
5.	0.73	a	b	c
6.	1.19	a	b	c
7.	0.24	a	b	c
8.	0.85	a	b	c
9.	0.69	a	b	c
10.	1.00	a	b	c

CHECKOUT

1. Write a decimal for each.

a 6 and 34 hundredths
b 26 and 5 tenths
c 1 and 365 thousandths
d 7 and 97 thousandths
e 24 thousandths
f six and twenty-four hundredths
g one hundred sixty-three thousandths
h four hundred twenty-one and seven-tenths

2. Write a decimal for each.

a $\frac{6}{10}$ **b** $\frac{23}{100}$ **c** $\frac{2}{100}$ **d** $\frac{48}{1000}$

3. Write a fraction for each.

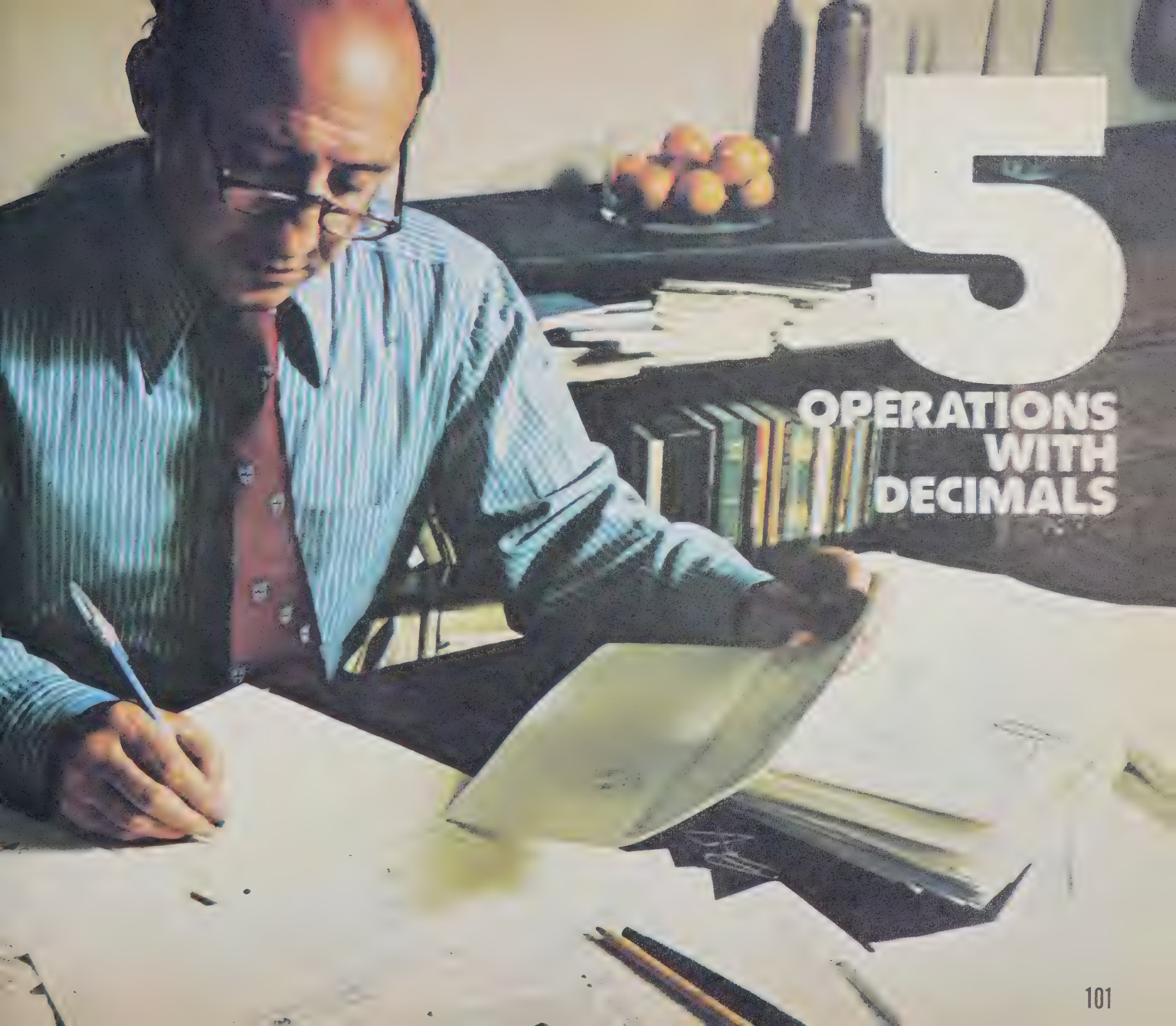
a 0.07 **b** 0.6 **c** 0.146 **d** 0.29

4. Round each to the nearest hundredth, tenth, and whole number.

a 4.638 **b** 27.134 **c** 1.975 **d** 0.847
e 2.603 **f** 10.738 **g** 4.455 **h** 3.505

5. Write the standard numeral for each.

a 10^3 **b** 4^2 **c** 2^3 **d** 10^4



5

OPERATIONS WITH DECIMALS

Temperatures will range f
and lower 90s to lows mostly in the
Local forecast: See Page 1.

Pollution readings

Citywide average air pollution le
ended at 9 a.m.:

Sulphur dioxide, 0.03 parts per m
Danger level is 0.11 ppm.

Haze, 0.84 coefficient of haze per 1
mal reading is 0.75 COH.

Carbon monoxide, 10 p
To report viol
City

Annual average weekly earnings

Year 1972 month	For-estry	Mining (incl. milling)	Manufacturing
			151.34
uary	165.90	183.01	151.51
ary	175.00	186.66	152.74
	173.50	187.28	155.29
	178.79	187.00	154.19

...ution measure provides
...average output levels of pollu-
...for automobiles manufactured
...975 should be less than 2.1 grams/
...or carbon monoxide, 0.25 grams/
...or hydrocarbons and 1.2 grams/
...nitrous oxide (or less than 0.25
...m 1976 onwards), with a
...ing the amount of
...es to one ten

$4.99

PATIENT-DAY REVENUE AND EXPENDITURE RATIOS OF OPERATING PUBLIC HOSPITALS.

	Revenue	Expenditure Total	Gross salaries and wages	Medical and surgical supplies	Drugs	Supplies and other expenses	Total
CANADA	50.21	57.49	42.05	1.95	1.88	13.86	59.7
General							
Allied special	74.54	94.77	70.80	2.76	3.17	25.51	102.2
Children's	32.95	38.03	26.76	0.41	0.47	10.64	38.2
Convalescent/rehabilitation	22.59	23.52	18.17	0.30	0.53	5.29	24.2
Chronic/extended care	65.62	81.01	60.84	1.82	2.20	19.92	84.7
Other	19.36	19.71	15.31	0.09	0.39	4.14	19.
Mental							

1 WHAT IS A DECIMAL? WHAT GOOD IS IT?

2 Each cost $1.35. You bought two. The tax for both was 14¢. How much did you pay in all?

3 How much difference is there between 0.0008 km and 0.012 km ?

4 She bought 5 MBI shares. Each share cost $16.87. How much did she pay?

5 Five people divided the cost of 1 share. It cost $9.65. About how much money would each have to pay?

GOAL

is to learn how to operate with decimals. Lots of people think decimals are easier to use than fractions. Maybe you will too.

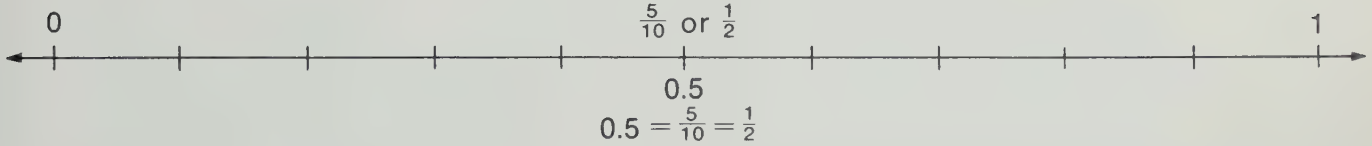
Back in the year 1585 a man named Simon Stevin published a book on the decimal numeration system. Stevin didn't invent decimals. And he didn't write them the way we do now. But many people first learned about decimals in his book.

Did you know that even today people have different ways to write a decimal?

In some European countries they write 1.1 as 1,1 or $1,_1$.

Perhaps some day everyone will write decimals the same way.

The same number can be named with different numerals. The decimal 0.5 means five-tenths of one. It can be put on a number line. In fact, 0.5 is another name for $\frac{1}{2}$.



$0.5 = \frac{5}{10} = \frac{1}{2}$

Do you suppose *every* fraction can be written as a decimal?

Say each of these decimals to yourself: 0.5 0.50 0.05
Think about the "denominator" of each of these fractions.

0.5 is five-tenths or $\frac{5}{10}$

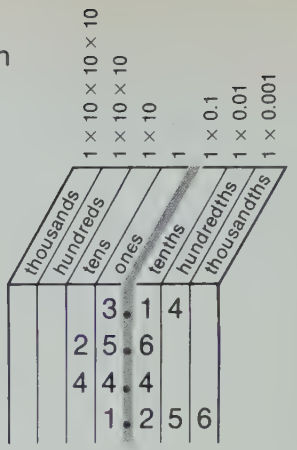
0.50 is fifty-hundredths or $\frac{50}{100}$

0.05 is five-hundredths or $\frac{5}{100}$

A Write each of these decimals as a fraction with a numerator and denominator.
0.2 0.20 0.25 0.75 0.40

The place-value chart shows that the number 3.14 can be thought of as 3 + 0.1 + 0.04. The value of each digit depends on its place in the numeral.
Another way of thinking about the number 3.14 is

$$(3 \times 1) + (1 \times 0.1) + (4 \times 0.01)$$

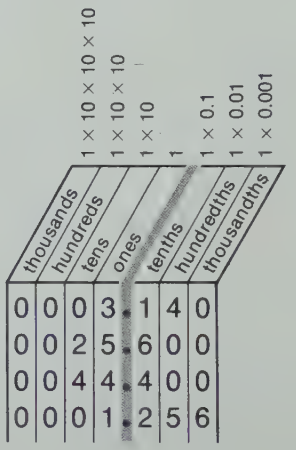


1. This example shows the place value of each digit in 25.6. What is the 2 multiplied by? the 5? the 6?

$$\begin{aligned} 2 \times \blacksquare &= 20.0 \\ 5 \times \blacksquare &= 5.0 \\ 6 \times \blacksquare &= 0.6 \\ &\overline{25.6} \end{aligned}$$

2. How many times larger is 4 tens than 4 ones?
4 tenths is what fraction of 4 ones?
Complete the following sentences.
$4 \times 10 =$? $\quad 4 \times 1 =$? $\quad 4 \times 0.1 =$?

This is the same chart as the one above. But there is a change in the way the numerals look. Now every available place has a zero.



3. Does a zero on the left of a number change the value of the number?

4. Does a zero on the right change the value of the number?

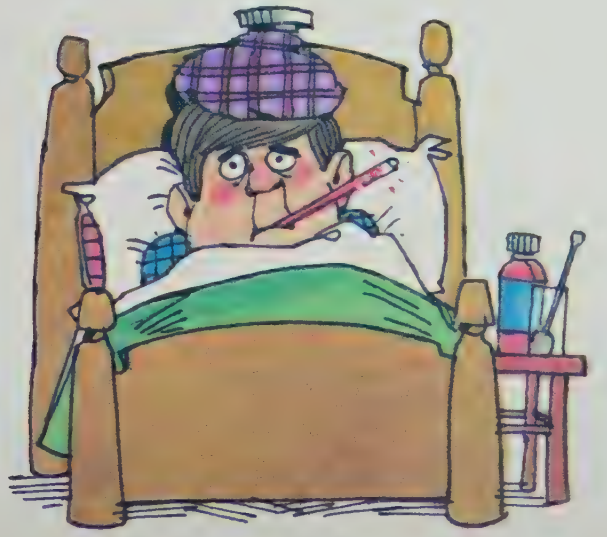
5. 3.14 means "three and fourteen-hundredths." Does 3.140 mean the same thing?

6. 25.6 means "twenty-five and six-tenths." Does 25.600 mean the same thing?

1

If you're not feeling well, someone may put a thermometer in your mouth. Normal body temperature is 37 degrees Celsius.
If your temperature is 37.2°C you may or may not be healthy. If your temperature is 37.7°C, something is probably wrong with you or the thermometer. There is a very small range in body temperature. That's why it is measured in tenths. Arrange in order from the lowest temperature to the highest temperature.

38.1	37.9	37.7	38.0
37.4	38.2	37.5	37.6



2

A. 0.289 km	D. 0.342 kg	G. 0.250 ℓ
B. 0.307 km	E. 0.269 kg	H. 0.198 ℓ
C. 0.282 km	F. 0.273 kg	I. 0.891 ℓ

a Which is the longer distance — A or B? How can you tell?

b Which is the greater mass — E or F? How can you tell this time?

c Which is the longer distance — A or C? Once again, how can you tell?

d Which is the greater volume —
G or H?
H or I?

e Order A, B, and C from longest distance to shortest distance.

f Order D, E, and F from greatest mass to least mass.

g Order G, H, and I from greatest volume to least volume.

3

Arrange these decimals in order from lowest to highest.

0.344	0.367	0.358	0.342
0.340	0.298	0.333	0.341

1 Can numbers with different place values be equivalent? THINK!

One tenth = 0.1 = $\frac{1}{10}$ Right?

One hundredth = 0.01 = $\frac{1}{100}$ Right?

One thousandth = 0.001 = $\frac{1}{1000}$ Are you sure that's right?

a Does 0.1 = 0.01? If you had $\frac{1}{10}$ of one dollar and I had $\frac{1}{100}$ of a dollar, would you trade with me?

b Does 0.10 = 0.01? If you had $\frac{10}{100}$ of one dollar and I had $\frac{1}{100}$ of one dollar, would you trade with me?

c Does 0.1 = 0.10? THINK! If you had $\frac{1}{10}$ of one dollar and I had $\frac{100}{1000}$ of one dollar, would we have the same amount of money?

d Does 0.01 = 0.001?

e Does 0.010 = 0.001?

2 Arrange these in order from smallest to largest.

a 12.35, 12.4, 12.355, 12.401

b 11.250, 11.330, 11.255, 11.200

Which set was easier to do? Can you figure out why?

PROGRESS CHECK

Copy and order each set of decimals from smallest to largest.

(1.) 2.1, 4.7, 3.6, 5.1

(2.) 0.9, 0.7, 0.6, 1.2, 0.5

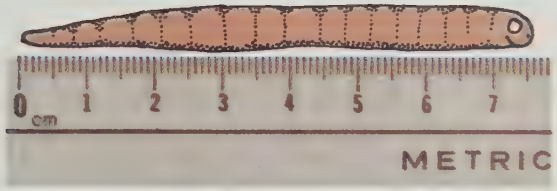
(3.) 0.106, 0.359, 0.201, 0.275

(4.) 3.015, 3.501, 3.105, 1.503

(5.) 0.79, 0.406, 9.3, 1.6

(6.) 2.704, 2.71, 2.695, 2.701

How long is the worm?



You could say 75 mm . In our system of measurement we never mix units of measure. If you wanted to find the length of the worm in centimetres, you would have to use a decimal fraction. Fractions related to measurement are always written in decimal form. Measuring in centimetres, the worm is 7.5 cm long.

1. Measure these lines. Write your answers first in millimetres and then in centimetres.

a _____ **b** _____

c _____ **d** _____

e _____ **f** _____

2. Measure the width and length of the following books. Write your answers first in millimetres and then in centimetres.

a this book **b** a reader **c** a workbook

d a library book **e** a dictionary **f** one other book

1. How many:

a millimetres in a metre?

b millimetres in a centimetre?

c centimetres in a metre?

d decimetres in a metre?

2. Supply the missing word.

a Decimetres are ? of a metre.

b Centimetres are hundredths of a ?.

c Grams are ? of a kilogram.

d ? are thousandths of a litre.

e ? are tenths of a centimetre.

f ? are tenths of a decimetre.

Think carefully about how accurately this measurement has been made.

4.50 m THINK:

Two decimal places.
That's hundredths.
Centimetres are hundredths of a metre. The measurement has been made to the nearest centimetre.

3. How accurately have these measurements been made?

a 7.2 cm **b** 4.500 kg **c** 8.75 m **d** 15.125 km

4. 4.65 m can be written 465 cm . Rewrite each measurement in question 3 as a smaller measure so that you don't need to use a decimal fraction.

5. The Olympic games bring together athletes from more than a hundred nations. They are played every four years. Swimming, running, speed skating, and skiing events are based on lengths measured in metric units. Check an almanac. Find out the last reported records set in the last Olympics.

Two hikers decided to go on a mountain-climbing trip. There were several stopping points along the way. The first morning they started at A and climbed 0.7 km to B. They rested at B, had lunch, and then climbed 0.2 km to C. They made camp for the night at C. How far up did they climb that day?

$$\begin{array}{r} 0.7 \\ +\ 0.2 \\ \hline 0.9 \end{array} \quad \begin{array}{r} \frac{7}{10} \\ +\frac{2}{10} \\ \hline \frac{9}{10} \end{array}$$

1. The next morning they climbed 0.9 km to D.
In the afternoon they climbed 0.6 km to E.
How far did they climb the second day?

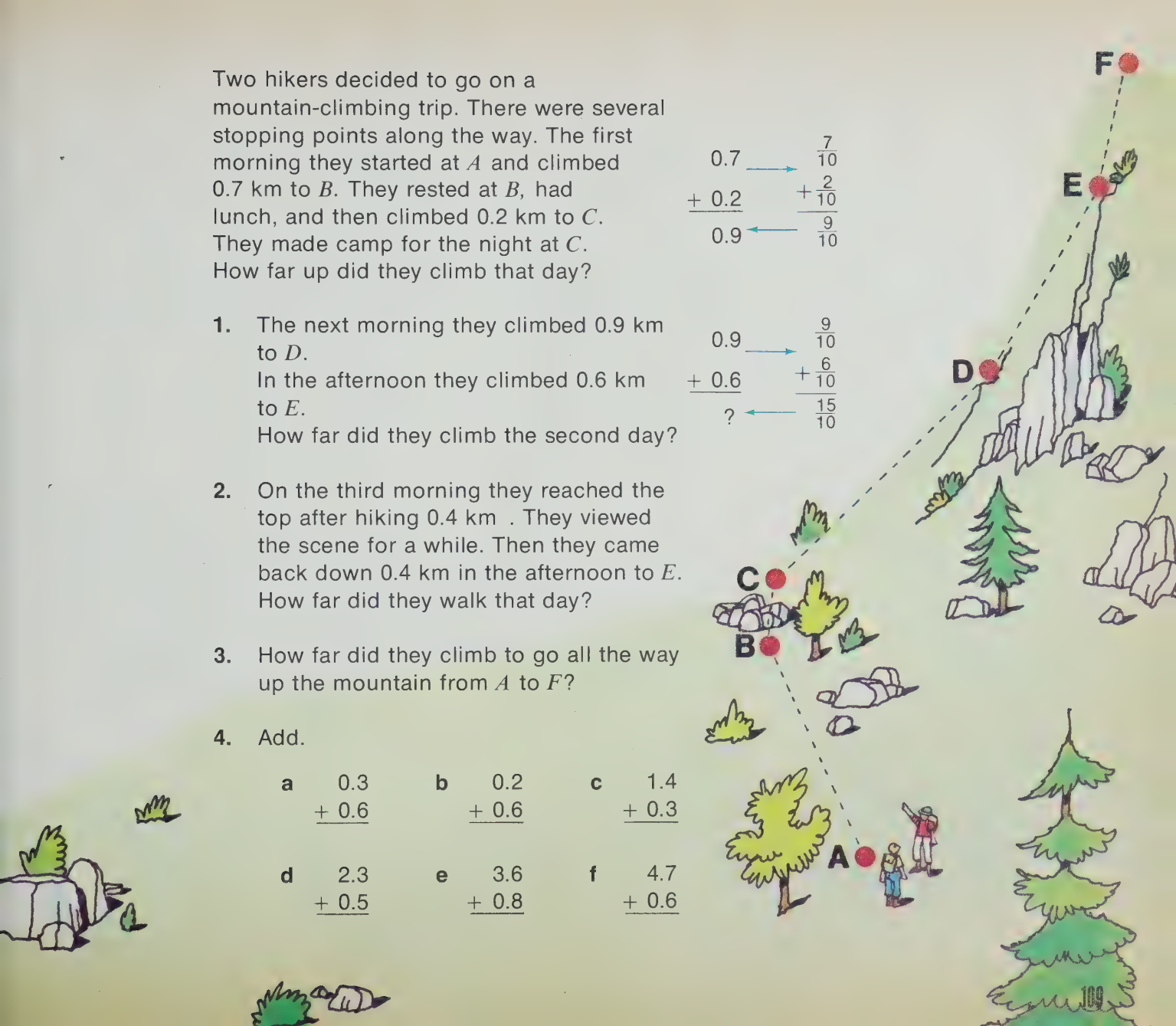
$$\begin{array}{r} 0.9 \\ +\ 0.6 \\ \hline ? \end{array} \quad \begin{array}{r} \frac{9}{10} \\ +\frac{6}{10} \\ \hline \frac{15}{10} \end{array}$$

2. On the third morning they reached the top after hiking 0.4 km . They viewed the scene for a while. Then they came back down 0.4 km in the afternoon to E. How far did they walk that day?

3. How far did they climb to go all the way up the mountain from A to F?

4. Add.

a	0.3 + 0.6	**b**	0.2 + 0.6	**c**	1.4 + 0.3
d	2.3 + 0.5	**e**	3.6 + 0.8	**f**	4.7 + 0.6



1. Coming down the mountain they went 1.5 km from E to C in one day.
In the morning they covered 0.6 km .

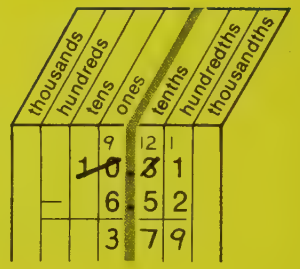
How far did they have left to go in the afternoon?

$$\begin{array}{rr} 1.5 & \frac{15}{10} \\ -\ 0.6 & -\frac{6}{10} \\ \hline 0.9 & \frac{9}{10} \end{array}$$

SUBTRACT

2. **a** $\begin{array}{r} 0.9 \\ -\ 0.5 \\ \hline \end{array}$ **b** $\begin{array}{r} 2.4 \\ -\ 0.3 \\ \hline \end{array}$ **c** $\begin{array}{r} 1.8 \\ -\ 0.4 \\ \hline \end{array}$ **d** $\begin{array}{r} 2.3 \\ -\ 0.6 \\ \hline \end{array}$ **e** $\begin{array}{r} 3.0 \\ -\ 0.7 \\ \hline \end{array}$

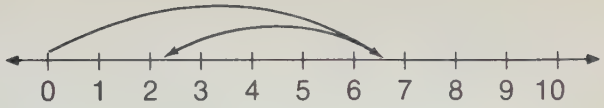
Sorry, but there really isn't much new to learn when it comes to adding and subtracting decimals. You had to pay attention to place value when you operated with whole numbers. You have to pay attention to place value with decimals, too.



Compute these. Watch the operation symbol!

	a	b	c	d	e	f	g	h
3.	2.3 + 0.4	4.7 − 2.5	3.7 + 0.8	4.9 + 1.6	6.2 − 1.7	5.4 − 1.4	6.3 + 1.7	7.60 − 4.06
4.	5.6 − 0.6	6.2 − 0.9	8.8 + 7.6	2.3 − 1.5	10.0 − 3.2	4.5 + 5.5	7.4 − 6.5	10.64 + 9.36

1. This number line shows this subtraction problem.



$$\begin{array}{r} 6.5 \\ -\ 4.25 \\ \hline \end{array} \longrightarrow \begin{array}{r} 6.50 \\ -\ 4.25 \\ \hline 2.25 \end{array}$$

Why do you suppose 6.5 was renamed as 6.50?

PRACTICE You may have to rename to subtract.

2.

a	b	c	d	e
4.6	3.35	6.8	3.62	3.07
− 2.14	− 1.41	− 2.42	− 1.91	− 2.45

3.

a	b	c	d	e
6.46	4.0	7.56	2.3	6.5
− 4.33	− 2.03	− 4.45	− 1.26	− 4.36

4. These two problems differ in three ways. What are these differences?

a

$$\begin{array}{r} 4.23 \\ +3.45 \\ \hline 7.68 \end{array} \longrightarrow \begin{array}{r} 4\frac{23}{100} \\ +3\frac{45}{100} \\ \hline 7\frac{68}{100} \end{array} \longleftarrow$$

b

$$\begin{array}{r} 7.643 \\ -2.381 \\ \hline 5.262 \end{array} \longrightarrow \begin{array}{r} 7\frac{643}{1000} \\ -2\frac{381}{1000} \\ \hline 5\frac{262}{1000} \end{array} \longleftarrow$$

add

Watch for a place-value pattern.

	a	b	c	d
1.	0.1 + 0.1	0.3 + 0.2	0.6 + 0.3	0.9 + 0.1

← How many tenths in one?

2.	0.5 + 0.5	0.9 + 0.9	1.9 + 0.1	9.1 + 1.9

← Make sure that answer is correct.

subtract

	a	b	c	d	e	f
3.	0.8 − 0.1	0.9 − 0.9	1.3 − 0.3	1.0 − 0.3	2.5 − 1.6	10.0 − 2.9
4.	21.8 − 6.2	35.6 − 12.4	64.5 − 21.3	43.2 − 26.7	60.2 − 13.5	20.0 − 16.1

There is an old rule that says—
Keep your decimal points in a straight line when you add or subtract.
Does this rule make sense? Why?

5. Compute these.

a 1.5 + 3.25 **b** 3.5 − 2.6 **c** 17.1 + 6.9 + 3.15

d $15 − $0.15 **e** $100 + $25.25 + $0.50 **f** $85.50 − $65

1. Have you been using your estimating skills?
Think money. That always helps.
Do not compute. Quickly write your answer.

	You owe	You pay with	Change you will get back
a	\$ 4.78	\$ 5	More than 25¢?
b	\$ 3.56	\$ 5	More than \$2.00?
c	\$ 7.68	\$10	More than \$2.00?
d	\$15.02	\$20	More than \$5.00?
e	\$ 4.49	\$ 5	More than 50¢?
f	\$ 6.50	\$10	More than \$4.00?
g	\$ 4.50	\$10	More than \$6.00?
h	\$ 9.28	\$20	More than \$11.00?

Add. Think money if it helps. Don't forget to estimate.

	a	**b**	**c**	**d**	**e**	**f**
2.	1.50	2.65	4.34	6.72	8.25	9.61
	+1.25	+1.15	+2.62	+5.28	+1.75	+1.29

Subtract.

3.	4.75	3.75	5.50	2.06	4.69	7.00
	−1.25	−1.26	−3.36	−1.92	−3.89	−6.69

4. Compute these.

a $6.5 + 3.56$ **b** $4 - 0.50$ **c** $1 + 0.1 + 0.01$ **d** $6.01 - 3.9$

Time to take a closer look at addition and subtraction of numbers.

1. Sometimes place value is especially important.
A fractional number can be written in either of two ways.
For example:

$$0.1 \text{ or } \frac{1}{10} \qquad 0.75 \text{ or } \frac{75}{100} \qquad 0.300 \text{ or } \frac{300}{1000}$$

a. Can 0.8 + 0.76 be rewritten as a problem with fractions? Would it be easier to compute?

$$\begin{array}{r} 0.8 \\ +\ 0.76 \\ \hline \end{array}$$

What should you do before you solve this one?
What did you do with the "denominators"?
Solve this problem.

b Can 0.9 − 0.29 be written as a problem with fractions? Would it be easier to compute?

$$\begin{array}{r} 0.9 \\ -\ 0.29 \\ \hline \end{array}$$

What must you do before you solve this one?
What did you do with the "denominators"?
Solve this problem.

2. Add 0.4 plus 0.22. Say the answer to yourself.
What place value was said?

3. Subtract 0.49 from 0.567.
What place value is indicated in the answer?

4. Add 0.9 plus 0.5.
What place value is in the answer?

5. Add 0.7 plus 0.55.
What place value is in the answer?

6. Subtract 0.56 from 0.9.
What place value is in the answer?

7. Subtract 0.35 from 2.2.
What place value?

8. Compute. Watch the operation signs.

a	**b**	**c**	**d**	**e**	**f**	**g**
1.5	2.17	0.6	1.31	3.1	4.05	3.65
+ 0.67	− 0.67	+ 0.71	− 0.6	+ 0.92	− 0.9	+ 0.05

PROGRESS CHECK

Add.

(1.) 2.76 + 3.13	(2.) 4.01 + 2.54	(3.) 3.23 + 1.58	(4.) 3.37 + 2.46	(5.) 6.85 + 1.16
(6.) 2.87 + 2.57	(7.) 5.05 + 4.68	(8.) 4.76 + 8.96	(9.) 16.32 + 4.79	(10.) 56.85 + 14.69

Subtract.

(11.) 4.89 − 1.43	(12.) 7.88 − 1.26	(13.) 6.61 − 3.24	(14.) 8.71 − 3.46	(15.) 5.15 − 4.17
(16.) 9.65 − 7.96	(17.) 8.15 − 5.78	(18.) 15.39 − 9.49	(19.) 10.00 − 5.74	(20.) 92.02 − 23.29

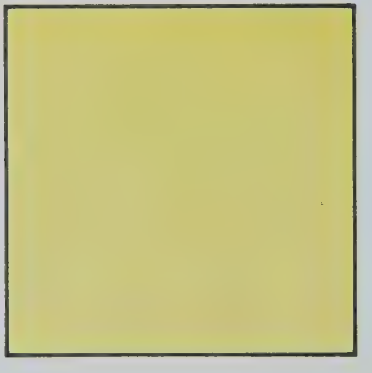
Compute. Watch out!

(21.) 3.723 + 1.246	(22.) 5.089 + 2.310	(23.) 4.681 − 2.150	(24.) 3.729 + 1.391	(25.) 5.960 + 2.580
(26.) 4.163 + 5.095	(27.) 9.278 − 1.709	(28.) 7.797 + 2.566	(29.) 6.437 − 3.779	(30.) 2.076 − 1.109

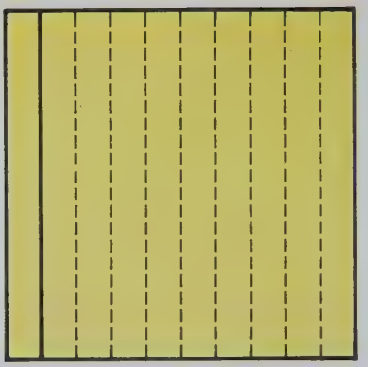
*(31.) Somebody made some errors. Can you find them?

3.465
+ 2.371
5.736

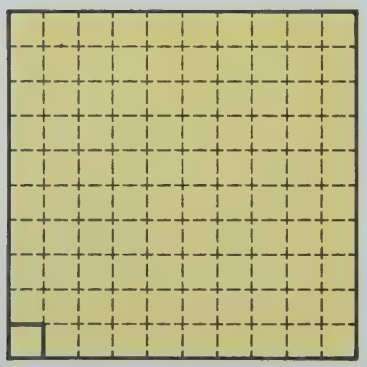
6.435
− 2.716
4.719



1 whole square region.



1 square divided into 10 parts.
1 of 10 parts. Could be written
1 tenth or $\frac{1}{10}$ or 0.1.



1 square divided into 100 parts.
1 of 100 parts. Could be written
1 hundredth or $\frac{1}{100}$ or 0.01.

1 square divided into 2 parts.
1 of 2 parts. Could be written $\frac{1}{2}$.
Could it also be written 5 tenths
or 0.5? Could it be 50 hundredths or
0.50?

$0.1 + 0.1 + 0.1 + 0.1 = 0.4$

$4 \times 0.1 = 0.4$

1. Answer these.

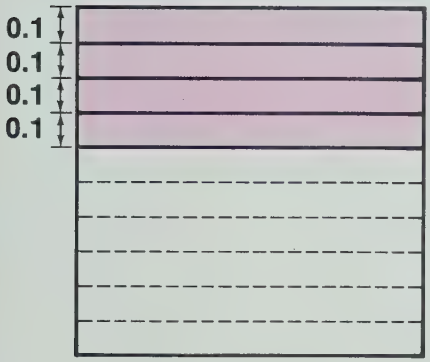
a $6 \times 0.1 = ?$

b $2 \times 0.1 = ?$

c $7 \times 0.1 = ?$

d $9 \times 0.1 = ?$

e $10 \times 0.1 = ?$



$0.3 + 0.3 + 0.3 = 0.9$

$3 \times 0.3 = 0.9$

2. Answer these.

a $1 \times 0.3 = ?$

b $2 \times 0.3 = ?$

Think carefully here.

c $4 \times 0.3 = ?$

d $5 \times 0.3 = ?$

$0.5 + 0.5 = 1.0$

$2 \times 0.5 = 1.0$

3. Answer these.

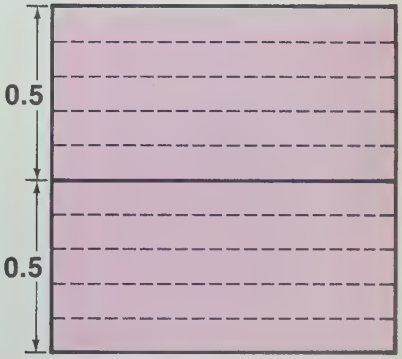
a $1 \times 0.5 = ?$

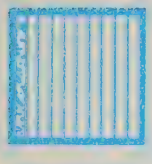
b $2 \times 0.5 = ?$

c $3 \times 0.5 = ?$

d $4 \times 0.5 = ?$

e $5 \times 0.5 = ?$

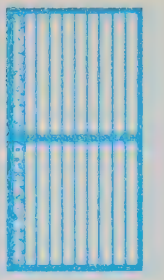




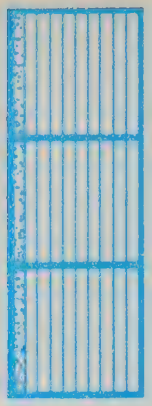
$1 \times 0.1 = 0.1$

What if more tenths were shaded?

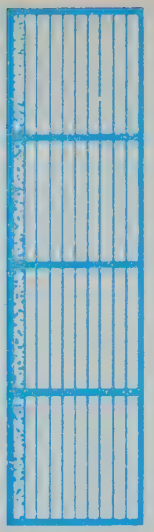
$1 \times 0.2 = ?$
$1 \times 0.3 = ?$
$1 \times 0.4 = ?$
$1 \times 0.5 = ?$
$1 \times 0.6 = ?$
$1 \times 0.7 = ?$
$1 \times 0.8 = ?$
$1 \times 0.9 = ?$
$1 \times 1.0 = ?$



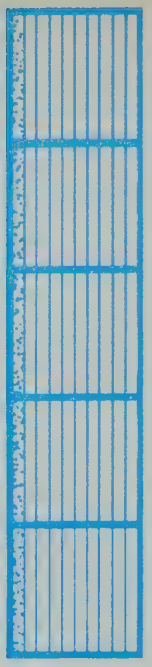
$2 \times 0.1 = 0.2$
$2 \times 0.2 = ?$
$2 \times 0.3 = ?$
$2 \times 0.4 = ?$
$2 \times 0.5 = ?$
$2 \times 0.6 = ?$
$2 \times 0.7 = ?$
$2 \times 0.8 = ?$
$2 \times 0.9 = ?$
$2 \times 1.0 = ?$



$3 \times 0.1 = 0.3$
$3 \times 0.4 = ?$
$3 \times 0.5 = ?$
$3 \times 0.6 = ?$
$3 \times 0.7 = ?$
$3 \times 0.8 = ?$
$3 \times 0.9 = ?$
$3 \times 1.0 = ?$



$4 \times 0.1 = 0.4$
$4 \times 0.7 = ?$
$4 \times 0.8 = ?$
$4 \times 0.9 = ?$
$4 \times 1.0 = ?$



$5 \times 0.1 = 0.5$
$5 \times 1.0 = ?$

Tenths times any whole number result in tenths in the answer. Prove it.

TRY THESE

1. 6×0.9 **2.** 7×0.2 **3.** 5×0.5 **4.** 9×0.1 **5.** 8×0.4

6. 12×0.2 **7.** 14×0.1 **8.** 15×0.3 **9.** 19×0.6 **10.** 20×0.1

11. 17×1.4 **12.** 55×2.3 **13.** 56×3.5 **14.** 123×1.2 **15.** 514×6.7

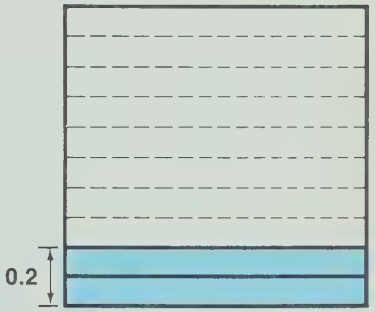
HERE'S A NEW PROBLEM

How much is 0.2 of 3?

You know what two-tenths is.
You know what three units are.
This is a multiplication situation.
But you could still use addition.

0.2 of 1

0.2 of 1



0.2 of 1

The pictures show $0.2 + 0.2 + 0.2$

But this is 3×0.2. You know that is 0.6. Does $3 \times 0.2 = 0.2 \times 3$?

Answer these.

1. $2 \times 0.2 = ?$ **2.** $4 \times 0.2 = ?$ **3.** $5 \times 0.2 = ?$ **4.** $10 \times 0.2 = ?$ **5.** $1 \times 0.2 = ?$

Take a closer look
at problems like this one.

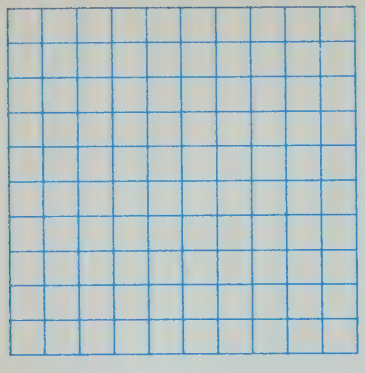
THINK

How many tenths in 1?
This problem deals with
two-tenths—5 of them.

Stack all the 5
two-tenths together.

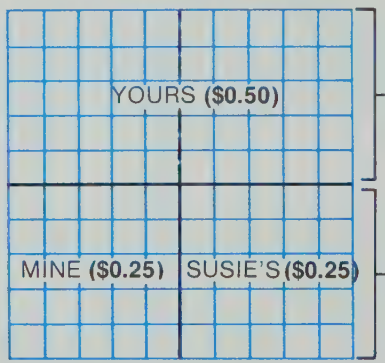
$0.2 + 0.2 + 0.2 + 0.2 + 0.2$

That makes ten-tenths
$\left(\frac{10}{10}\right)$, or 1 whole unit.



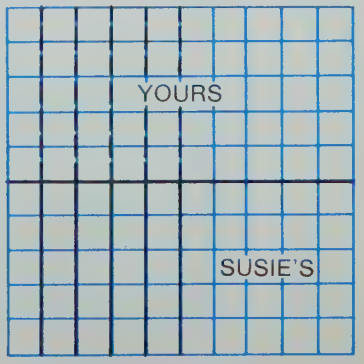
How many cents in 1 dollar?
Pretend we have square cents.
Here they are all lined up in 10 rows
of 10 square cents in each row.
Make sure that there are 100 pictured.

The cents are mine. But you say you
want half of them. You're a nice
person, so I'll give you half of them.
(You can have half of my square
cents anytime!)



This is yours. How many cents do you get?
What fraction of the dollar do you get?
Can your amount be written $0.50?

This is mine. Susie says she would like
half of mine. She's nice too. So she
gets half. How many cents do I have left?



Tom, June, Olga, and Pat would like
to share with me too. They would
like me to give them a share. They are
really nice!

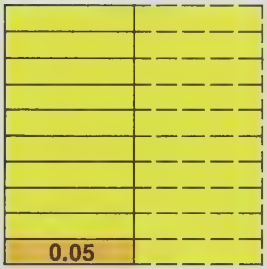
	MINE?
PAT	($0.05)
OLGA	($0.05)
JUNE	($0.05)
TOM	($0.05)

I had $1.00. I gave you $0.50. Susie got
$0.25, and then Tom, June, Olga, and
Pat came along.
How much do I have left?

I went out and earned another dollar. You guessed it!
I found a lot of nice people again. Can you figure out
what happened this time from the drawing on the right?
It is too painful for me to talk about. You tell the story.

The picture could also be used as a model for
decimals. The first picture shows halves. Each part
is marked 0.5, and that is a name for halves.

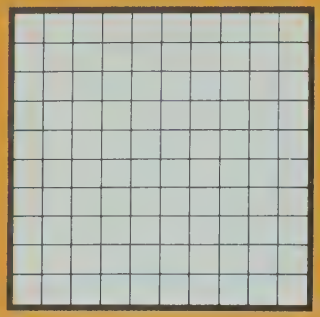
The region was also marked into ten equal parts.
Each of the ten parts represents 0.1 of the total square region.
The shaded part shows 0.1 of 0.5 parts. $0.1 \times 0.5 = \underline{?}$



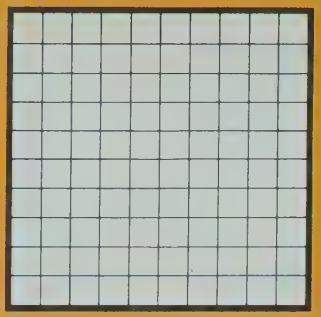
Now your job is to find the product of 0.7×0.2.
The top picture shows the whole unit in
five equal parts. Each part shows 0.2.

The problem says you want 0.7 of 0.2.
0.7 of 0.2 is how many hundredths?

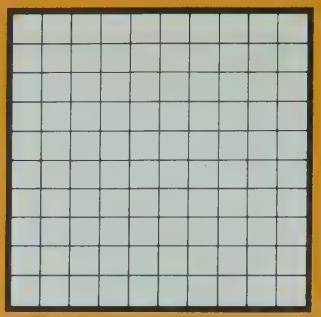
$$0.7 \times 0.2 = \underline{?}$$



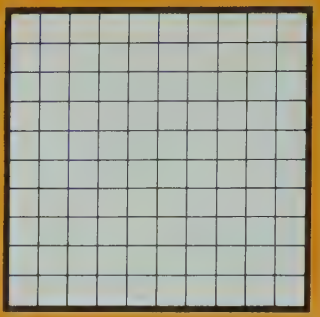
1 square



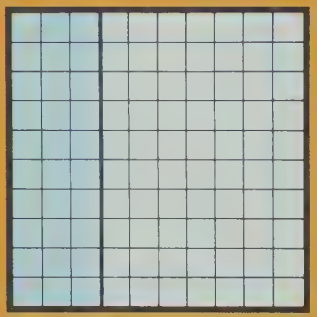
1 square



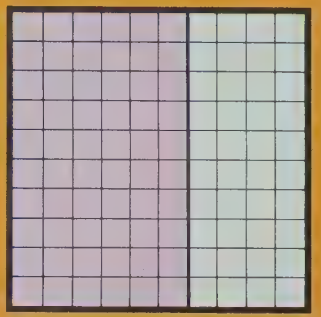
1 square



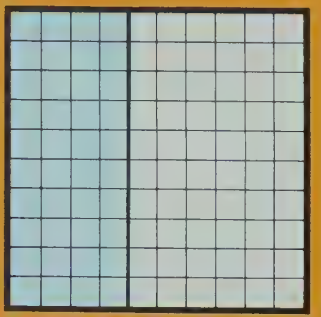
1 square



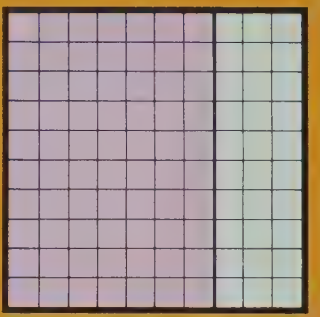
0.3 square



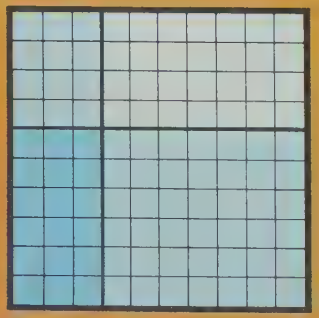
0.6 square



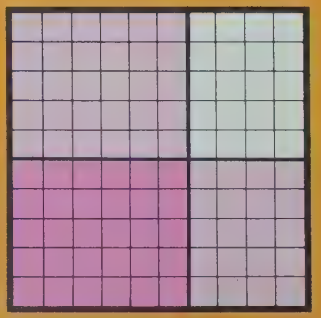
0.4 square



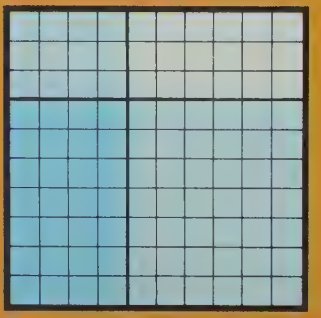
0.7 square



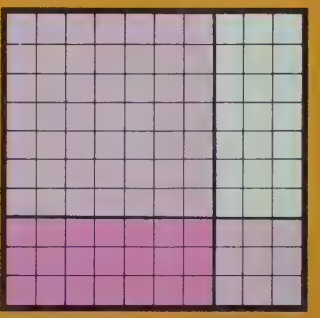
0.6 of 0.3 square
$0.6 \times 0.3 = 0.18$



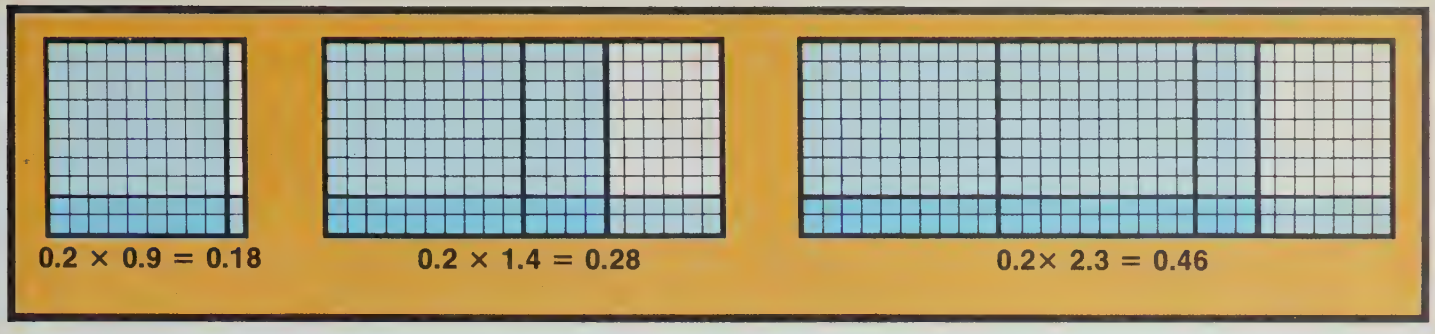
0.5 of 0.6 square
$0.5 \times 0.6 = ?$



0.7 of 0.4 square
$0.7 \times 0.4 = ?$



0.3 of 0.7 square
$0.3 \times 0.7 = ?$



What if more tenths were shaded?

0.3 × 0.9 = ?	0.3 × 1.4 = ?	0.3 × 2.3 = ?
0.6 × 0.9 = ?	0.5 × 1.4 = ?	0.4 × 2.3 = ?
0.8 × 0.9 = ?	0.7 × 1.4 = ?	0.6 × 2.3 = ?
0.9 × 0.9 = ?	0.8 × 1.4 = ?	1.0 × 2.3 = ?

Any number with tenths in its place value times another number with tenths in its place value results in hundredths in the answer.

Prove it. Try these.

1. 0.2 × 0.6	**2.** 0.7 × 0.8	**3.** 0.9 × 0.6	**4.** 0.5 × 0.2	**5.** 0.3 × 0.3
6. 1.2 × 0.2	**7.** 2.1 × 0.6	**8.** 1.5 × 0.3	**9.** 1.7 × 0.8	**10.** 2.5 × 0.7
11. 21.3 × 0.2	**12.** 14.1 × 0.4	**13.** 52.7 × 0.6	**14.** 62.7 × 0.4	**15.** 35.6 × 0.9

1 You know how to compute with money. Go shopping.

	You want	Each costs	You have	Can you buy them?	How much money is left?
a	3 rolls of film	$1.25	$5.00	?	?
b	2 packages of flash bulbs	$0.79	$2.00	?	?
c	3 tins of polish	$0.65	$3.25	?	?
d	2 hamburgers	$0.55	$1.25	?	?
e	12 lemons	$0.08	$1.00	?	?

2 You have to buy these things. You are in charge of a field day.

	You need	Each costs	You have	How much more do you need?
a	24 train tickets	$1.50	$24.00	?
b	24 tickets to the game	$0.75	$15.00	?
c	19 lunch tickets (5 people will bring their own)	$1.10	$20.00	?

d How much money will be spent in all for this field day?

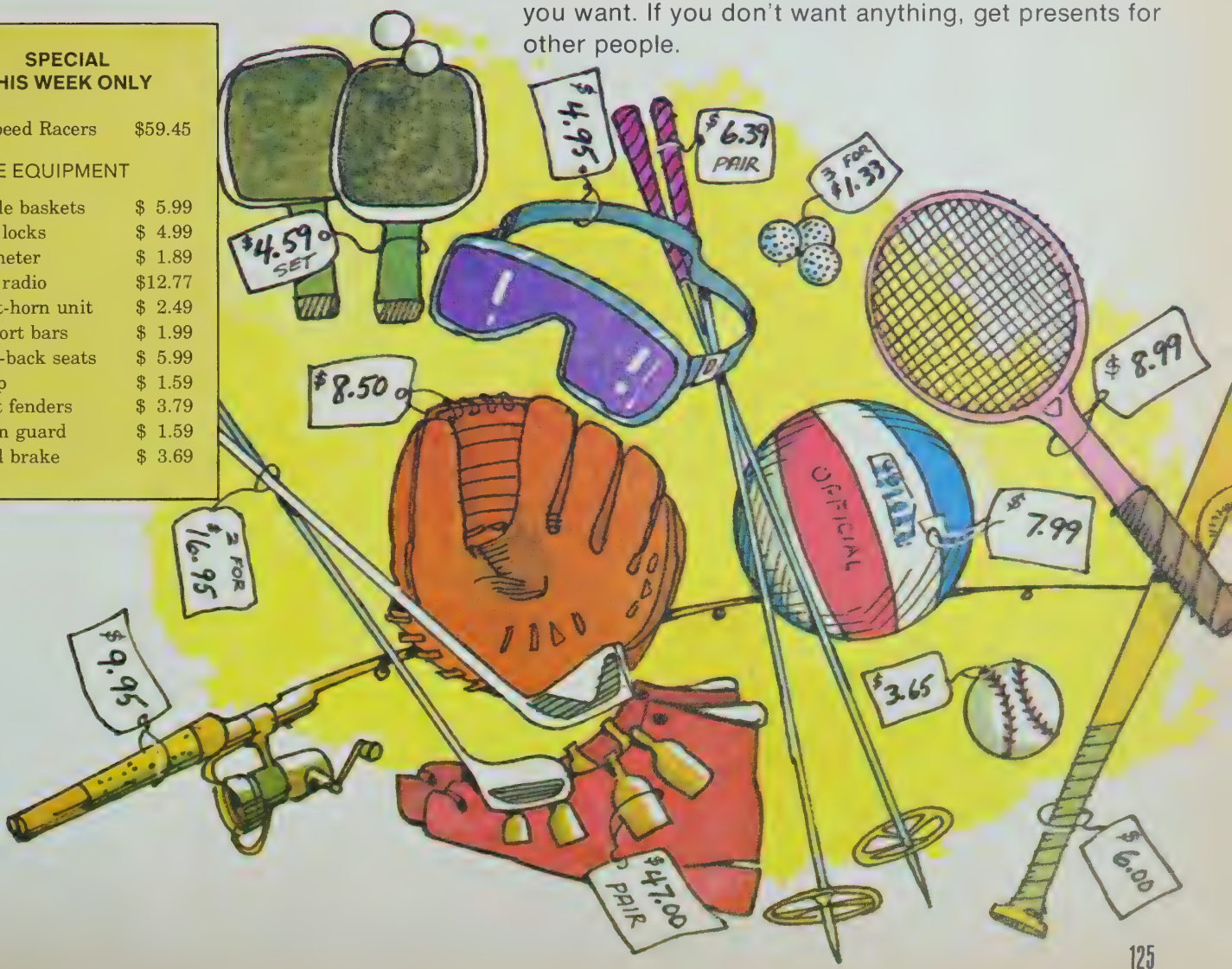
There is a sale at the sporting-goods store. You decide to go. You win the door prize. You get $50 worth of gear. Spend as much of the $50 as you can. Get what you want. If you don't want anything, get presents for other people.

SPECIAL
THIS WEEK ONLY

10 Speed Racers	$59.45

BIKE EQUIPMENT

Saddle baskets	$ 5.99
Bike locks	$ 4.99
Odometer	$ 1.89
Bike radio	$12.77
Light-horn unit	$ 2.49
Support bars	$ 1.99
High-back seats	$ 5.99
Pump	$ 1.59
Sport fenders	$ 3.79
Chain guard	$ 1.59
Hand brake	$ 3.69



PROGRESS CHECK

Here's what you were supposed to have learned so far.
Did you? Prove it.

Write each as a decimal.

(1.) $\frac{2}{10}$ (2.) $\frac{2}{100}$ (3.) $\frac{2}{1000}$ (4.) $\frac{20}{10}$ (5.) $\frac{20}{100}$ (6.) $\frac{20}{1000}$

(7.) Arrange these decimals in order from smallest to greatest.

10.1 10.01 1.15 11.1 11.12 100.0

Round each decimal to the nearest tenth.

(8.) 2.567 (9.) 10.16 (10.) 1.782 (11.) 1.021 (12.) 2.135

Compute. Watch the operation signs.

(13.) $3.7 + 0.9$ (14.) $10.3 - 1.2$ (15.) $8.3 - 0.4$ (16.) $3.526 + 0.758$ (17.) $0.6 - 0.37$

(18.) 0.2×0.6 (19.) $1.5 + 0.78$ (20.) 62.7×1.8 (21.) 15.4×0.9 (22.) 2.5×0.1

Keep your eyes open for decimals. Start a decimal collection from newspaper clippings, facts you hear on TV, stories from magazines. Copy information you might see in a shop. Save labels from grocery items that show decimals being used. Keep your eyes and ears open.

The four members of the Junior Business Club got their first job on Saturday. They helped Ms. Wilton with her garden clean-up. Ms. Wilton paid them $6. The $6 was to be split among the four of them.

The math sentence is $\$6 \div 4 = \underline{?}$

To compute, you would write $4\overline{)\$6}$.

First try:

$$\begin{array}{r} \$1 \text{ R2} \\ 4\overline{)\$6} \\ -4 \\ \hline 2 \end{array}$$

Answer: Each pays $1, but $2 remains to be divided. This answer isn't good enough.

Second try: Each member was given $1. That left $2 still to be paid out. Then Tricia, the president, suggested they each get another $0.25.
How much more was paid out?
How much was still left to be paid out?
How much did they each get in the end?

$$\begin{array}{r} \$1.50 \\ 4\overline{)\$6.00} \end{array}$$

Is this true?

Is $6 the same amount of money as $6.00?

Ricky, the treasurer, wanted to make sure $1.50 was correct. He wrote

$$\begin{array}{r} \$1.50 \\ \times \quad 4 \\ \hline \$6.00 \end{array}$$

It checks out.

Ricky thought there had to be a way to do problems like this without so much trouble. Do you agree?

Ricky decided to write it as a division problem again. He just worked it like a whole-number division problem.

```
 1 50
   50
 1 00
4)6.00
-4 00
 2 00
-2 00
    0
```

Then he went back and put in the decimal points, so that it looked like this:

```
$1.50
 0.50
 1.00
4)6.00
-4 00
 2 00
-2 00
    0
```

When the Junior Business Club got their next job, the club had grown to 8 members. (Everybody wanted to get in on the act.) They worked all afternoon for Mr. Rees and were paid $10. How much was each person's share?

Ricky did the division this time.

He divided without paying attention to the decimal point.

Then he put the decimal points where he thought they belonged.

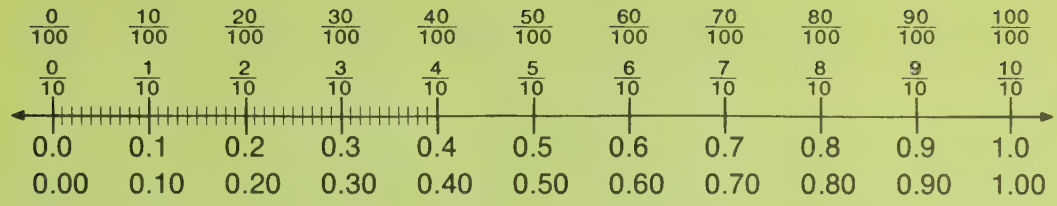
```
 1.25
    5
 0.20
 1.00
8)10.00
- 8 00
  2 00
 -1 60
   40
  -40
    0
```

Check his work. Was he correct?

```
 1.25
×   8
```

It's your turn to do division.

1. $5\overline{)8.00}$ **2.** $6\overline{)15.00}$ **3.** $4\overline{)4.80}$ **4.** $7\overline{)28.00}$ **5.** $8\overline{)8.80}$ **6.** $3\overline{)9.30}$



1. Do $\frac{10}{100}$, $\frac{1}{10}$, 0.1, and 0.10 mark the same point on the number line? Are they names for the same number?

2. Do $\frac{30}{100}$, $\frac{3}{10}$, 0.03, and 0.30 mark the same point on the number line? Are they names for the same number?

3. The decimal 0.5 tells that the number line between 0 and 1 has been divided into 10 equal parts.

a The decimal 0.50 tells that the number line between 0 and 1 has been divided into ? equal parts.

b Do 0.5 and 0.50 mark the same point on the number line?

c Would 0.500 also mark the same point?

d Would 0.5000 also mark the same point?

4. Do 1, 1.0, 1.00, and 1.000 mark the same point on the number line?

5. Do these quotients name the same number? $2\overline{)4}$ $2\overline{)4.0}$ $2\overline{)4.00}$

Look at this division situation:

$$10\overline{)12} \quad 1\ \text{R}2$$

Do you think we could annex a decimal point and a zero to the right of 12 and not change the value of 12?

What would happen to the quotient and the remainder if we did that?

$$\begin{array}{r|l} & 1.2 \\ & 0.2 \\ & 1.0 \\ 10\overline{)} & 12.0 \\ -\ 1 & 10.0 \quad 1.0 \times 10 \\ \hline & 2.0 \\ - & 2.0 \quad 0.2 \times 10 \\ \hline & 0 \end{array}$$

Notice—the tenths are all in the same column. The ones also are in a single column.

Hey! The remainder is 0.

Can this be right? Check the problem.

$$\begin{array}{r} 1.2 \\ \times\ 10 \\ \hline \end{array}$$

You know $5 \div 2$ should have a remainder. Try annexing a decimal point and a zero (or zeros) and see what happens.

$$\begin{array}{r} 2.5 \\ 0.5 \\ 2.0 \\ 2\overline{)5.0} \\ -\ 4.0 \quad 2.0 \times 2 \\ \hline 1.0 \\ -\ 1.0 \quad 0.5 \times 2 \\ \hline 0 \end{array}$$

Check the problem:

$$\begin{array}{r} 2.5 \\ \times\ 2 \\ \hline \end{array}$$

Annex a decimal point and zero if this is necessary to avoid a remainder. Divide and check.

1. $4\overline{)10}$ **2.** $5\overline{)18}$ **3.** $2\overline{)11}$ **4.** $6\overline{)27}$ **5.** $8\overline{)12}$ **6.** $5\overline{)32}$

$$
\begin{array}{r}
0.2 \\
1.0 \\
4\overline{)5.0} \\
-\,4.0 \\
\hline
1.0 \\
-\,0.8 \\
\hline
0.2
\end{array}
$$

1.0×4

0.2×4

Now what? A zero has been annexed, but there is still a remainder.

Can you annex another zero and not change the value of 5.0? Try it.

$$
\begin{array}{r}
? \\
0.05 \\
0.20 \\
1.00 \\
4\overline{)5.00} \\
-\,4.00 \\
\hline
1.00 \\
-\,0.80 \\
\hline
0.20 \\
0.20 \\
\hline
0
\end{array}
$$

1.00×4

0.20×4

0.05×4

You can avoid remainders in many division problems. Sometimes you have to annex only one zero, sometimes more.

Your practice problems are arranged so that you won't have to annex more than three zeros. But please remember that there are some division problems that would need more.

In fact, there are some division problems that will have a remainder no matter how many zeros are annexed. Maybe you'll find one some day!

Try these for practice.

1. $8\overline{)26}$ **2.** $4\overline{)6}$ **3.** $6\overline{)39}$ **4.** $8\overline{)11}$ **5.** $2\overline{)1}$ **6.** $5\overline{)2}$

7. $8\overline{)1}$ **8.** $4\overline{)1}$ **9.** $4\overline{)3}$ **10.** $5\overline{)4}$ **11.** $8\overline{)5}$ **12.** $10\overline{)1}$

1 Annexing zeros without paying attention to the decimal point is unwise.

a Do 3, 3.0, 3.00, 3.000 name the same number?

b Do 3, 0.3, 0.03, 0.003 name the same number?

2 **Look at these division problems.**

$2\overline{)4}$ Take both × 2. $4\overline{)8}$ Is the quotient the same?

Take both × 3. $6\overline{)12}$ Is the quotient the same?

Take both × 10. $20\overline{)40}$ Same quotient?

Take both × 100. $200\overline{)400}$ Same quotient?

If you multiply both numbers by the same number, it seems that the quotient doesn't change. Test the idea on these problems.

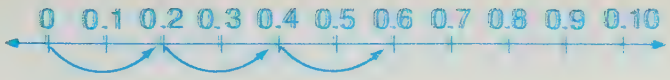
3 $4\overline{)8}$ $40\overline{)80}$ $400\overline{)800}$ $4000\overline{)8000}$ Are the quotients the same?

4 $5\overline{)10}$ $50\overline{)100}$ $500\overline{)1000}$ $5000\overline{)10000}$ Are the quotients the same?

5 What happens to the quotient if only one number is multiplied by another number? Does it stay the same?

$5\overline{)10}$ $5\overline{)100}$ $5\overline{)1000}$ $5\overline{)10000}$

HOW MANY 0.2s IN 0.6?



It just doesn't make sense to draw a number line every time you want to do a simple division problem. There is a way to compute the quotient.

You know how to divide whole numbers. You can use that skill. The only thing you must do is figure out a way to make 0.2 a whole number.

Is there a number you can multiply times 0.2 to make it a whole number?

$1 \times 0.2 = 0.2$ That won't work.

O.K., try again. $10 \times 0.2 = 2.0$ THAT'S IT!

$0.2\overline{)0.6}$ ← What about that part of the problem?

↑ If 0.2 is multiplied by 10, 0.6 must also be multiplied by 10.

$0.2\overline{)0.6}$ **Think** $10 \times 0.2 = 2$
$10 \times 0.6 = 6$

Both numbers have been multiplied by 10. The quotient of $0.2\overline{)0.6}$ and $2\overline{)6}$ will be the same. Now the problem is $2\overline{)6}$.

Rename so that you can divide by a whole number. Write the new problem. Then compute.

1. **a** $0.4\overline{)0.8}$ **b** $0.4\overline{)8}$ **c** $0.3\overline{)0.6}$ **d** $0.3\overline{)0.06}$

2. **a** $0.3\overline{)60}$ **b** $0.3\overline{)6}$ **c** $0.4\overline{)80}$ **d** $0.4\overline{)800}$

What can you do here? $0.14\overline{)0.28}$

What number times 0.14 is equal to 14?

$10 \times 0.14 = 1.40$ No!

What about $100 \times 0.14 = 14.00$? That's it! 14.00 names 14.

$0.14\overline{)0.28}$ **THINK** $100 \times 0.14 = 14$

$100 \times 0.28 = 28$

Now the problem is $14\overline{)28}$

What is the quotient?

Rename so that you can divide by a whole number.
Write the problem. Then compute.

	a	b	c	d
1.	$0.25\overline{)0.75}$	$0.25\overline{)7.5}$	$0.25\overline{)75}$	$0.25\overline{)0.075}$

You are on your own. Now you know how the model for whole-number division works. All you have to do is make sure you divide by a whole number.

	a	b	c	d
2.	$0.06\overline{)0.18}$	$0.3\overline{)0.9}$	$0.27\overline{)0.81}$	$0.5\overline{)0.85}$
3.	$0.2\overline{)1.4}$	$0.5\overline{)2.5}$	$0.7\overline{)4.2}$	$0.3\overline{)1.2}$
4.	$0.70\overline{)2.8}$	$0.15\overline{)3.00}$	$0.10\overline{)10}$	$0.1\overline{)1}$

5. Don't let this one fool you. Operate in the same way. $2.5\overline{)5}$

Someone discovered a way to write the action of multiplying by 10 or 100 or 1000. Rather than rewrite the problem, he used an arrow to show the placement of the decimal. It can help remind you about the decimal placement in the quotient, too.

$2.3\overline{)3.22}$

Usually you rewrite the problem.
Multiply both times 10.
Use an arrow instead of rewriting.

$2.3\overline{)3.22}$

$0.12\overline{)0.516}$

By what number were both these parts multiplied? Find the quotient.

PROGRESS CHECK

Compute the quotients. Check each quotient to see that your placement of the decimal point makes the answer reasonable.

(1.) $0.5\overline{)2.5}$ (2.) $0.7\overline{)0.49}$ (3.) $0.4\overline{)0.072}$

(4.) $0.6\overline{)13.8}$ (5.) $1.3\overline{)6.5}$ (6.) $3.2\overline{)86.4}$

*(7.) $0.23\overline{)3.68}$ *(8.) $0.35\overline{)0.0105}$ *(9.) $0.07\overline{)0.0063}$

*(10.) $1.35\overline{)3.510}$ *(11.) $2.65\overline{)9.805}$ *(12.) $0.03\overline{)0.0051}$

1

The digits for the quotients are given for each problem below, BUT there is no hint about the value of the digits. Estimate the quotients. Put the decimal point in the correct place in the quotients.

a $26.4 \div 6 = 44$ **b** $38.4 \div 3 = 128$ **c** $79.48 \div 4 = 1987$

d $19.2 \div 8 = 24$ **e** $1.33 \div 7 = 19$ **f** $40.08 \div 12 = 334$

g $3.164 \div 14 = 226$ **h** $485.1 \div 21 = 231$ **i** $74.88 \div 32 = 234$

2

In each example below, the decimal point is put in the quotient directly above the decimal point in the dividend. Are the quotients reasonable answers? Check to make sure the answers are correct.

a
$$\begin{array}{r} 7.5 \\ 0.3\overline{)2.25} \\ -21 \\ \hline 15 \\ -15 \\ \hline \end{array}$$

b
$$\begin{array}{r} 34.2 \\ 0.07\overline{)2.394} \\ -21 \\ \hline 29 \\ -28 \\ \hline 14 \\ -14 \\ \hline \end{array}$$

c
$$\begin{array}{r} 1.4 \\ 1.2\overline{)1.68} \\ -12 \\ \hline 48 \\ -48 \\ \hline \end{array}$$

You align decimal points when you subtract. Does it make sense to align decimal points in your division computation too?

3

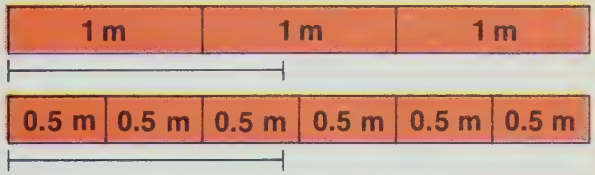
You are on your own. Try these.
Make sure the quotient is reasonable.

a $0.2\overline{)5.32}$ **b** $0.05\overline{)2.55}$ **c** $0.03\overline{)0.243}$

Any measurement you make is as accurate as your measuring device. Pretend you have a measuring stick 3 metres long. It has no markings except metres.

You could say the mark is more than 1 m and less than 2 m . Which is it closer to?

If 0.5 m were marked, you could say the mark is more than 1.0 m and less than 1.5 m . Which is it closer to?



If someone came along with a metrestick marked in centimetres, the mark could be measured to the nearest centimetre.

Someone else could come along with a metrestick marked in millimetres, and so on, and so on, and so on.

If you are computing measurements, you operate in a very special way.

1.530 m ← This is measured to the nearest millimetre.
+1.4 m ← This wasn't. If it had been measured to the nearest millimetre, it would be written 1.400 m . Does that mean you can't add?

You can add. But both measurements should be the same.
You can't improve the 1.4 m measure.
But you can round the 1.530 to tenths.

1.530 rounds to 1.5 m
+1.4 m
O.K. now!

This is the way you should work with measurements if you want to be accurate. When you are doing computation practice, you can annex zeros if you need to.

If you are doing computation practice, you are free to annex zeros to the right of any digit.

$$\begin{array}{rcl} 1.7 & \longrightarrow & 1.70 \\ +\,3.85 & \longrightarrow & +\,3.85 \end{array}$$ O.K.?

$$\begin{array}{rcl} 1.7\ \text{m} & \longrightarrow & 1.7\ \text{m} \\ +\,3.85\ \text{m} & \longrightarrow & +\,3.9\ \text{m} \end{array}$$ O.K.?

$$\begin{array}{rcl} 9.8 & \longrightarrow & 9.800 \\ -\,5.736 & \longrightarrow & -\,5.736 \end{array}$$ O.K.?

$$\begin{array}{rcl} 9.8\ \text{m} & \longrightarrow & 9.8\ \text{m} \\ -\,5.736\ \text{m} & \longrightarrow & -\,5.7\ \text{m} \end{array}$$ O.K.?

Decide if you should round before you compute. Then compute. Look out! Some of them are tricky.

1. He was to find out how much gold wire the goldsmith had in stock. One box was marked 10.55 cm . Another was marked 3.5 cm . The third box was marked 6.4 cm . About how much gold wire was in the three boxes?

2. The car odometer read 1782.5 km when she started her drive. It read 1925.0 km when she arrived. How far had she driven?

3. One label on a package of trim said 100 cm . Another package label said 1 m . She bought both of them. How much trim did she buy?

*4. Read this one carefully. A carton contains forty 25 g packages of spice. Another carton contains two 25 g packages of spice. How many kilograms of spice in all?

International travel

Travel between Canada and other countries grew at a slower rate in 1971. Both receipts and payments from international travel increased by less than 5%, compared with gains of close to 15% a year earlier, and receipts rose at a slightly higher rate than payments, reducing the deficit on travel account. The growth in travel in 1971 was dampened to a greater degree in North America than in Europe, where rapid growth rates were experienced for the second consecutive year. Travel earnings for both Canada and the United States advanced by less than 7%, while European countries registered substantial gains of about 20%.

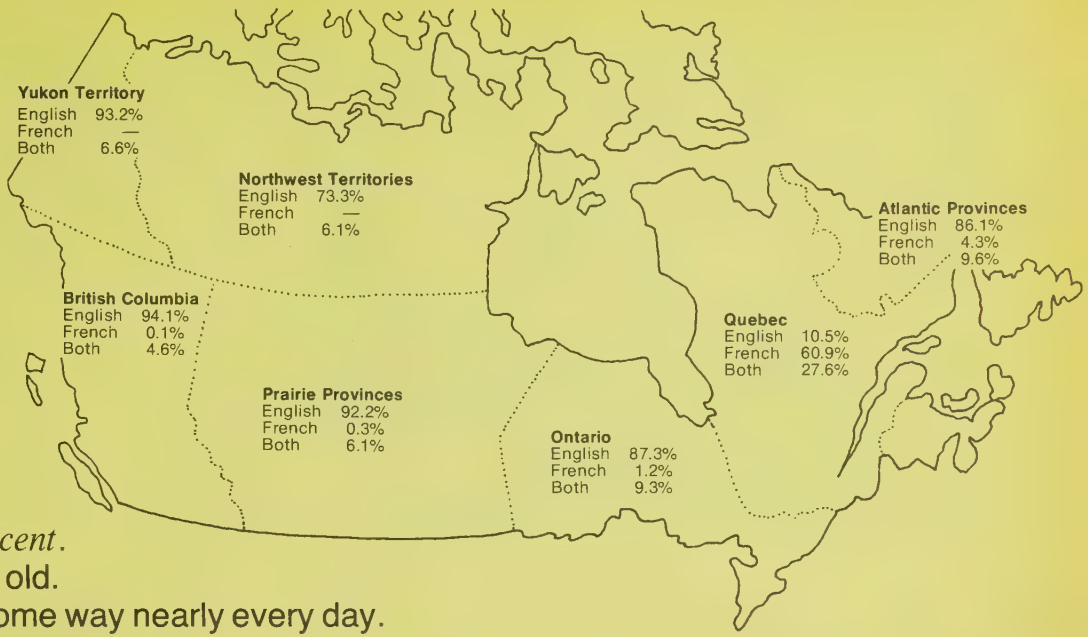
50% Off

Age distribu	
	19
Age group	
0-4 years	2 197
5-9 years	2 300
10-14 years	2 093
15-19 years	1 837
20-24 years	1 461
25-29 years	1 241
30-34 years	1 241
35-39 years	1 286
40-44 years	1 257
45-49 years	1 089
50-54 years	988
55-59 years	816
60-64 years	663
65-69 years	531

Employment

Over the 1966-71 period, the industrial composite index of employment for Canada rose by 5.9%. Among the industry divisions showing gains over this period, service led with a 34.1% advance, followed by finance, insurance, and real estate (21.3%), trade (15.0%), mining (7.4%), and transportation, communication, and other utilities (6.5%). Declines occurred in forestry (25.1%), construction (10.4%) and manufacturing (1.5%) during the same period. Compared with 1970, the industrial composite index for 1971 was up 0.6%.

Percentage distribution of the population speaking one or both of the official languages (preliminary figures for 1971).



% is a symbol for the word *percent*.
The idea is nearly 2000 years old.
We still see percent used in some way nearly every day.
Look at the pictures on this page.
List the different ways people use percent.

66, 1971.

1966	1971
%	%
11.0	8.4
11.5	10.4
10.5	10.7
9.2	9.8
7.3	8.8
6.2	7.3
6.2	6.1
6.4	5.9
6.3	5.9
5.4	5.7
4.9	4.9
4.1	4.4
3.3	3.6
2.7	2.9

Certificates and Savings Accounts

9.00%

new effective annual yield on

7.75% a year

their spaceships. They have reason: last year, Soviet output of goods and services rose less than 2%, the smallest gain in a decade. That contrasts with a 1972 rise of 9.7%, or 6.5% after subtracting price increases, in the U.S. gross nation- predictably, the So-

15% to 50% OFF
on a Select Group of men's Suits, Sport Coats and Slacks
20% OFF

The sign says you pay 10% less than the selling price.

But what does 10% mean?

The word *percent* comes from Latin. It means "in each hundred." Percent is really another way to write a decimal for hundredths.

10% is the same as 0.10 $\left(\frac{10}{100}\right)$.

Suppose you went to the sale.
You bought a book marked $1.00.
10% of $1 is $0.10.
You would get the book for $1.00 − $0.10, or $0.90.

No matter what the price was, you would get 10 cents taken off every dollar you spend.

Time to look at other percents. Remember percent means hundredths.

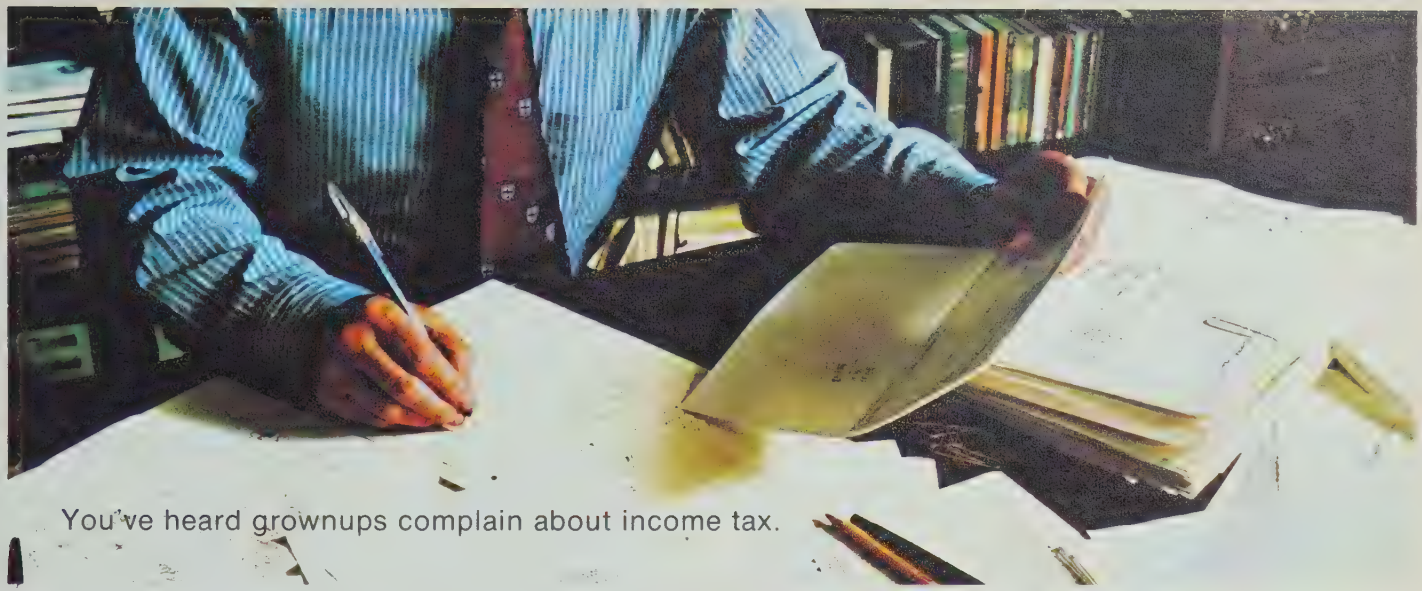
5% is the same as 0.05 $\left(\frac{5}{100}\right)$.

That's an important digit! Why?

37% is the same as 0.37 $\left(\frac{37}{100}\right)$.

Write each of the following percents as a decimal.

	a	b	c	d	e	f	g	h
1.	15%	80%	40%	35%	12%	98%	30%	4%
2.	25%	10%	9%	1%	5%	3%	15%	99%



You've heard grownups complain about income tax.

1. What is an income tax?
   - **a** Who has to pay income tax?
   - **b** What is the tax money used for?

2. Some provinces have their own income tax. Does yours? Find out.

3. Most places have a sales tax. Do you pay any money in tax when you buy something for 10¢? for 25¢? for $1.00? If you pay a sales tax, find out how much.

4. If you sell something, do you charge tax?

5. Lorna lives in a place where they have a 5% sales tax. She must pay $0.05 every time she buys something that costs $1. How much tax would she pay if she bought something for $2? $3? $5? $10?

6. Do you think Lorna would pay a tax on something that costs less than $1? Would she pay $0.05 or more? Why?

Do some research. Interview an adult. Find out what kind of taxes that person pays.

1. Suppose there was only one question on a test. Your score was 100%. How many did you get right? How many wrong?

2. There were 10 questions on a test. Your score was 100%. How many right? How many wrong?

3. There were 100 questions on a test. Your score was 100%. How many right? How many wrong?

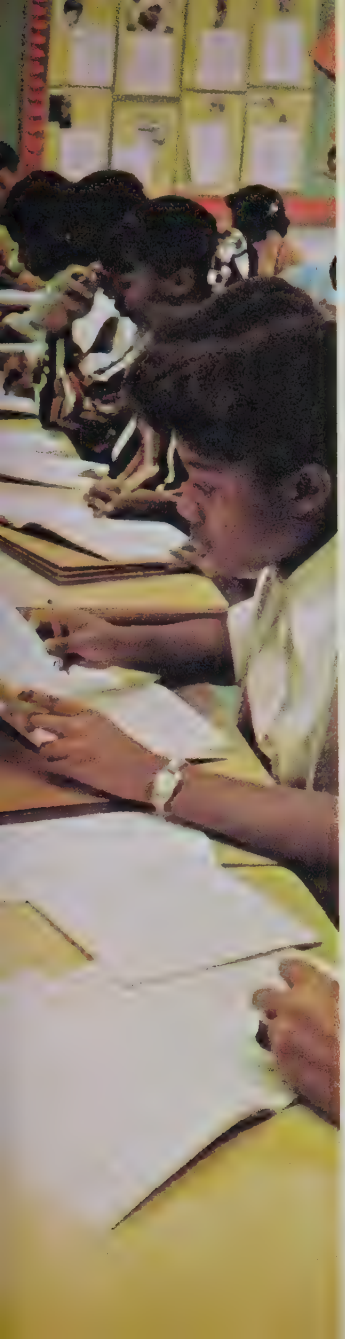
4. There were 100 questions on the test. Look at these scores. Tell how many questions each person got right and how many each got wrong.

Allan 78%	Betty 95%	Carol 77%
Darcey 82%	Ellen 88%	Frank 90%
Gene 98%	Hank 80%	Irma 100%
John 92%	Karen 85%	Lee 99%
Mary 75%	Nancy 94%	Owen 82%

5. Percent is used in other ways. Think carefully. What information is missing?

   a Ron said he got 60% in his history test. How many questions did he get right?

   b Sally said she got a 10% increase in her salary. How much salary does she get now?



Percent is a good way to describe situations. But you can't compute with percent. You must rename the percent as a decimal. Then compute.

Sylvia said, "I got 30 words right in my test last week.
In this week's spelling test I got 20% more."

How many words did she get right this week?

THINK 30 last week. What's 20% more? Rename as a decimal and multiply.

$\times$ 0.20

6.00 That's 6 more words.
30 words plus 6 more.
She had 36 words right this week.

Elma said, "I only had 20 words right last week.
But I improved by 40% this week."

How many words did she get right this week?

THINK 20 last week. What's 40% more? Rename and multiply.

$\times$ 0.40

8.00 That's 8 more. You finish the problem.

1. Sara got a 15% raise. She was earning $100 a week. How much will she earn after the raise?

2. Dave had 500 bricks for the wall. He needed 50% more. How many bricks did he need in all?

3. A year ago his mass was 70 kg . It has increased by 20%. What is his mass now?

CHECKOUT

1. Add.

a $0.2 + 0.7$ **b** $0.26 + 0.56$ **c** $0.75 + 0.25$ **d** $1.2 + 0.8$ **e** $0.9 + 0.1$

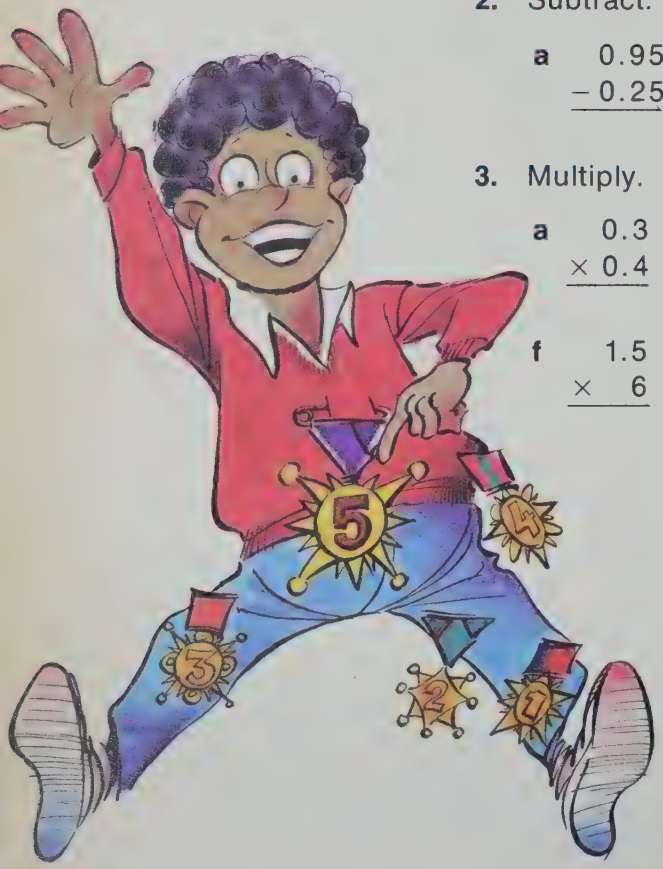
2. Subtract.

a $0.95 - 0.25$ **b** $0.36 - 0.09$ **c** $0.5 - 0.12$ **d** $1.05 - 0.76$ **e** $3.00 - 0.48$

3. Multiply.

a 0.3×0.4 **b** 0.2×0.4 **c** 0.04×5 **d** 0.03×3 **e** 0.67×8

f 1.5×6 **g** 2.8×0.2 **h** 0.23×0.7 **i** 0.23×0.3 **j** 0.05×0.5



4. Writing the correct digits does not mean you have the correct answer. Your answer must also have the correct place value. Multiply again. The emphasis is on place value here.

a $1 \times 1 = ?$ **b** $0.1 \times 0.1 = ?$

c $1 \times 0.1 = ?$ **d** $0.1 \times 0.01 = ?$

e $1 \times 0.01 = ?$ **f** $0.1 \times 0.001 = ?$

g $1 \times 0.001 = ?$

Would you believe place value is important?

5. Divide.

a $4\overline{)3}$ **b** $5\overline{)9}$ **c** $0.4\overline{)1.2}$ **d** $1.2\overline{)6}$ **e** $0.5\overline{)75}$

6

AREA AND PERIMETER

Area is a common word.
It is used in many ways.
Your goal is to learn how
it is used in mathematics.
As a bonus you'll get to
know about perimeter, too.

Bill explained that his community was 24 square blocks. Was this a complete description of the size of his community?

Assume that it had the shape of a rectangle. Use graph paper and show the different rectangular shapes it might have. (There can be more than one.)

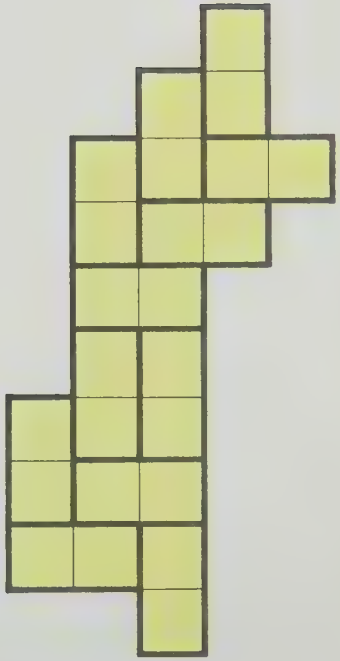
Suppose that you are going to walk the outside boundaries of each of the shapes. How many blocks would you walk on the first rectangular shape? How many blocks on the second? How many blocks on each of the other rectangular shapes that you have drawn?

Bill didn't say that his community was shaped like a rectangle, did he? What are the other shapes it could have? It must have 24 square blocks, remember. (There are a lot more shapes than one now.)

1. Did you remember something about factors and multiples when you tackled the problem on other shapes for Bill's community?

2. If you didn't, take another look at your work. Did you put Bill's community together with square shapes that are multiples of 2? multiples of 3? of 4? of 6? of 8? of 12?

3. For example, one shape made up of two square-block units might look like this:



4. Are there other shapes that contain 24 square blocks?
Keep on building, using multiples of two square blocks.

5. Walk the outside boundaries of this shape. How many blocks did you walk? How many square blocks are there inside the boundary?

6. Now it's your turn. Prove that you can build another shape made up of two square-block units. Remember—you need a total of 24 square blocks.

7. Walk around each of the shapes you have constructed. How far is it around the outside boundary of each shape?

The amount of surface is called *area*.
The distance around the boundary is called *perimeter*.

1. It appears that two shapes can have the same area but different perimeters. Can you have two shapes with the same perimeter but different areas?

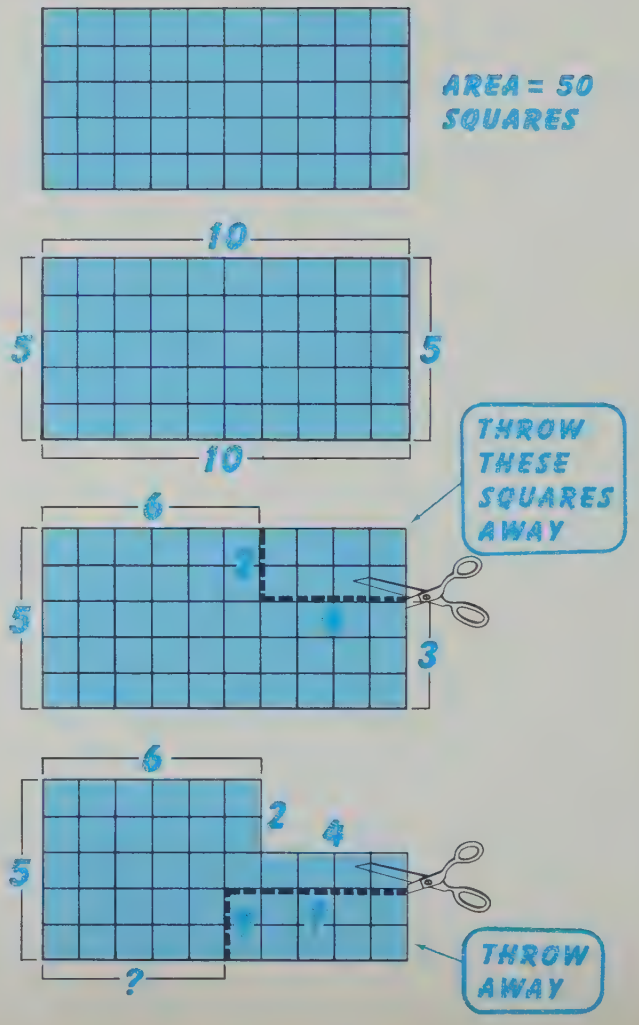
2. You'll need graph paper and scissors for this one. Start with a rectangular piece of paper 5 squares wide and 10 squares long.

   a What is the number of squares contained in this shape?

   5 rows of 10, or 50

   b What is the number of squares around?

   10 + 5 + 10 + 5, or 30



3. Now cut away part of the rectangle.

   a Now how many squares for the area?

   b What is the number of squares around?

4. Cut some more squares away.

   a Now how many squares for the area?

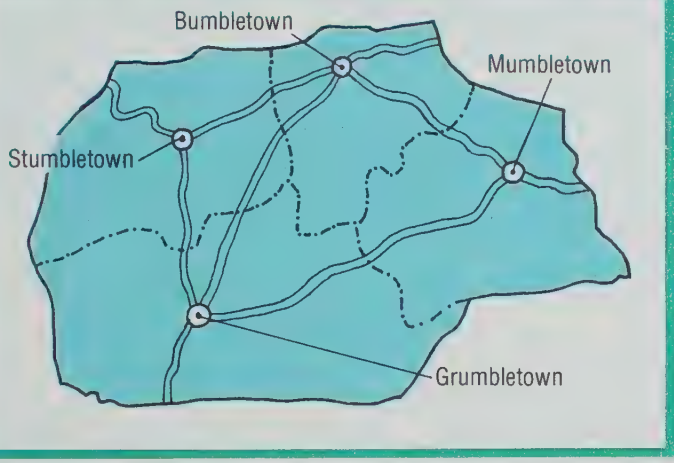
   b What is the perimeter?

5. How small can you make the area and keep the perimeter at 30 units?

The idea of a perimeter is used in many situations. You don't have to build a fence before you have a perimeter. You don't have to walk around some city blocks. You might plan a trip and have the path of that trip be "the distance around."

Mr. Trumble lives in Bumbletown. He wants to visit Mumbletown, Grumbletown, and Stumbletown on his next trip. His time is short. He does not want to travel more than 1000 km round trip. He may not get to all the cities he wants if he holds to this rule.

Mr. Trumble checks his road map to find the distances.



	Bumbletown	Mumbletown	Grumbletown	Stumbletown
Bumbletown	0	306	459	263
Mumbletown	306	0	569	494
Grumbletown	459	569	0	251
Stumbletown	263	494	251	0

Trace the map. Plan Mr. Trumble's trip for him. How many cities can he visit? Does he have a choice? Which cities can he visit? What path should he take? How many kilometres does he travel in all? How many kilometres less than 1000 is his journey?

1. George Greenbean owns a large farm. One day a farmhand tells George that his fence in the northwest corner (point B) needs fixing. George and the farmhand drive from the farmhouse to the broken fence and fix it. They drive on the dirt roads by the fence line. They take the shortest route.

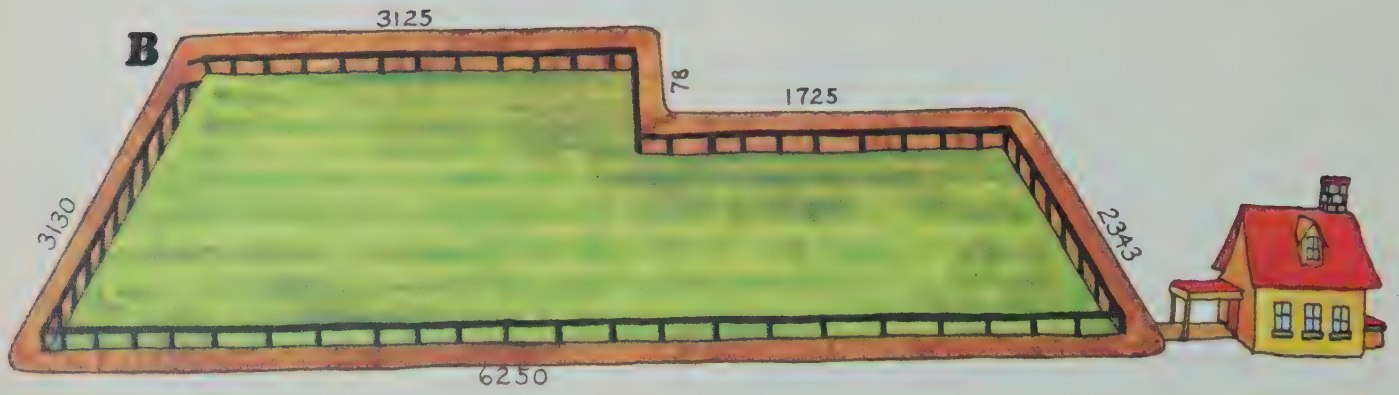
a Which route appears to be shorter?

b How many metres did they drive to point B?

c How many metres shorter is this path than the other path to point B?

d If they drive back the way they came, how many metres will they drive in all?

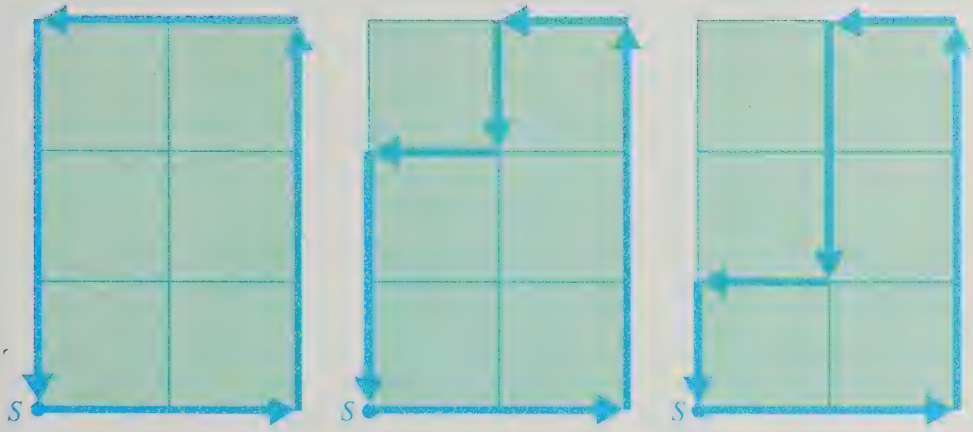
e If they drive back the other way, how many metres will they drive in all?



2. Finding the answers to the questions above meant that you had to know how to add. Check your addition skills with these problems. Work for accuracy and speed.

	a	b	c	d	e
SET 1	2746 +3251	3104 +5087	5658 +3274	2546 +7454	2556 +1765
SET 2	121 32 + 43	256 310 + 25	62 158 +5213	225 132 371 + 50	267 113 1506 + 251

Tom, Dick, and Harry decided to have a footrace around six square blocks. They started at point S. Tom got ahead right away. Dick wanted to win, so he decided to cheat. Poor Harry was way behind. He saw Dick cheat, so he decided to cheat more. Tom still won.



Tom's path Dick's path Harry's path

Each of the paths is different. Each path is a perimeter. Each path goes around a different number of squares. A polygon is formed by each perimeter. The amount of surface inside the polygon is an area.

1. Each side of each square in the picture represents 100 m .

a How far did Tom run?

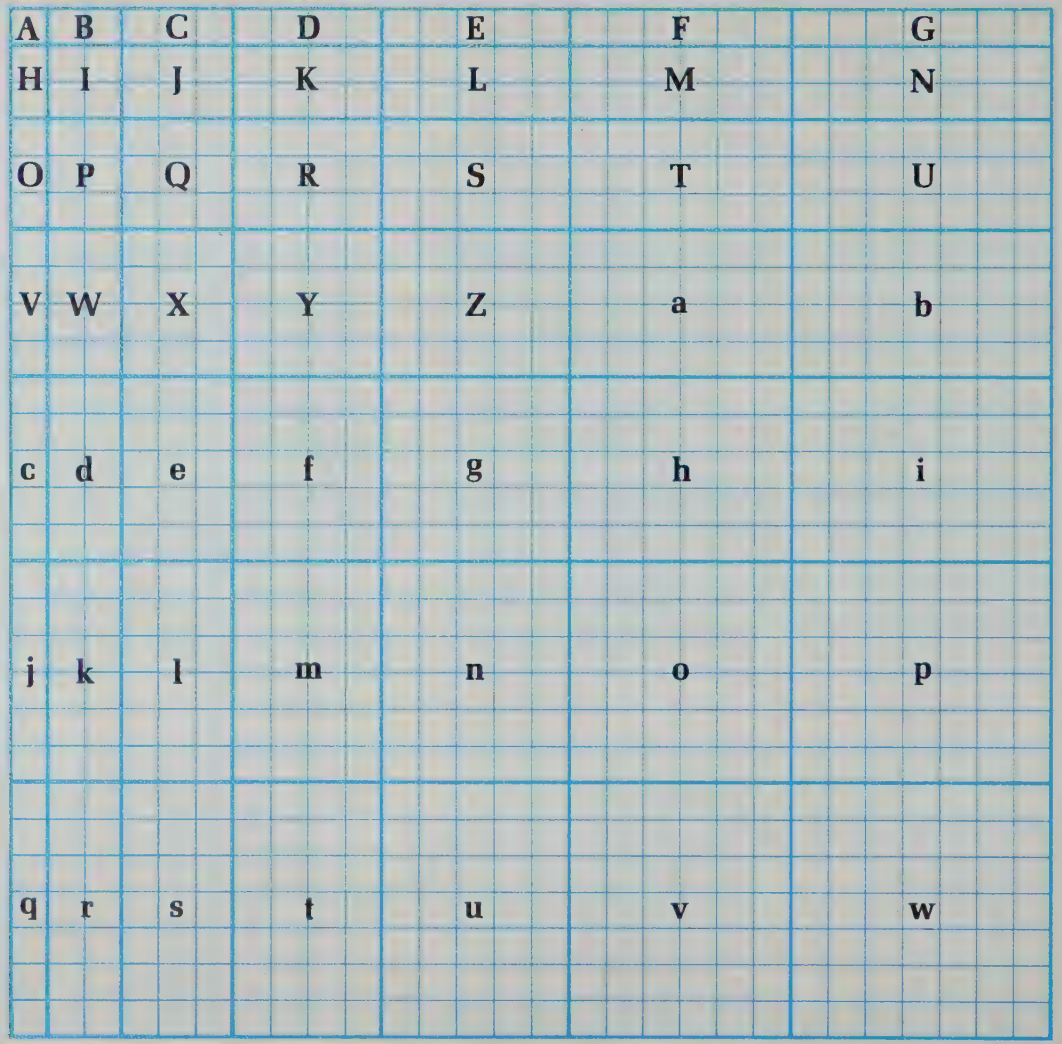
b How far did Dick run?

c How far did Harry run?

d Did Tom, Dick, or Harry run around the greatest number of 100 m squares?

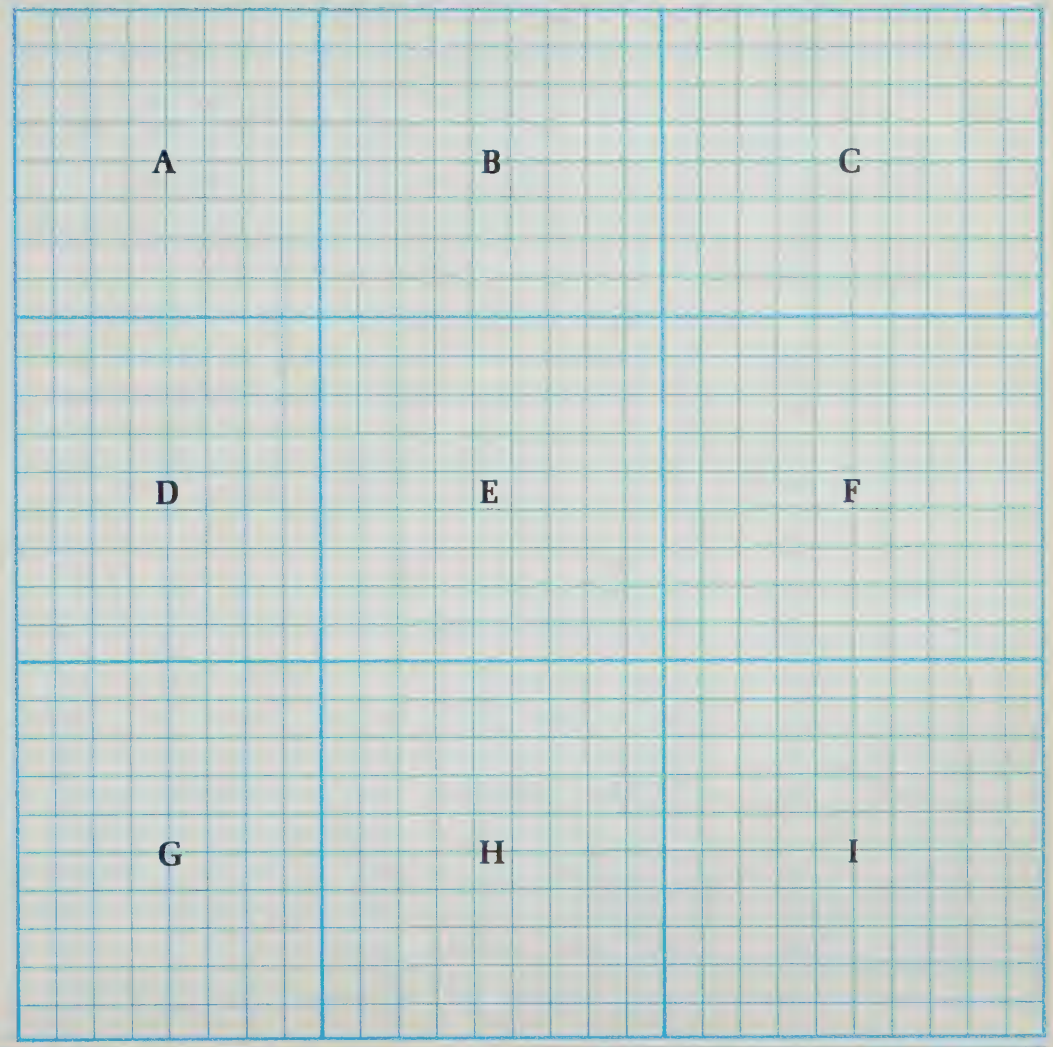
1. Find the area of each rectangular block. If you discover a pattern, keep it a secret.

2. Write the pairs of rectangles that have the same area.



1. Look at the nine rectangular regions on this page. Are any the same size? Which ones?

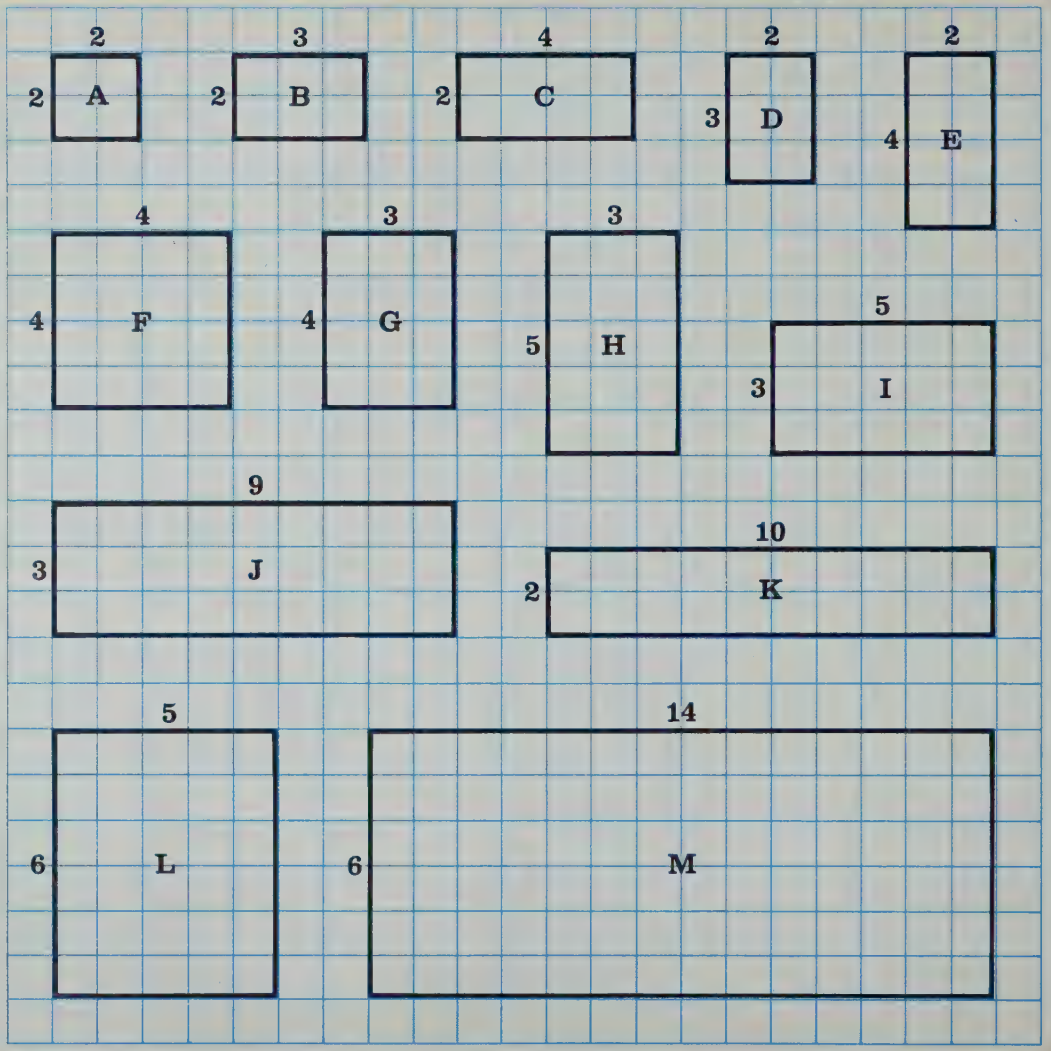
2. What is the area of the total set of nine rectangles?



To find the perimeter of any figure, you add the lengths of the sides.

Area is always in square units of measure. To find the area, you can count the number of squares a figure covers. OR maybe you have found a better way to find the area than counting.

Find the perimeter and the area of each figure. You name the unit of measure.



1. You must know how to multiply if you compute area. How are your multiplication skills? Do these problems. Work for accuracy and speed.

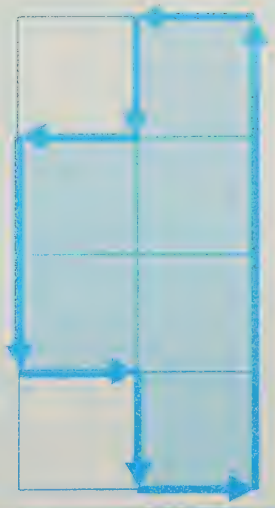
	a	b	c	d	e
SET 1	57 × 8	79 × 6	86 × 7	78 × 9	96 × 4
SET 2	40 × 75	19 × 18	36 × 36	39 × 95	62 × 58
SET 3	647 × 481	518 × 926	3967 × 207	2740 × 509	3958 × 471

2. Mr. Jones bought 60 m of fence. Use graph paper and show at least five different rectangular pieces of land that he could enclose with that length of fence.

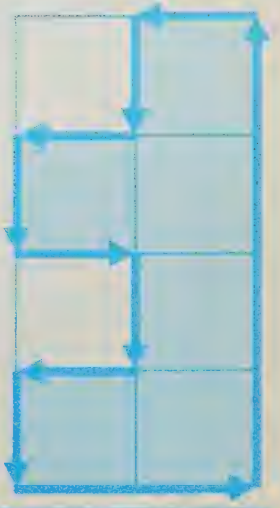
3. Name the perimeter and the area of each of the figures.

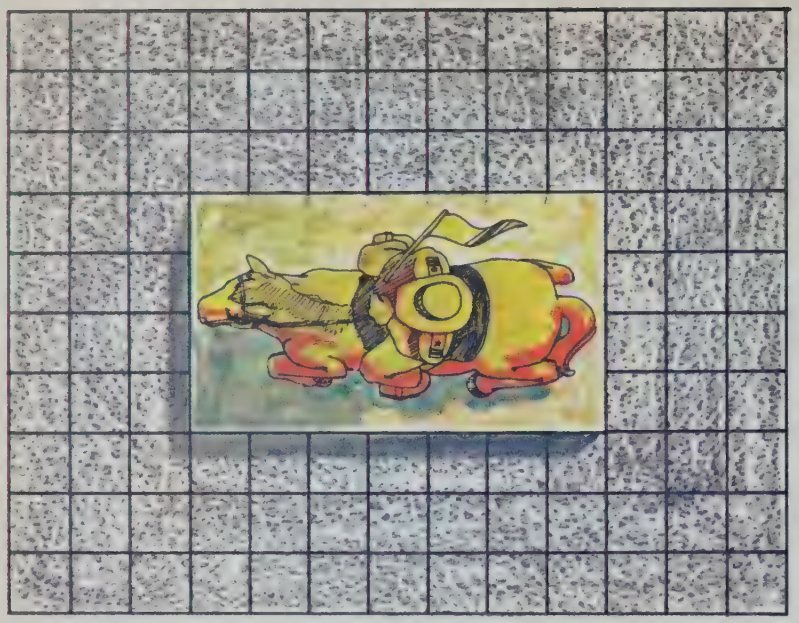
a

b



c





There is a long way and several short ways to do the next problem. Counting is the long way. That way will get an answer. It's O.K. But try to find one of the short ways.

The people of the town had wanted a statue for many years. They finally got one. They paved the area around the statue and placed benches for people to sit and admire it. The paved area was 3 m wide around the statue. The base of the statue was 4 m by 7 m .

This picture shows the shape of the paved area.

1

How many square metres of paved area did the town need?

*2

If the paved area cost $2.50 a square metre, how much did it cost in all?

Today you are a problem solver. Help Mr. Smith.

Mr. Smith had a very big garden. And he had a dog. The dog needed an outdoor pen. Mr. Smith thought the dog needed at least 36 square metres (36 m^2).

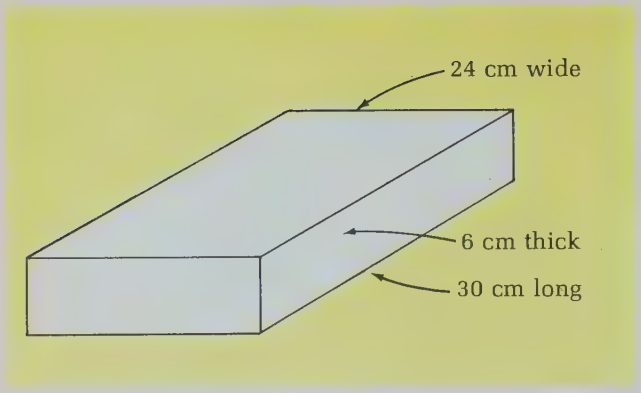
1. Use graph paper and show at least two possible ways he could build the pen.

Then tell how many metres of fencing Mr. Smith needs for the pen.

Mr. Smith can buy a ready-made pen measuring 4 m by 9 m for $64.99.
Or he can buy fencing and build the pen himself. Fencing costs 60¢/m .
The four posts he needs cost $5.09 each.
He will need a gate too.
(He can't lift the dog into the pen.)
The gate comes complete with posts and all hardware. The gate costs $12.95.

2. How can he get the pen for the least amount of money?

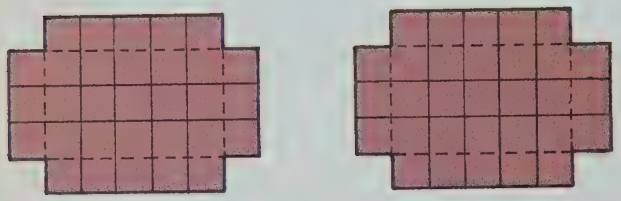
3. Should Mr. Smith's time be considered as part of the cost?



Today you are a boxmaker.

Your job is to find out the size and shape the box you are going to make should be. You have to save cardboard, of course. Extra cardboard costs money. You plan by making models. You start with a flat pattern. Then you cut it out and make the box.

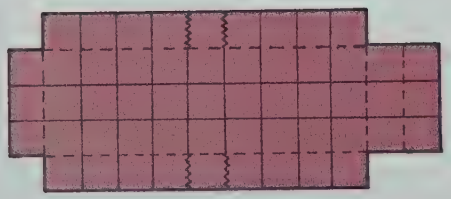
Your first customer comes to you with a book. He plans to mail 10 000 of the books. He needs a box to put each one in.



One box you might pick has a lid and a base.
The box is cut from flat cardboard.
The pattern looks like this.

Let one square show about 6 cm .

1. The lid would need to be a little larger. Why?



A one-piece, fold-up box pattern would look like this.

~~~~ shows cut marks

- - - - - - shows fold marks

2. Which box uses less cardboard?

3. Which box would you suggest using for the books? Why?
~~~~

You know one pattern for a square box looks like this when it is laid out flat. This kind of box would be taped together.

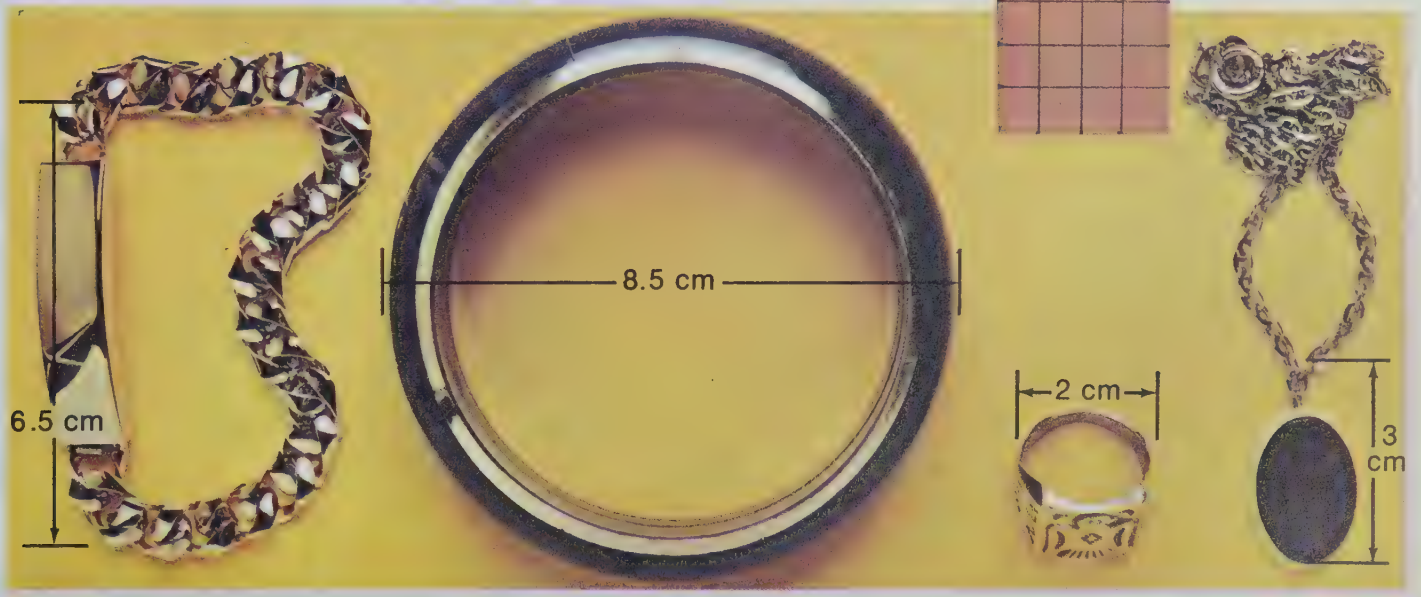
1 square represents 1 square centimetre ($1\ cm^2$)

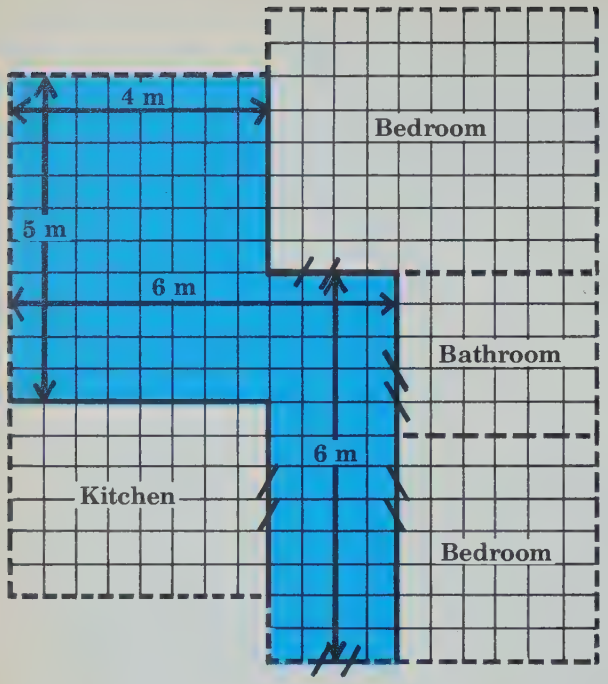
This could hold something round like a ball.

It could be used for packing a fancy cup you are giving as a gift.

It could hold another box.

Use graph paper. Let each square be 1 cm long and 1 cm wide. Plan a box for the following objects. Show what each box would look like before it was folded together. Make the box actual size.





Today you are a rug salesman.

1. A man comes in with this floor plan. He says he wants to buy carpeting for the living room and the hallway (the shaded part). He says this is an accurate drawing.

You really want to sell him the carpeting.

a Can you figure out how many square metres of carpet he needs?

b Do you suppose you can talk him into buying some for the big bedroom, too? How many square metres in the big bedroom?

The man decides he wants to put down carpet squares. They measure 25 cm^2 . He could do that himself. You have three different prices for carpet squares:

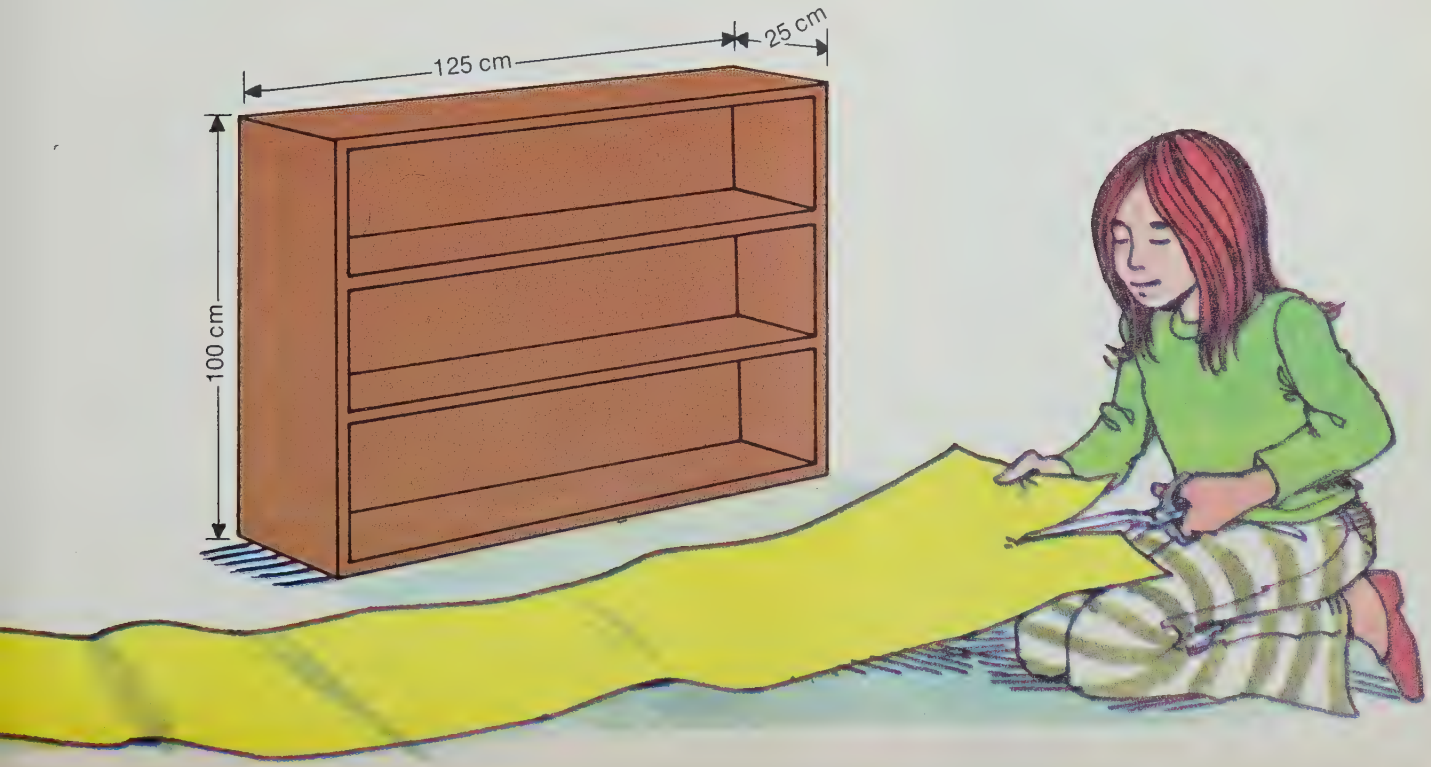
Solid color, flat texture	38¢ each
Two-tone, textured	59¢ each
Thick shag pile	75¢ each

He wants you to figure all three costs. Remember — it's your job.

You have an old bookcase. You think it would look great if you covered it with self-stick paper. You have to figure out how much paper it would take.

You have to cover the top and sides. You have to put paper on the inside and on the shelves too. About how much paper 50 cm wide will you need to buy?

You have only $5. The paper cost 75¢/m .
Do you need more money? How much more?



Today you are a real estate agent.

You buy and sell land and buildings. This is a very busy day. You hope to sell a farm to one person. You are going to show another person some land in town where she could build a house.

Land in the country is sold by the hectare. A hectare is a square unit of land measure. To give you an idea of how big it is, think of the size of the field inside an athletic track. That's a lot of land!
The farm you want to sell is 120 hectares (ha).

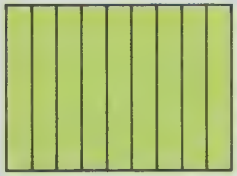
1. The land costs $920/ha . How much will the farmland cost? When you tell the man this, he says he wants time to think it over.
2. Use squared paper. Let 1 square represent 5 ha . (24 squares will represent all of the land.) What might be the shape of the land in the farm?
3. The land in town is 30 m across the front. It is 40 m in depth. How many square metres of land is this? Show the shape of the land on squared paper. Let the side of one square represent 5 m .
4. The woman wants to build a house that has 300 square metres of living space. Does she have room to build that big a house? Prove your answer on your graph paper. Show the area of the land and where the house could be built. You sell the land!

The skills of finding area and perimeter are used in many situations. There are no right answers to the next questions. There are reasonable answers. You have to use some good thinking to come up with any kind of answer.

1. Wood moulding is sold by the metre. You want to make a new frame for a window. The window measures 115 cm by 175 cm . About how many metres of moulding should you buy?

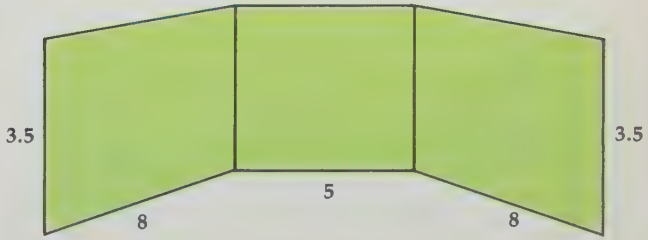
Think

2. You want to put wall paper on one wall. The paper is 50 cm wide. There is 12.5 m in a roll. The wall you want to cover is 3.5 m high and 5 m wide. How many rolls of paper do you have to buy?



Think

3. It says on the paint tin that its contents, 4 ℓ, will cover 50 m^2. You want to paint the other three walls in the room. It is a rectangular room. You know one of the walls is 3.5 m by 5 m . Each of the other two walls is 3.5 m by 8 m . Will 4 ℓ of paint be enough?



Think

4. Do you have to know area or perimeter to buy enough fabric to cover a chair?

TODAY YOU ARE A STUDENT.

You will need graph paper again.

Your job is to take five squares.

Fit the five squares into twelve different patterns.

At least one side of each square must match another side.

Each side touches at least one other side.

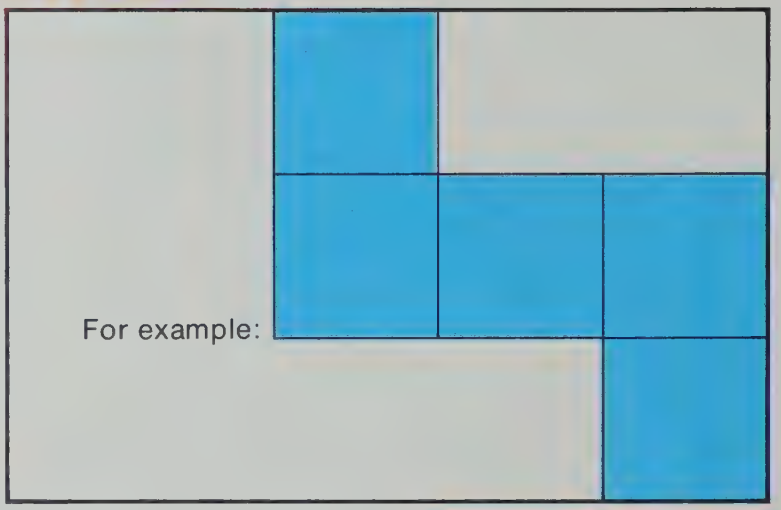
After you have found your twelve different patterns, cut each out.

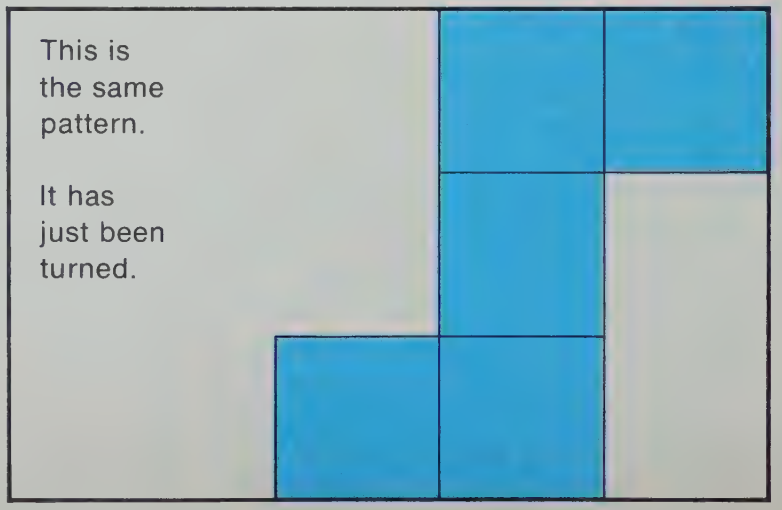
Make a rectangular puzzle board from graph paper. One side should be 6 squares high. The other side should be 10 squares wide. 60 squares in all.

Now all you have to do is fit your twelve pieces onto the playing board.

GOOD LUCK!

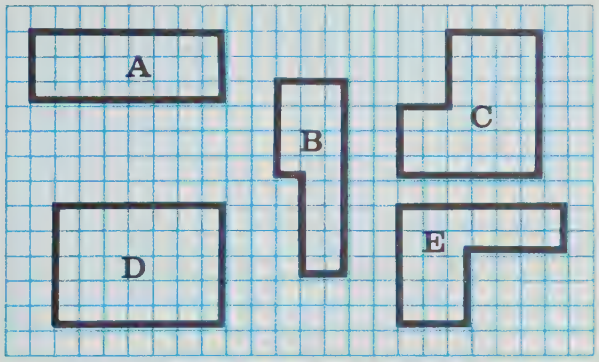
This is HARD!





CHECKOUT

1. Pretend the pictures below show garden plots. Pretend the side of each small square equals 1 m . You want to fence one plot. Give the letter name of the plots that could be enclosed with 24 m of fencing.



2. Now pretend that the pictures above show parts of a city park that need grass seed. You have enough to cover 24 m^2. Give the letter name of the parts that could be sown with seed.

Add.

	a	b	c	d	e
3.	27 +64	354 + 23	426 +152	577 +128	675 +238

Multiply.

4.	37 × 4	53 ×70	67 ×23	123 ×213	687 ×254

FRACTIONS

There are times when you must operate with fractions, whether you like it or not. They really aren't that hard. Your goal is to check out your skills in multiplication with fractions.

Where did the idea of fractions come from? Symbols for fractional parts were in use in Egypt a long time ago. The Egyptians already had numbers that told "how many." But they also needed numbers that told about parts of a whole or a unit. They had to keep records, so they also made up symbols for fractions.

This is how the Egyptians wrote $\frac{1}{4}$:

←This symbol meant "a part."

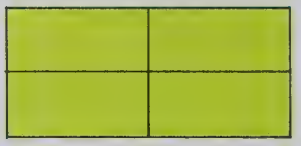
←These marks represent the number 4.

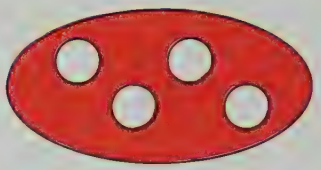
Our symbols for fractions aren't too much different.

the numerator is a number → 1 ← it tells how many parts you're thinking of

the denominator is a number → 4 ← it tells the number of parts in a unit

The number line, the region, and the set
are the models that can help us think fractions.





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1. We use fractions to describe real situations, too.
What fractional part is—

a 3 months out of 12 months?

b 30 minutes out of 60 minutes?

c 3 out of 4 questions wrong on a test?

d 10 cents out of 1 dollar?

e 25 cents out of 50 cents?

f 1 out of 3 parts of a pie?

NOW LOOK OUT FOR THESE.

g 1 month out of 1 year?

h 1 day out of 1 week?

i 1 hour out of 1 day?

j 5 cents out of 1 dollar?

k 6 hours out of 1 day?

l 1 minute out of 1 hour?

m 1 year out of an 80-year-old man's life?

n 1 year out of the years you have lived?

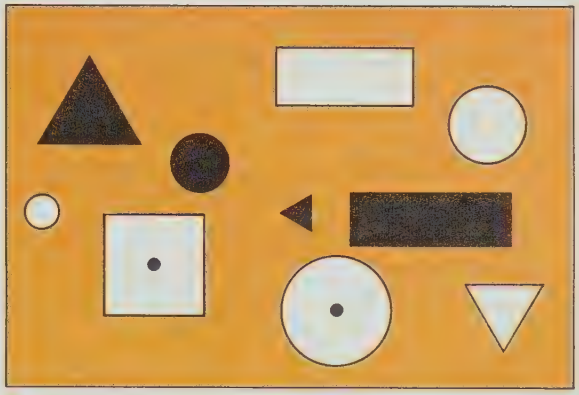
1. How many equal segments from 0 to 3?

a This number line shows three units. Each unit is divided into equal-size parts. Point B could be named a fraction. What fraction?

b What fraction does point C name? point D? point E?

2. Draw a number line to show the fraction $\frac{7}{4}$.

3. The next questions refer to the set of shapes on the right. Name the fractions.



a What part of the set is triangles?

b What part of the set is rectangles?

c What part of the set is shaded figures?

d What part of the set is figures with dots?

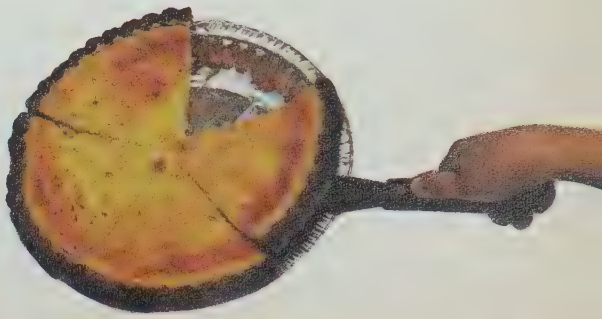
e What part of the set is circles?

f What part of the set is figures with straight sides?

g What part of the set is hexagons?

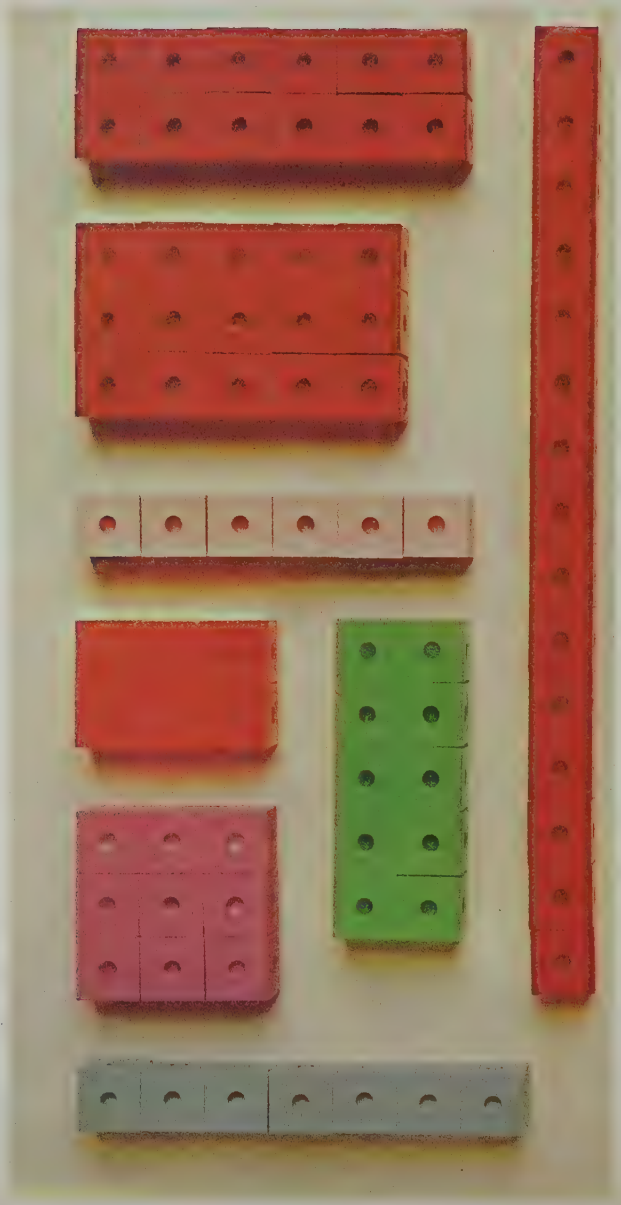
4. Which would you rather have—
$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, or $\frac{5}{10}$ of a dollar?

5. Bill looked at the place where the piece of pie had been. He said, "I would like to have had 6 times more than the piece I had." How much pie would Bill like to have had in all?



6. His sister said she would be satisfied with $\frac{1}{2}$ of the pie that was left. How much did she want?

Pretend that you just invented a lotion that grows hair. You decide to sell your wonderful lotion. You plan to ship it in boxes of 12. You know that you will put only one layer of bottles in each box, and you'll have to have dividers so that the bottles won't break. How many differently shaped boxes could you use?



1. Pretend you decided not to sell boxes of 12. Describe the boxes (how many rows, how many in each row) you might use to ship a total of—

a	6	**b**	4	**c**	14
d	2	**e**	9	**f**	3
g	27	**h**	15	**i**	5
j	20	**k**	16	**l**	18
m	7	**n**	10	**o**	1

We can call the *total number* in each box a *product*. To get a product, we multiply the number of rows times the number in each row.
The numbers we multiply are called factors.

p Can any of the products named above be named by other factors? Which ones? What are the other pairs of factors that can name each product?

Every counting number can be named as the product of at least two factors.

1. Which numbers between 1 and 18 can be named by only *one* pair of factors?

2. The number 1 can be used as a factor for any counting number. A number that can be named only by the factors 1 and another number is so special that it has a special name. Such numbers are called prime numbers. You found the prime numbers between 1 and 18 in the first problem. Do you suppose there are prime numbers greater than 18? What about 19?

   $2 \times \underline{?} = 19$ No.
   $3 \times \underline{?} = 19$ No.
   $5 \times \underline{?} = 19$ No.
   Why wasn't 4 as a factor investigated?
   Why wasn't 6 as a factor investigated?
   $7 \times \underline{?} = 19$ No.
   Why can the questions stop with the factor 7?

   Is 19 a prime number?

3. Is 20 a prime number? 21? 22? 23? 29?

4. Is 31 a prime number? 33? 37? 39?

5. Some numbers can be expressed as a product of three or more factors.

   For example: $8 = 2 \times 2 \times 2$

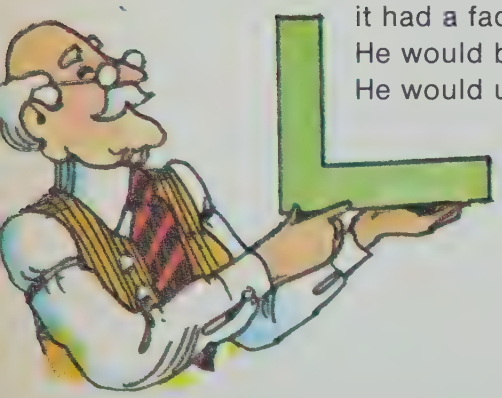
   a Express 12 as a product of three factors. Is each of the factors a prime number?

   b Express 18 as a product of three factors. Is each of the factors a prime number?

If you were very lucky, you would have a great-great-grandfather around. And if you were even luckier, he would remember his days in school when he was about your age. Factoring a number was almost a game. Everyone had learned the prime numbers. It became a race to see who could factor a large number the fastest.

Our work will not deal with large numbers. But maybe it can be almost like a game. The primes you'll need to remember are 2, 3, 5, and 7.
(If you were going to work with numbers greater than 100, you'd need more.)

Your great-great-grandfather would have factored this way: He would pick a number—60, for example. He would see it was an even number. He would know it had a factor of 2. He would start taking the prime factors "out" of 60.
He would begin with 2. The factors would be taken out through division.
He would use a special symbol for this special division.



$2\overline{)60}$	⌊ is an upside-down division symbol.
$2\overline{)30}$	He would take "out" all the 2-factors.
$3\overline{)15}$	Then he'd take "out" the 3-factors.
$5\overline{)5}$	No more 3s. But there's a 5-factor.
1	His job would be finished.

Check his method.
Does $2 \times 2 \times 3 \times 5 = 60$?

Here is the factoring of 28:

$2\overline{)28}$
$2\overline{)14}$ (No 3s. No 5s.)
$7\overline{)7}$
1

Does $2 \times 2 \times 7 = 28$?

Try to factor 11.

$?\,|\,11$ No 2-factors. No 3-factors.
No 5-factors. No 7-factors.
11 must be a prime number!

Factor 50.

$2\,|\,50$
$?\,|\,25$ No more 2-factors. No 3-factors.
$?\,|\,?$ What's next?
1

1. Factor 72.

$2\,|\,72$
$2\,|\,36$
$?\,|\,18$
$?\,|\,?$
$?\,|\,?$
$?$

2. Factor 100.

$2\,|\,100$
$?\,|\,50$
$?\,|\,25$
$?\,|\,?$
$?$

You can imagine the kind of race there might have been if the number to be factored were as large as 395.

3. How many prime factors in 8?

4. How many prime factors in 12?

5. How many prime factors in 18?

6. How many prime factors in 15?

7. How many prime factors in 22?

WATCH OUT!

***8.** What are ALL the factors of 24?

1. Now think about ALL possible factors of a number. Copy and complete the list.

2. Use the list. What are the factors that are common to 4 and 8?
Which is the largest factor common to 4 and 8?
This number is called the *greatest common factor*.

3. What are the common factors of 10 and 20?
Name the greatest common factor of 10 and 20.

4. Name the greatest common factor of each of these pairs of numbers. Use your list for help.

   a 5 and 10
   b 3 and 12
   c 4 and 16
   d 6 and 12
   e 2 and 8
   f 5 and 15
   g 6 and 18
   h 9 and 15
   i 10 and 18
   j 12 and 18

1. The factors of 12 are 1, 2, 3, 4, 6, 12.
The factors of 15 are 1, 3, 5, 15.
What factors are common to both numbers?

2. What are the common factors of 12 and 16?

12 1, 2, 3, 4, 6, 12

16 1, 2, 4, 8, 16

What is the greatest common factor of 12 and 16?

3. What is the greatest common factor for each pair of numbers? The list on the right will help.

a 14 and 18 **b** 8 and 10
c 20 and 10 **d** 8 and 20

4. What is the greatest common factor for the numerator and denominator of the fraction $\frac{8}{20}$? Can both the numerator and the denominator be divided by the same number? What is the greatest common factor of 8 and 20?

$$\frac{8}{20} = \frac{8 \div 2}{20 \div 2} = \frac{4}{10}$$

common factor

$$\frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \frac{2}{5}$$

greatest common factor

Which factor results in the simplest fraction name for the number?

FACTORS

8 ➡ 1, 2, 4, 8

10 ➡ 1, 2, 5, 10

14 ➡ 1, 2, 7, 14

18 ➡ 1, 2, 3, 6, 9, 18

20 ➡ 1, 2, 4, 5, 10, 20

1. Complete each set using a common factor and the greatest common factor as indicated.

	A common factor	Greatest common factor
a	$\frac{12}{18} = \frac{12 \div 2}{18 \div 2} = \frac{?}{?}$	$\frac{12}{18} = \frac{12 \div ?}{18 \div ?} = \frac{?}{?}$
b	$\frac{16}{24} = \frac{16 \div 2}{24 \div 2} = \frac{?}{?}$	$\frac{16}{24} = \frac{16 \div ?}{24 \div ?} = \frac{?}{?}$
c	$\frac{20}{24} = \frac{20 \div 2}{24 \div 2} = \frac{?}{?}$	$\frac{20}{24} = \frac{20 \div ?}{24 \div ?} = \frac{?}{?}$

2. Use the greatest common factor to find the simplest fractions.

a $\frac{15}{35}$ **b** $\frac{20}{60}$ **c** $\frac{16}{28}$ **d** $\frac{27}{36}$ **e** $\frac{48}{18}$ **f** $\frac{42}{36}$ **g** $\frac{25}{40}$ **h** $\frac{21}{35}$

3. Which of these fractions have a numerator and a denominator that have 1 as the greatest common factor?

a $\frac{3}{7}$ **b** $\frac{3}{8}$ **c** $\frac{4}{6}$ **d** $\frac{5}{12}$ **e** $\frac{2}{9}$ **f** $\frac{6}{12}$ **g** $\frac{3}{12}$ **h** $\frac{7}{11}$

4. Write six fractions using numbers in both the numerator and the denominator that have 1 as the greatest common factor.

5. Write six fractions using numbers in the numerator and the denominator that have a common factor greater than 1.

6. In 4 and 5 did you use prime numbers for both the numerator and denominator of any fraction? What is the greatest common factor of any two prime numbers?

1. Which of these fractions is in simplest form?

a $\frac{1}{2}$ **b** $\frac{7}{14}$ **c** $\frac{11}{12}$ **d** $\frac{22}{24}$ **e** $\frac{3}{4}$ **f** $\frac{6}{12}$

2. Does $\frac{1}{2}$ name the same fractional part as $\frac{7}{14}$?

a Does $\frac{11}{12}$ name the same fractional part as $\frac{22}{24}$?

b Does $\frac{3}{4}$ name the same fractional part as $\frac{6}{12}$?

3. If two or more fractions name the same fractional part, they are equivalent.

a Sometimes you need an equivalent fraction with a numerator and a denominator larger than those of the original fraction.

For example: $\frac{1}{2} + \frac{1}{4} = \underline{?}$

You need a common denominator.
You need an equivalent fraction.
The denominator must be 4.

Here's a "how to do it" review.

$\frac{1}{2} = \frac{?}{4}$ $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

Had you needed a denominator still larger, this is what you'd do.

$\frac{1}{2} = \frac{?}{12}$

Think ↑—2 times what factor = 12? $\frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12}$

b Sometimes you need an equivalent fraction with a numerator and a denominator smaller than those of the original.

For example: $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$ You need to rename the answer.
↑—You need an equivalent fraction.

Here's another "how to do it" review.

$\frac{4}{6} = \frac{?}{?}$

Think ↑—What is the common factor of 4 and 6? $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

When writing an equivalent fraction with a denominator greater than the one given, compare the denominators.

By what number has the denominator been multiplied?

$\frac{5}{6} = \frac{?}{12}$ $\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

6 times what number gives 12?

Take that number times the numerator.

1. Complete.

a $\frac{2}{5} = \frac{?}{10}$ **b** $\frac{3}{4} = \frac{?}{12}$ **c** $\frac{5}{10} = \frac{?}{100}$ **d** $\frac{4}{5} = \frac{?}{10}$

e $\frac{3}{5} = \frac{?}{20}$ **f** $\frac{2}{3} = \frac{?}{15}$ **g** $\frac{3}{4} = \frac{?}{8}$ **h** $\frac{5}{6} = \frac{?}{24}$

2. More practice. Complete.

a $\frac{1}{4} = \frac{?}{20}$ **b** $\frac{1}{2} = \frac{?}{10}$ **c** $\frac{2}{3} = \frac{?}{12}$ **d** $\frac{3}{8} = \frac{?}{24}$

e $\frac{4}{7} = \frac{?}{14}$ **f** $\frac{1}{12} = \frac{?}{24}$ **g** $\frac{3}{4} = \frac{?}{16}$ **h** $\frac{2}{3} = \frac{?}{9}$

3. Apply this skill: Addition and subtraction problems require renaming. Watch the operation symbols.

a $\frac{2}{5} + \frac{3}{10}$ **b** $\frac{5}{6} + \frac{1}{12}$ **c** $\frac{5}{10} - \frac{1}{100}$ **d** $\frac{4}{5} - \frac{1}{10}$

e $\frac{3}{5} - \frac{1}{4}$ **f** $\frac{2}{3} - \frac{1}{5}$ **g** $\frac{3}{4} - \frac{3}{8}$ **h** $\frac{1}{6} + \frac{1}{4}$

i $\frac{1}{4} + \frac{1}{10}$ **j** $\frac{1}{2} - \frac{2}{5}$ **k** $\frac{2}{3} - \frac{1}{4}$ **l** $\frac{3}{8} + \frac{1}{6}$

To find the simplest name for any fraction, *divide* both numerator and denominator *by the same number.*

THINK

What is the GREATEST common factor of numerator and denominator?

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

2 is the greatest common factor.

$$\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$$

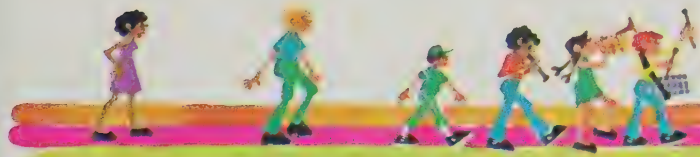
5 is the greatest common factor.

1. Write the simplest name for each fraction.

a $\frac{4}{8}$ **b** $\frac{3}{6}$ **c** $\frac{6}{10}$ **d** $\frac{10}{10}$ **e** $\frac{6}{15}$ **f** $\frac{16}{28}$ **g** $\frac{15}{30}$ **h** $\frac{12}{16}$

2. More practice. Write the simplest name for each fraction.

a $\frac{15}{18}$ **b** $\frac{10}{24}$ **c** $\frac{18}{20}$ **d** $\frac{8}{10}$ **e** $\frac{10}{16}$ **f** $\frac{16}{20}$ **g** $\frac{12}{18}$ **h** $\frac{9}{15}$



PROGRESS CHECK

Apply this skill: Rename each answer in these addition and subtraction problems.

(1.) $\frac{7}{8} - \frac{1}{8}$ (2.) $\frac{5}{6} - \frac{2}{6}$ (3.) $\frac{9}{10} - \frac{3}{10}$ (4.) $\frac{3}{10} + \frac{7}{10}$ (5.) $\frac{2}{5} + \frac{3}{5}$ (6.) $\frac{3}{4} + \frac{3}{4}$

You'll need a common denominator on these. Use two renaming skills.

(7.) $\frac{1}{2} + \frac{3}{9}$ (8.) $\frac{3}{8} + \frac{1}{6}$ (9.) $\frac{2}{4} + \frac{2}{5}$ (10.) $\frac{6}{10} - \frac{1}{5}$ (11.) $\frac{5}{6} - \frac{1}{3}$ (12.) $\frac{1}{2} - \frac{1}{10}$

When you rename a mixed number as a fraction, focus on the denominator of the fraction part.

THINK I must rename the whole number in the same number of fractional parts as the denominator of the fraction.

$1\frac{3}{4} = \frac{4}{4} + \frac{3}{4}$ or $\frac{7}{4}$

Rename 1 as $\frac{4}{4}$, and add the $\frac{3}{4}$ already there.

$2\frac{2}{3} = \frac{6}{3} + \frac{2}{3}$ or $\frac{8}{3}$

Rename the 2 as $\frac{6}{3}$, and add the $\frac{2}{3}$ already there.

1. Rename as a fraction.

a $1\frac{1}{3}$	**b** $5\frac{1}{2}$	**c** $2\frac{3}{5}$	**d** $4\frac{3}{4}$	**e** $1\frac{3}{10}$
f $3\frac{7}{8}$	**g** $4\frac{4}{5}$	**h** $6\frac{5}{8}$	**i** $2\frac{5}{6}$	**j** $2\frac{7}{10}$
k $2\frac{1}{5}$	**l** $2\frac{1}{2}$	**m** $4\frac{1}{3}$	**n** $1\frac{3}{8}$	**o** $3\frac{1}{4}$

When you have a fraction with a numerator that is larger than the denominator, focus on the denominator. That's your start.

THINK Take out any whole numbers, and the parts that are left are a fraction.

$\frac{7}{4} = 1\frac{3}{4}$

How many four-fourths? Just 1, and $\frac{3}{4}$ remain.

$\frac{22}{5} = 4\frac{2}{5}$

How many five-fifths? There are 4, and $\frac{2}{5}$ remain.

2. Rename as a whole or a mixed number.

a $\frac{3}{2}$	**b** $\frac{10}{3}$	**c** $\frac{12}{4}$	**d** $\frac{15}{8}$	**e** $\frac{13}{4}$
f $\frac{5}{3}$	**g** $\frac{11}{4}$	**h** $\frac{17}{10}$	**i** $\frac{15}{4}$	**j** $\frac{15}{6}$

The main application of renaming mixed numbers as fractions and renaming fractions as mixed numbers is found in multiplication.

Multiply. Write each product in simplest form.

1. $3 \times \frac{2}{7}$ **2.** $2 \times \frac{6}{8}$ **3.** $5 \times \frac{3}{16}$ **4.** $7 \times \frac{2}{3}$

5. $1 \times \frac{5}{6}$ **6.** $4 \times \frac{1}{4}$ **7.** $4 \times \frac{3}{4}$ **8.** $6 \times \frac{2}{3}$

The answers are on page 193.
Check your work.

Barry got paid for working $\frac{2}{3}$ of a day for 3 days each week. How many full days of pay did he get?

You can solve by adding. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{2+2+2}{3} = \frac{6}{3}$ or 2 days of pay

Or you can solve by multiplying. $3 \times \frac{2}{3} = \frac{3 \times 2}{3} = \frac{6}{3}$ or 2 days of pay

1. Solve these by adding. Write the simplest name for the sum.

a $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \underline{?}$ **b** $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \underline{?}$ **c** $\frac{5}{8} + \frac{5}{8} = \underline{?}$

2. Solve these by multiplying. Write the simplest name, please.

a $4 \times \frac{2}{7} = \frac{4 \times 2}{7} = \underline{?}$ **b** $2 \times \frac{5}{8} = \underline{?}$ **c** $3 \times \frac{3}{4} = \underline{?}$

3. Find matching problems in 1 and 2. Do your answers match too?

Go on to the next page.

1. Multiply.

a $5 \times \frac{1}{6}$ **b** $4 \times \frac{2}{9}$ **c** $3 \times \frac{3}{11}$ **d** $2 \times \frac{3}{10}$

e $2 \times \frac{2}{3}$ **f** $3 \times \frac{4}{5}$ **g** $8 \times \frac{2}{9}$ **h** $5 \times \frac{7}{10}$

You should be getting good. Try these problems. Write the simplest name for your answers.

2. Mike always spent $\frac{3}{4}$ of his weekly pocket money of $1.60. How much did he spend in February?

3. Sleepy spent $\frac{13}{24}$ of every day sleeping. During 2 days he slept a total time equal to how many days?

4. Kathy spent $\frac{1}{4}$ of an hour every day on her music practice. How many hours did she spend on music practice during 4 days?

5. Mary spent $\frac{1}{4}$ of an hour walking to school each morning, and the same time in going home in the afternoon. How much time did she spend each week going to and from school?

6. The bakery sold pies whole or by the portion. In one day the bakery sold five $\frac{1}{2}$-portions and nine $\frac{1}{4}$-portions. How many pies did they sell in all?

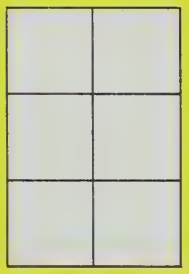
Multiply. Write each product in simplest form.

1. $\frac{1}{8} \times 5$ **2.** $\frac{2}{5} \times 2$ **3.** $\frac{2}{3} \times 6$ **4.** $\frac{2}{3} \times 3$ **5.** $\frac{1}{2} \times 5$ **6.** $\frac{3}{7} \times 3$ **7.** $\frac{7}{8} \times 5$ **8.** $\frac{3}{4} \times 6$

The answers are on page 193. Check your work.

You know this.

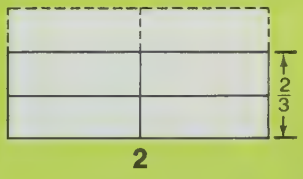
$2 \times 3 = 6$



$3 \times 2 = 6$

Does the order of factors change the product when you operate with fractions?

$2 \times \frac{2}{3} = \frac{2 \times 2}{3} = \frac{4}{3}$ or $1\frac{1}{3}$ square units



$\frac{2}{3} \times 2 = \frac{2 \times 2}{3} = \frac{4}{3}$ or $1\frac{1}{3}$ square units

Multiply. Write each product in simplest form.

1. $3 \times \frac{1}{7} = \frac{3 \times 1}{7} = \underline{?}$ **2.** $\frac{1}{7} \times 3 = \frac{1 \times 3}{7} = \underline{?}$ **3.** $5 \times \frac{2}{9} = \frac{5 \times 2}{9} = \underline{?}$ **4.** $\frac{2}{9} \times 5 = \frac{2 \times 5}{9} = \underline{?}$

5. $2 \times \frac{1}{2} = \frac{2 \times 1}{2} = \underline{?}$ **6.** $\frac{1}{2} \times 2 = \frac{1 \times 2}{2} = \underline{?}$ **7.** $4 \times \frac{2}{3} = \frac{4 \times 2}{3} = \underline{?}$ **8.** $\frac{2}{3} \times 4 = \frac{2 \times 4}{3} = \underline{?}$

Go on to the next page.

Multiply

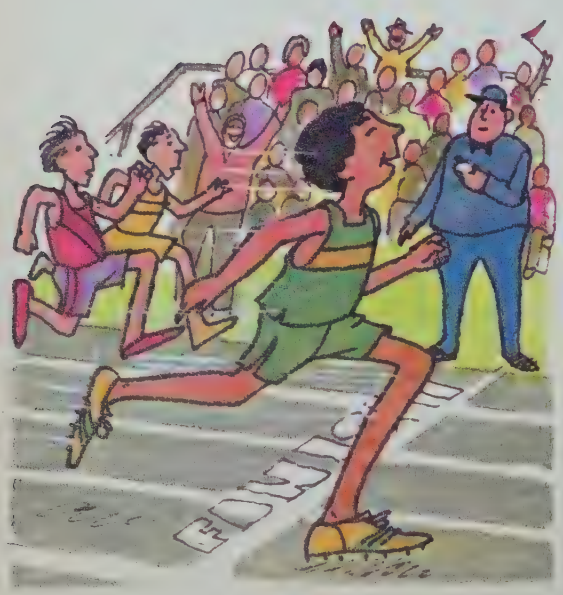
1. **a** $\frac{3}{4} \times 2 = \frac{3 \times 2}{\square} = \underline{?}$ **b** $\frac{2}{3} \times 7 = \frac{\square \times \triangle}{3} = \underline{?}$ **c** $\frac{7}{8} \times 5$

d $\frac{9}{10} \times 3$ **e** $\frac{7}{6} \times 4$ **f** $\frac{11}{10} \times 10$ **g** $\frac{1}{5} \times 5$ **h** $\frac{2}{5} \times 5$

i $\frac{3}{5} \times 5$ **j** $\frac{4}{5} \times 5$ **k** $\frac{1}{10} \times 5$ **l** $\frac{3}{10} \times 5$ **m** $\frac{7}{10} \times 5$

n $\frac{9}{10} \times 5$ **o** $\frac{1}{3} \times 6$ **p** $\frac{2}{3} \times 6$ **q** $\frac{1}{4} \times 8$ **r** $\frac{3}{4} \times 8$

Continue. Write the simplest name for your answers.



2. Kay had \$3. She spent $\frac{1}{3}$ of it. How much did she spend ?

3. Chris had 12 problems to do. She completed $\frac{3}{4}$ of them before her favorite TV show came on. How many problems had she done?

4. Steve ran the race in 7 minutes last year. This year he ran it in $\frac{3}{4}$ of that time. How many minutes did he run the race in this year?

5. Four people each worked $\frac{3}{4}$ of an hour overtime. How many hours of overtime must the company pay for?

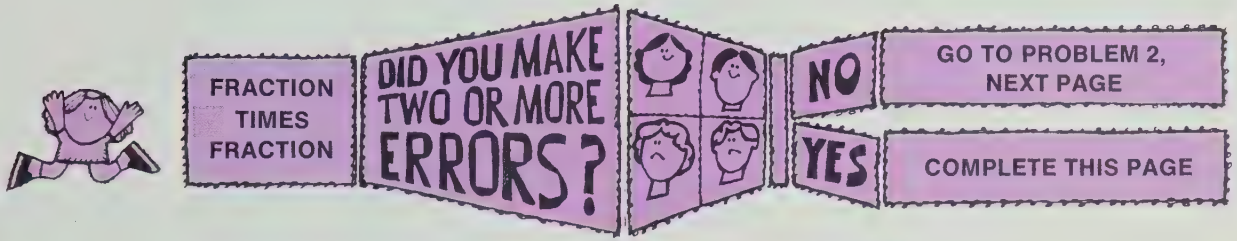
6. She worked $\frac{1}{2}$ an hour overtime every day for 6 days. How many hours overtime did she work?

Multiply. Write each product in its simplest form.

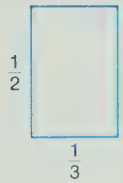
1. $\frac{1}{4} \times \frac{1}{4}$ **2.** $\frac{1}{2} \times \frac{1}{2}$ **3.** $\frac{1}{4} \times \frac{3}{4}$ **4.** $\frac{3}{4} \times \frac{3}{4}$

5. $\frac{2}{3} \times \frac{1}{3}$ **6.** $\frac{1}{4} \times \frac{2}{3}$ **7.** $\frac{2}{5} \times \frac{2}{3}$ **8.** $\frac{3}{5} \times \frac{4}{5}$

The answers are on page 193.
Check your work.

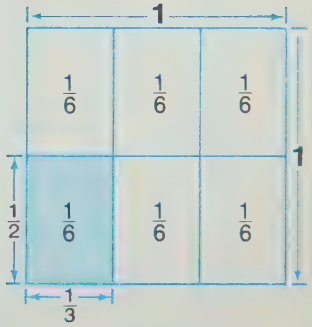


The area of this rectangle is equal to the product of $\frac{1}{2} \times \frac{1}{3}$. The rectangle pictured is part of a larger square.



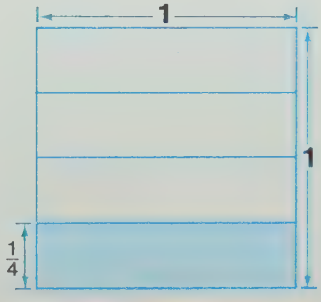
The area of the large square is 1×1, or 1 square unit. The entire unit has been divided into 6 rectangles of equal size. Each rectangle is $\frac{1}{2} \times \frac{1}{3}$

Therefore the area of one of the six rectangles is $\frac{1}{2} \times \frac{1}{3}$ or $\frac{1}{6}$ of a square unit.



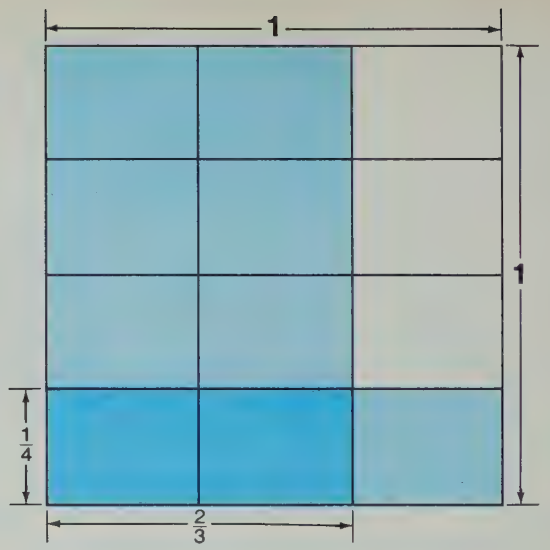
What is the area of a rectangle $\frac{1}{4} \times \frac{2}{3}$?
Get out a sheet of paper.

Step 1. Draw a unit square.
Step 2. Divide it into 4 equal parts. Then shade 1 of the 4 parts. This shows $\frac{1}{4} \times 1$.



Go on to the next page.

Step 3. Divide the square vertically into
3 equal parts.
Then shade 2 of the 3 parts.



The part of the square with double shading is $\frac{1}{4} \times \frac{2}{3}$.
Its area is 2 of the 12 parts or $\frac{2}{12}$ of the unit square.

So $\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$ ←—Look again at the equation.

Does $\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12}$? Can $\frac{2}{12}$ be renamed?

What is the area of a region $\frac{2}{3} \times \frac{3}{5}$?

$$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \underline{?}$$

1. Multiply. Write each product in simplest form.

a $\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \underline{?}$ **b** $\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2} = \underline{?}$ **c** $\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \underline{?}$

d $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \underline{?}$ **e** $\frac{4}{7} \times \frac{1}{3} = \frac{4 \times 1}{\square \times \triangle} = \underline{?}$ **f** $\frac{3}{5} \times \frac{5}{8} = \frac{\square \times \triangle}{5 \times 8} = \underline{?}$

g $\frac{6}{7} \times \frac{1}{4} = \underline{?}$ **h** $\frac{4}{5} \times \frac{5}{4} = \underline{?}$ **i** $\frac{1}{6} \times \frac{3}{4} = \underline{?}$

2. Half of the cake was eaten. He ate $\frac{1}{2}$ of that.
What part of the whole cake did he eat?

3. She decided it would take her $\frac{3}{4}$ of an hour to get there.
She spent $\frac{1}{2}$ of that time waiting for a bus.
What fractional part of an hour did she wait?

Multiply. Write each product in simplest form.

1. $\frac{2}{3} \times 1\frac{1}{4}$ **2.** $2\frac{1}{3} \times \frac{2}{7}$ **3.** $1\frac{1}{4} \times 2\frac{1}{2}$ **4.** $1\frac{1}{3} \times 2\frac{1}{4}$ **5.** $2\frac{1}{5} \times 1\frac{1}{2}$

6. $4 \times 1\frac{1}{3}$ **7.** $1\frac{1}{10} \times \frac{10}{10}$ **8.** $1\frac{1}{3} \times \frac{3}{4}$

Check your answers on page 193.

The action of multiplying mixed numbers is exactly the same as it is for multiplying fractions. Change the mixed number to a fraction and go!

$1\frac{1}{2} \times \frac{1}{2} = \underline{?}$ *Rename* $1\frac{1}{2}$ as a fraction: $\frac{3}{2}$ $\quad \frac{3}{2} \times \frac{1}{2} = \frac{3 \times 1}{2 \times 2} = \frac{3}{4}$

Look at another problem.

$2\frac{2}{3} \times 1\frac{1}{4} = \underline{?}$ *Rename* both mixed numbers as fractions.

$\frac{8}{3} \times \frac{5}{4} = \frac{8 \times 5}{3 \times 4} = \frac{40}{12}$ or $3\frac{4}{12}$ or $3\frac{1}{3}$

Be prepared The renaming sometimes takes longer than the multiplication itself. Multiply. Write each product in simplest form.

1. $1\frac{1}{4} \times 2\frac{1}{3}$ **2.** $3\frac{1}{9} \times 1\frac{3}{4}$ **3.** $2\frac{2}{3} \times 1\frac{1}{2}$ **4.** $3\frac{1}{4} \times 1\frac{3}{4}$ **5.** $1\frac{1}{2} \times 1\frac{4}{5}$

6. $2 \times 5\frac{5}{6}$ **7.** $3\frac{1}{3} \times 2\frac{1}{6}$ **8.** $2\frac{3}{8} \times 2\frac{1}{2}$ **9.** $6\frac{1}{5} \times 1\frac{2}{7}$ **10.** $1\frac{3}{4} \times \frac{1}{8}$

sidetrack

To try these riddles on your friends, print the word in capital letters as you answer each sentence.

Riddle:

a The last $\frac{3}{4}$ of the word SHOW.

b The first $\frac{3}{5}$ of the word CANNY.

c The first $\frac{3}{8}$ of the word YOURSELF.

d The first $\frac{5}{6}$ of the word LEAVEN.

e The middle $\frac{3}{7}$ of the word HOTHEAD.

f The last $\frac{4}{5}$ of the word BROOM.

g The first $\frac{1}{2}$ of the word WITHDRAW.

h The middle $\frac{3}{7}$ of the word OUTWORK.

i The $\frac{1}{1}$ of the word LEGS.

j The middle $\frac{3}{5}$ of the word HANDY.

k The first $\frac{2}{3}$ of the word COMELY.

l The last $\frac{4}{7}$ of the word OUTBACK.

m The first $\frac{1}{2}$ of the word WITHERED.

n The first $\frac{3}{8}$ of the word SIXTYISH.

Answer:

a The last $\frac{1}{3}$ of the word SHABBY.

b The last $\frac{8}{10}$ of the word UPBRINGING.

c The middle $\frac{1}{5}$ of the word CHAIR.

d The first $\frac{5}{8}$ of the word CHAIRMAN.

e The first $\frac{1}{2}$ of the word BACKBONE.

Go on to the next riddle.

Riddle:

a The first $\frac{1}{2}$ of the word WHATEVER.

b The first $\frac{4}{5}$ of the word MUSTY.

c The first $\frac{3}{5}$ of the word YOURS.

d The middle $\frac{3}{5}$ of the word PADDY.

e The first $\frac{1}{2}$ of the word TOPS.

f The first $\frac{1}{2}$ of the word NINETEEN.

g The last $\frac{1}{2}$ of the word INTO.

h The first $\frac{4}{9}$ of the word MAKESHIFT.

i The second $\frac{1}{4}$ of the word FLITTING.

j The first $\frac{3}{5}$ of the word SIXTY.

Answer:

a The first $\frac{3}{6}$ of the word THEORY.

b The first $\frac{2}{3}$ of the word LETTERING.

c The last $\frac{1}{6}$ of the word SEVENS.

d The first $\frac{3}{7}$ of the word FORTUNE.

e The middle $\frac{1}{7}$ of the word HOUSING.

f The first $\frac{4}{5}$ of the word PLUSH.

g The middle $\frac{2}{4}$ of the word PIXY.

h The last $\frac{5}{7}$ of the word REMAKES.

i The first $\frac{3}{7}$ of the word SIXTEEN.

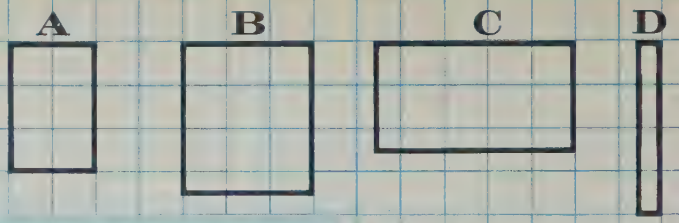
PROGRESS CHECK

Multiply. Write the simplest name for each product.

1. $5 \times \frac{2}{5}$ **2.** $\frac{3}{10} \times 5$ **3.** $\frac{3}{4} \times \frac{2}{3}$ **4.** $1\frac{1}{2} \times \frac{4}{5}$ **5.** $1\frac{1}{3} \times 2\frac{1}{4}$ **6.** $2\frac{1}{2} \times 1\frac{3}{4}$

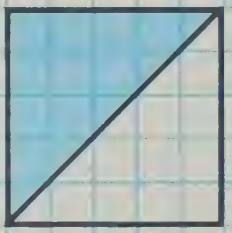
7. $\frac{2}{5} \times 2\frac{1}{2}$ **8.** $2\frac{1}{5} \times 2\frac{1}{2}$ **9.** $3\frac{1}{6} \times 1\frac{1}{4}$ **10.** $2\frac{3}{5} \times 1\frac{2}{3}$ **11.** $3\frac{2}{3} \times 1\frac{1}{2}$ **12.** $1\frac{1}{2} \times 4\frac{5}{6}$

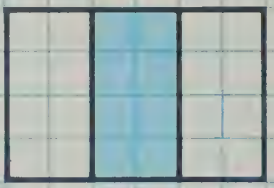
Here are four polygons. Each has a different size and shape. Count to find the units of length and the units of width. Then COMPUTE to find the area.

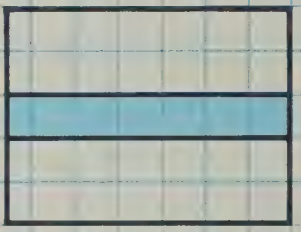


What's the length? What's the width? Compute the area of the whole polygon. Then tell the area of the shaded part of each polygon.

1.

2. 

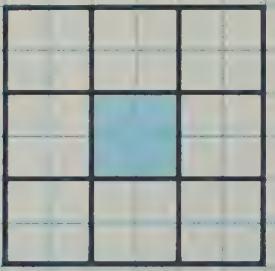
3. 

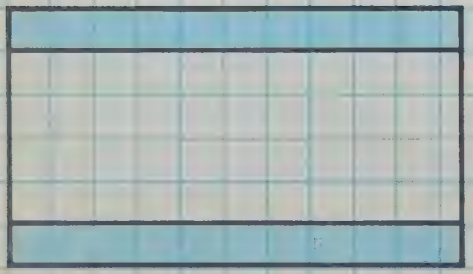
4. 

5.

6.

7.

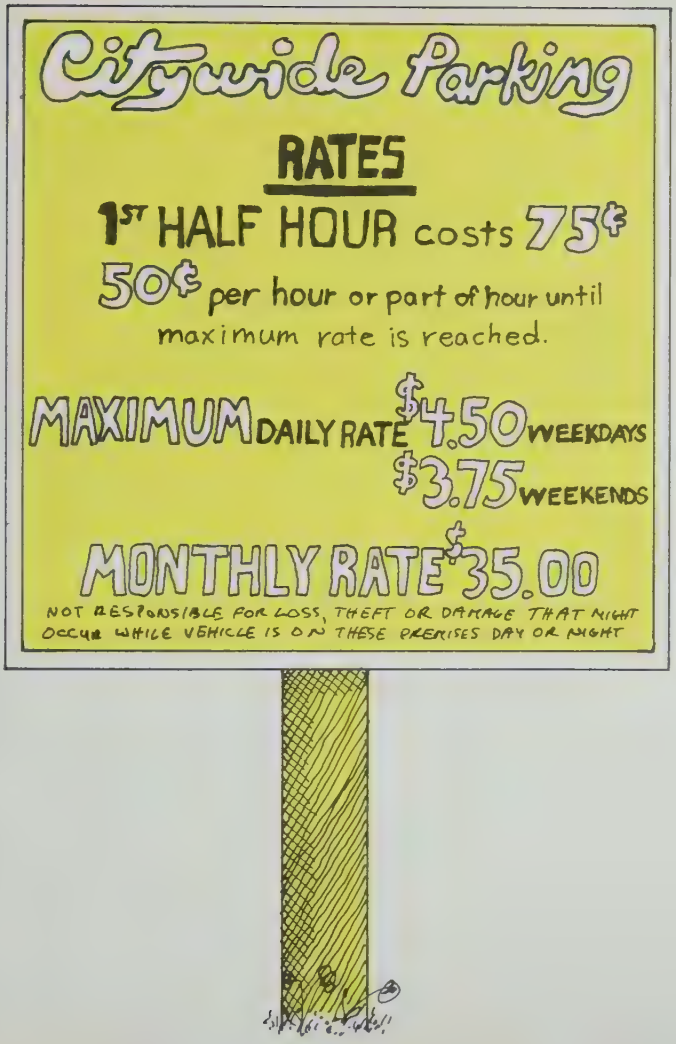
8. 

9. 

10.

11.

12.


Citywide Parking
RATES
1ST HALF HOUR costs 75¢
50¢ per hour or part of hour until maximum rate is reached.
MAXIMUM DAILY RATE $4.50 WEEKDAYS
$3.75 WEEKENDS
MONTHLY RATE $35.00
NOT RESPONSIBLE FOR LOSS, THEFT OR DAMAGE THAT MIGHT OCCUR WHILE VEHICLE IS ON THESE PREMISES DAY OR NIGHT

1. To park a car in a large city is quite expensive. Fortunately Ms. Bakuska has the monthly rate. If she works an average of 20 days a month, how much does she pay per day?

2. How much would you pay for 20 weekdays at the daily rate?

3. If you parked for 3 h on Saturday, how much would parking cost you? Are you under or over the maximum rate?

4. Compute the parking costs for these people.

   a Mr. Lei: $5\frac{1}{4}$ h on Tuesday.

   b Miss Thibaudeau: $1\frac{3}{4}$ h on Friday.

   c The McConnell family: 6 h on Wednesday.

   d Mrs. Shipton: $6\frac{1}{4}$ h on Sunday.

   e Mr. and Mrs. Logan: $8\frac{1}{2}$ h on Monday.

   f Ms. Westington: Wednesday, Thursday, and Friday for $3\frac{1}{4}$ h each day.

5. Why do parking costs vary?

CHECKOUT

Pick any five rows of problems to complete. Multiply. Write the simplest name for each product.

1.	**a**	$3 \times \frac{2}{3}$	**b**	$\frac{4}{5} \times 6$	**c**	$\frac{2}{3} \times \frac{1}{4}$	**d**	$5\frac{3}{5} \times 2\frac{4}{5}$
2.	**a**	$4 \times \frac{3}{4}$	**b**	$\frac{2}{3} \times 4$	**c**	$\frac{5}{6} \times \frac{7}{5}$	**d**	$2\frac{4}{5} \times 3\frac{2}{5}$
3.	**a**	$6 \times \frac{6}{7}$	**b**	$\frac{8}{9} \times 3$	**c**	$\frac{5}{12} \times \frac{4}{5}$	**d**	$3\frac{1}{3} \times 1\frac{5}{8}$
4.	**a**	$9 \times \frac{5}{9}$	**b**	$\frac{2}{11} \times 5$	**c**	$\frac{1}{5} \times \frac{2}{3}$	**d**	$1\frac{5}{6} \times 3\frac{1}{4}$
5.	**a**	$5 \times \frac{4}{5}$	**b**	$\frac{10}{11} \times 5$	**c**	$\frac{4}{5} \times \frac{3}{8}$	**d**	$\frac{3}{8} \times 4\frac{1}{4}$
6.	**a**	$7 \times \frac{2}{7}$	**b**	$\frac{3}{16} \times 4$	**c**	$\frac{6}{10} \times \frac{2}{3}$	**d**	$\frac{5}{6} \times 3\frac{1}{9}$
7.	**a**	$10 \times \frac{7}{10}$	**b**	$\frac{2}{5} \times 6$	**c**	$\frac{1}{7} \times \frac{3}{3}$	**d**	$\frac{3}{4} \times 2\frac{2}{5}$
8.	**a**	$8 \times \frac{7}{8}$	**b**	$\frac{3}{4} \times 12$	**c**	$\frac{1}{10} \times \frac{2}{7}$	**d**	$\frac{2}{3} \times 4\frac{3}{4}$

If you make two or more errors on an **a** problem, do all the other **a**'s for practice. If you make two or more errors on **b**, do other **b**'s for practice. You know the pattern. If you need the practice, do it now.

ANSWERS

ANSWERS TO PROBLEMS ON PAGE 181.

1. $\frac{6}{7}$	**2.** $1\frac{1}{2}$	**3.** $\frac{15}{16}$	**4.** $4\frac{2}{3}$
5. $\frac{5}{6}$	**6.** 1	**7.** 3	**8.** 4

ANSWERS TO PROBLEMS ON PAGE 183.

1. $\frac{5}{8}$	**2.** $\frac{4}{5}$	**3.** 4	**4.** 2
5. $2\frac{1}{2}$	**6.** $1\frac{2}{7}$	**7.** $4\frac{3}{8}$	**8.** $4\frac{1}{2}$

ANSWERS TO PROBLEMS ON PAGE 185.

1. $\frac{1}{16}$	**2.** $\frac{1}{4}$	**3.** $\frac{3}{16}$	**4.** $\frac{9}{16}$
5. $\frac{2}{9}$	**6.** $\frac{1}{6}$	**7.** $\frac{4}{15}$	**8.** $\frac{12}{25}$

ANSWERS TO PROBLEMS ON PAGE 187.

1. $\frac{5}{6}$	**2.** $\frac{2}{3}$	**3.** $3\frac{1}{8}$	**4.** 3
5. $3\frac{3}{10}$	**6.** $5\frac{1}{3}$	**7.** $1\frac{1}{10}$	**8.** 1

8
PROBABILITY

Would you like to know everything that's going to happen to you tomorrow? Probably not! Most people like surprises. But almost everyone tries to guess about the future. The mathematics of *probability* can help. **Your goal is to learn how probability can help you in the world around you.**

Insurance companies use probability so that they know what to charge for insurance. Scientists use probability to predict what atoms will do. A baseball catcher uses probability to decide what pitch to call for. You use probability too.

What Are the Chances?

that your favorite team will win the next series?
that you'll get a good mark in English?
that it will rain during the picnic?
that you'll win first prize in the raffle?

You don't need magic to figure out the chances (or probability) that certain things will happen. A few facts can help you predict more accurately.

What Do You Need to Know?

before you can decide whether your team is likely to win the series?
before you can feel you'll probably get a good mark in English?
before you can figure the odds of rain during the picnic?
before you know if you have a decent chance to win the raffle?

Have you ever been to a carnival? Carnival games are based on probability. They are also set up to make money for the operators. Players will probably miss the balloon more often than they'll hit it with the dart. They'll probably miss the cup more often than they'll get the ball into it. If a game is too easy, the operator will lose money. If it looks too hard, no one will play.

In order to make money, the operator has to know what he's doing. He has to make sure that his guesses will be correct often enough for him to make a profit. How many seasons of the year are there? Which season would you guess?

1

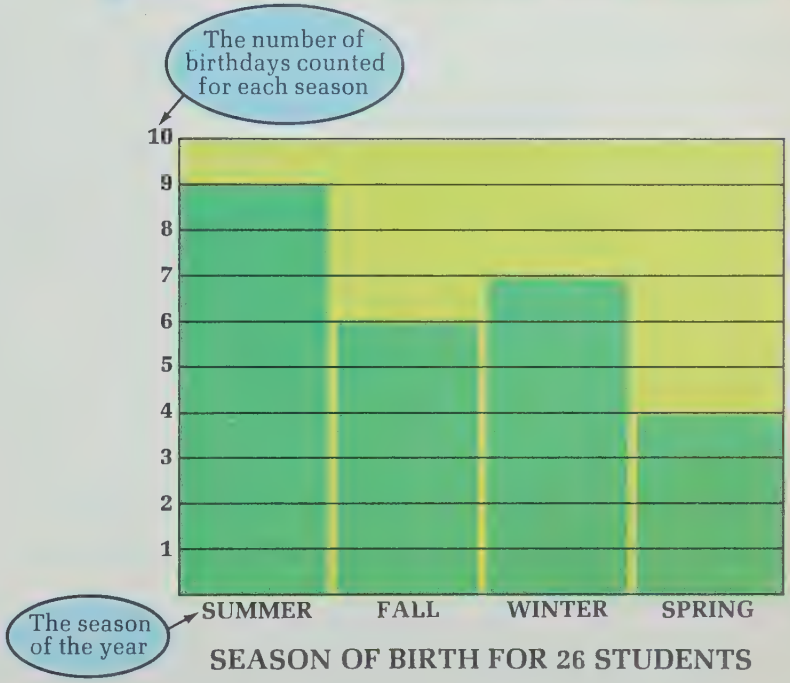
Find how many students in your class have birthdays in each of the four seasons. Your results might look like this:

Summer (June, July, August) HHT IIII
Fall (September, October, November) HHT I
Winter (December, January, February) HHT II
Spring (March, April, May) IIII

This is called a *tally chart*.

2

Make a bar graph showing your results. A bar graph showing the tally above would look like this:



SEASON OF BIRTH FOR 26 STUDENTS

3

If the bar graph had been made for the same students a year ago, would it have looked the same? If it's made a year from now, will it look the same?

4

Would a bar graph showing the seasons of birth for the class next door look the same? Would it look similar?

Go on to the next page.

5 In which season is a person in your class most likely to celebrate a birthday? (Look for the highest bar on the graph.) In which season is a person in your class least likely to celebrate a birthday?

a If the operator guesses just one season for each person in your class, which should it be?

b How many times would he be correct?

6 Pretend each of the 26 people in the class in our example pays 50¢ for a guess. Each incorrect guess costs the barker 65¢. (He has to pay for the stuffed rabbit.) Suppose the barker guesses summer as the season of birth for each person. How much money would he make on this class?

—How many guesses? 26
—What is the cost to the students? $26 \times 50¢$, or ?
—How many correct guesses? ?
—How many incorrect guesses? ?
—What is the cost to the barker? ?
—What is the barker's profit? ? − ?, or ?

Do the same calculations for your own class.

7 Where might you find more information about the seasons of birth for your community? for the whole country? Do you think it is likely that the barker studied information of this kind for your community before the carnival opened?

In the next pages you'll play a carnival game called the Wheel of Fortune. Probability will help you determine your chances of winning. (It can't guarantee that you *will* win, however!)

WHEEL I

Here's the first Wheel of Fortune. It's divided into two sections, A and B. These sections are exactly the same size. A spinner is fastened at the center of the wheel. O.K.! The spinner is spun. Round and round it goes. Where it will stop, nobody knows. But we do know that it will stop on section A or section B. (If it stops on the line, it's spun again.)

There are two *outcomes* on this wheel:
The spinner stops on section A.
The spinner stops on section B.
Since sections A and B are the same size, the spinner is as likely to stop on A as on B.
A and B are *equally likely* outcomes.

One spin has 2 equally likely outcomes. The probability of stopping on A is 1 out of 2, or $\frac{1}{2}$. (Probability is usually expressed with a fraction.) What does the numerator of this fraction stand for? the denominator? What is the probability of stopping on B?

If you were going to bet on the outcome of a spin, would you always bet on A? always on B? If you make many bets and always bet on the same outcome, you can expect to win about $\frac{1}{2}$ of the time. If you make only a few bets, however, you can't be sure of winning $\frac{1}{2}$ of the time.

Get ready to play

WHEEL I

Here are the rules:
Each bet costs 50¢.
If you win, you get 75¢.
Bets are placed before the spinner is spun.
You may not change your bet after the spinner starts to spin.

Now let's look at an example of how money changes hands in this game.

1 You bet on A 10 times in a row. This will cost you $10 \times 50¢$, or \$? .

2 About how many times will the spinner stop on A in 10 spins? Let's say it stops on A 5 times. Then you win $5 \times 75¢$, or \$? .

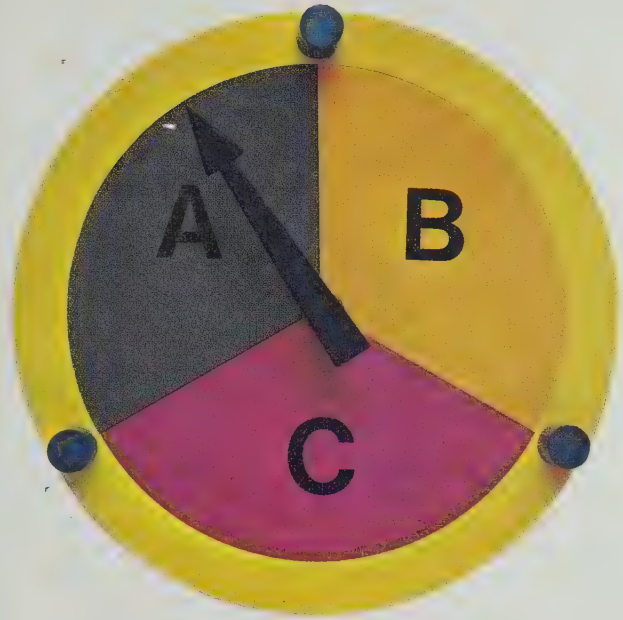
3 After 10 spins, how much money have you gained or lost? You have paid \$5.00 and won only \$3.75. This means you lost money. How much?
\$5.00 − \$3.75 = ?

You lost! This game, like all carnival games, favors the operator over the player.

4 Of course the spinner might have stopped on A more than 5 times. How many times would it have to stop on A in order for you to gain money?

5 Could the spinner stop on B 10 times?
Would this be very likely?
How much would you lose if this happened?

Let's look at other Wheels of Fortune to learn more about probability.



WHEEL II

1 Name three outcomes for wheel II.

2 Sections A, B, and C are all the same size. So the probability that the spinner will stop on A is 1 out of 3, or $\frac{1}{3}$. What is the probability that it will stop on B? on C? Are these outcomes equally likely?

3 One section of the wheel is shaded black. The other two are color. What is the probability of stopping on the black section? What is the probability of stopping on either of the color sections? Are the outcomes "black" and "color" equally likely?

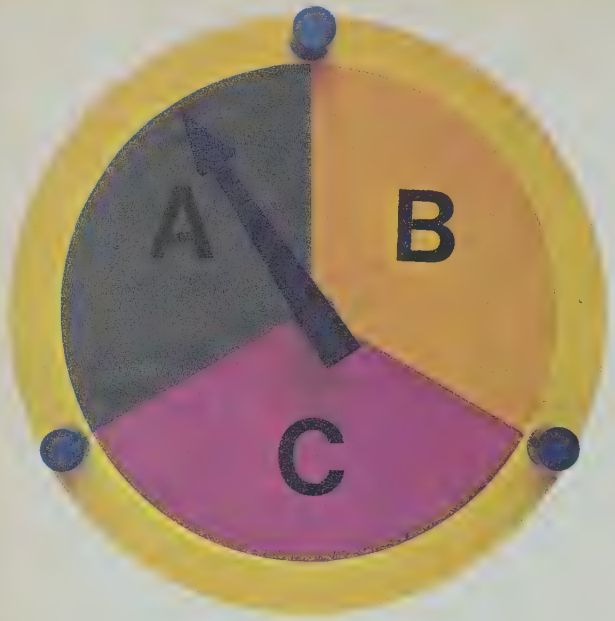
4 A bet on wheel II costs 10¢. For each bet on black you win 20¢ if the spinner stops on A, the black section. You get nothing if it stops on a color section. Suppose you bet on black 12 times and win $\frac{1}{3}$ of the time. How many times do you win? Let's see how much you gain (or lose).

Cost of 12 bets: $12 \times 10¢$, or ?

Your winnings: $4 \times 20¢$, or ?

Which is more?

How much more? ? − ?, or ?



WHEEL II

1 A *fair game* would give you and the operator an equal chance to make money. Do you think the game with wheel II is fair? How much money did you lose after playing 12 games? What do you think would happen if you played longer?

2 Each time you bet on black, you pay 10¢. If you play many games, you can expect to win on $\frac{1}{3}$ of the spins. Each time you win, you get 20¢; but it costs 10¢ to play, so you're only 10¢ ahead when you win. If you play many games, you can expect to lose on $\frac{2}{3}$ of the spins. You lose 10¢ $\frac{2}{3}$ of the time. You gain 10¢ $\frac{1}{3}$ of the time. Is the game fair?

3 Suppose you bet on black 24 times and you win $\frac{1}{3}$ of the time.

Cost of 24 bets:	$24 \times 10¢$, or ?
Your winnings:	$8 \times 20¢$, or ?
How much do you lose?	? − ?, or ?

4 Make a copy of wheel II on a piece of cardboard. Make sure the sections are the same size. Cut a spinner out of cardboard. Fasten it to the center of the wheel (with a pin, pencil point, or similar object) so that it spins freely. Predict the number of times black will win in 12 spins. Then spin it 12 times and compare the results with your prediction.

WHEEL III

1 Name five outcomes for wheel III.

2 The sections are all the same size.
What is the probability of landing on E?

3 Sections B and E are shaded black. A, C, and D are color. Suppose you bet on black. For which outcomes would you win?

4 What is the probability of landing on black?

5 A bet on wheel III costs 10¢. For each bet on black, you win 30¢ if the spinner stops on a black section. You get nothing if it stops on a color section. Do you think this is a fair game?

6 On wheel III black wins $\frac{2}{5}$ of the time. Black loses $\frac{3}{5}$ of the time. Let's say you bet on black 20 times and win 8 times. How much money will you gain (or lose)?

Cost of 20 bets:	$20 \times 10¢$, or ?
Your winnings:	$8 \times 30¢$, or ?
Which is more?	
How much more?	? $-$?, or ?

You can see that you have *gained* money by betting as you have. Would you be wise to continue betting on black at wheel III? How much money would you expect to gain if you bet on black 100 times in a row?

7 Do you think the operator would lose money with wheel III? Is this a fair game?

WHEEL IV

1. What are the possible outcomes for wheel IV?

2. Which outcome would you want if you bet on black?

3. The sections are all the same size. What is the probability of landing on B?

4. What is the probability of stopping on black?

5. What is the probability of stopping on a color section?

6. A bet on black costs 10¢. You win $1.00 if the spinner stops on black and nothing if it stops on color. Do you think this is a fair game?

7. Make a wheel like wheel IV. Predict the number of times black will win in 16 spins. Then spin it 16 times and compare the results with your prediction. Spin it 16 more times and see if you get black a different number of times.

Here are two more Wheels of Fortune:

WHEEL V

WHEEL VI

1 On wheel V the probability of stopping on black is $\frac{0}{6}$ or 0.
Would it be smart to bet on black? Why not?

2 On wheel VI the probability of stopping on black is $\frac{6}{6}$ or 1.
Would the operator accept bets on black? Why not?
Would it be smart to bet on color? Why not?

3 It's important to know whether a game is fair.
Ask yourself these questions before you play a game of chance:

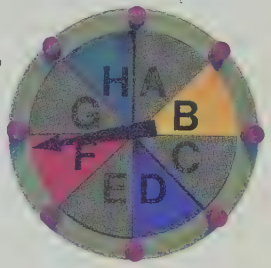
How many chances do I have to win?
How many chances do I have to lose?
Is the prize worth the amount I will probably have to pay for it?

In each of the following games the sections for each game are all the same size. The payoff is the same, too. If you bet on black and win, you get 50¢. If you bet on color and win, you get 50¢.

WHEEL VII

There are 2 chances for black to win and 3 chances for black to lose. The *odds of winning* are 2 to 3. Is it smart to bet on black?

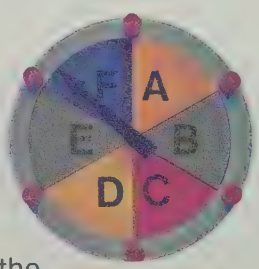
WHEEL VIII



How many chances for black to win on this wheel? How many chances for black to lose? The *odds* of black's winning are 4 to 4. We also say that the *odds are even* that black will win. (That's a funny sentence—"The odds are even!") This is a fair game. It would be smarter to bet on black here than on wheel VII. But could you lose anyway?

When the odds are even, some people say that the odds are 50-50. That means if you played 100 times, the rules of probability say that you can expect to win 50 times and lose 50 times. Of course the actual results could be different.

1

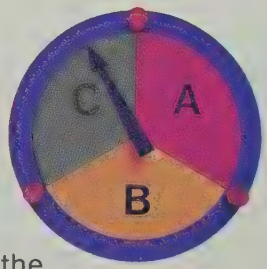


Bet black.

a What is the probability of stopping on black?

b What are the odds that you will win?

2

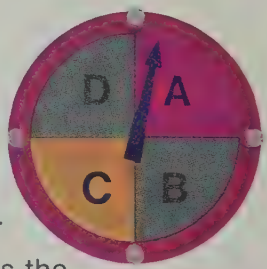


Bet black.

a What is the probability of stopping on black?

b What are the odds that you will win?

3



Bet color.

a What is the probability of stopping on color?

b What are the odds that you will win?

Find a cube.
Write 1 on one face, 2 on another face, 3 on another, 4 on another, 5 on another, and 6 on the last face.

You'll need a sheet of paper to record your tests.

Roll until 3 comes up on top of the cube.

What is the probability that you'll get 3 on the first throw?

Record how many throws you need to get 3.

Try again (trial 2).
Throw until you get 3.
Record the number of throws.

Throw for 3 again (trial 3).
Record the number of throws.

Complete 7 more trials.
In each trial you roll until you get 3.

1. Draw a bar graph to show the number of throws needed to get 3 in each trial. Is there more than one way to make the graph?

2. What is the most common number of throws?

***3.** Find the total number of throws for all 10 trials. Divide the total by 10. According to the rules of probability, your answer should be close to 6. Why?

The Marble Draw is another game based on probability

1. A jar contains six marbles. Each marble has a number on it. You are blindfolded. Then you draw one marble from the jar.

   a What are the possible outcomes? (Name the different possible choices.)

   b What is the probability of drawing marble 4?

   c What are the odds of drawing marble 4?

   d What is the probability of drawing marble 1?

   e What are the odds of drawing marble 1?

Another jar contains six marbles—two red, three yellow, and one green. Again you draw a marble blindfolded.

Just so we can tell the red marbles apart, let's mark one of them with a 1 and the other with a 2. We do the same for the yellow marbles.

1. Do "red," "yellow," and "green" describe all possible outcomes? Do these words tell you how many red, yellow, and green marbles there are? What is a better way to describe all possible outcomes?

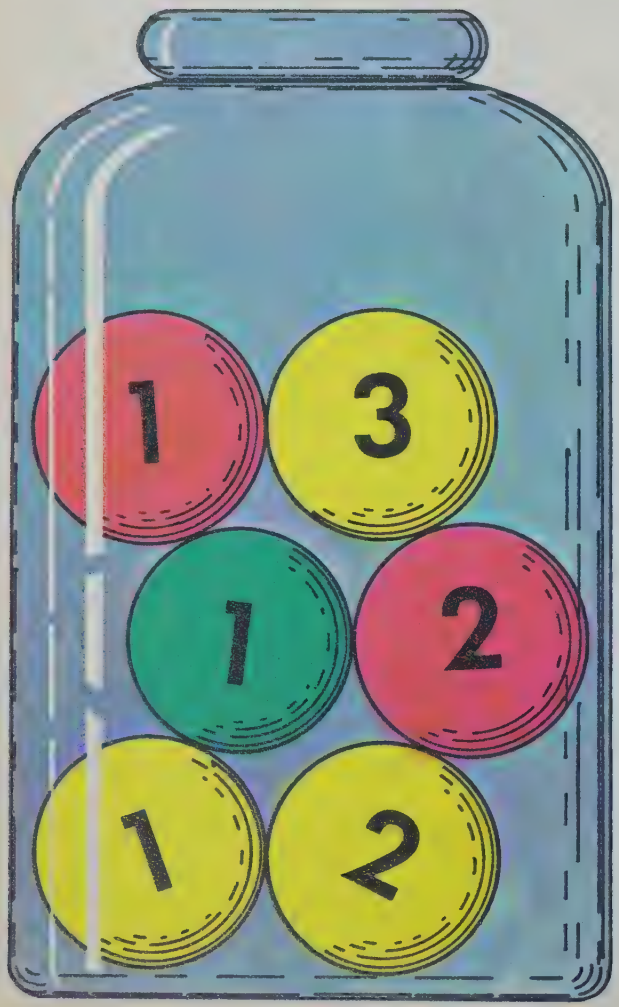
   a What is the probability of drawing red 1?
   b What is the probability of drawing red 2?
   c What is the probability of drawing a red marble?
   d What are the odds of drawing a red marble?

2. The probability of drawing a red marble is equal to the probability of drawing red 1 plus the probability of drawing red 2. Let's see what happens with yellow.

   a The probability of drawing yellow 1 is ?.
   b The probability of drawing yellow 2 is ?.
   c The probability of drawing yellow 3 is ?.
   d What is the probability of drawing a yellow marble?

3. What are the odds of drawing a green marble?

4. What are the odds of drawing a yellow marble?



A ticket to the Marble Draw costs 25¢.
You win 90¢ if you draw a green marble, 30¢ if you draw a red marble, and nothing if you draw a yellow marble.

1. You buy 1 ticket. What is the probability of winning 90¢? ofwinning 30¢? of winning nothing?
2. You buy 6 tickets. You will draw from the full jar 6 times.
   - **a** What is the probability of drawing yellow on one try? Then you would probably draw yellow how many times in 6 tries?
   - **b** What is the probability of drawing red on one try? Then you would probably draw red how many times in 6 tries?
   - **c** What is the probability of drawing green on one try? Then you would probably draw green how many times in 6 tries?
   - **d** How much money are you likely to win on the 6 tickets?

     yellow = 3 × 0¢, or ?
     red = 2 × 30¢, or ?
     green = 1 × 90¢, or ?
     Total = ? + ? + ?, or ?

   - **e** How much money does it cost you to buy 6 tickets?
3. How much money are you likely to win or lose betting 6 times on the Marble Draw? ? − ? = ?
4. Is this a fair game? Why?

When an outcome is certain to happen, its probability is 1.

When an outcome is impossible, its probability is 0.

When there are only two equally likely outcomes, the probability of either is $\frac{1}{2}$.

$$\text{Probability of a favorable outcome} = \frac{\text{number of favorable outcomes}}{\text{number of all possible outcomes}}$$

Tell whether the probability is 1, 0, or $\frac{1}{2}$ for each of these.

1. You are older today than yesterday.

2. You will draw the longer of two straws.

3. A tossed coin will land heads up.

4. A six-year-old will swim 1 km in 51 s next week.

5. The numerals from 1 through 6 are written on the faces of an ummarked cube, one on each face. The cube is tossed, and the face with 7 on it comes up on top.

A probability must be at least 0. It can't be greater than 1. But it might be any number from 0 to 1. What is the probability of each of the following?

1. There are 9 cards. Each card is marked with a different numeral—1 through 9. The card you draw is marked with 5.

2. You roll one die. When it stops, the face with 6 dots is on top.

3. The names of 7 persons are written on 7 cards. (A different name is on each card.) Of these persons, 4 are boys and 3 are girls. The cards are put in a hat, and you draw one without looking. You draw the name of a girl.

Discuss each of the following situations. How might you calculate the probability of the given outcome?

4. You enter a room with 7 persons in it: 4 are boys and 3 are girls. The first person to speak to you is a girl.

5. You will be called on to answer the next question in class.

6. The next traffic light you see will be green.

7. Tomorrow will be a cloudy day.

Can you think of ways probability must be used by a businessman to help him run a good business?

The rules of probability can help you save money on long-distance telephone calls.

You have a choice when you call long distance.

You can dial the call yourself.

You can ask the operator to call person-to-person.

If you dial the call yourself, you must pay for it if someone answers. If the person you want isn't around, you've wasted your money.

If you call person-to-person, you pay only if the person you want to talk to is there.

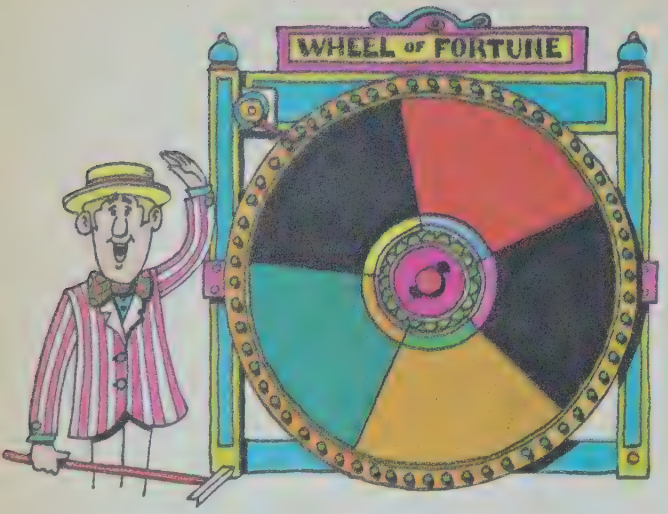
Dialing a long-distance call costs less, but . . .

Jill has to call Tom twice a week on business. The odds are even that Tom will be near the phone when Jill calls. Should Jill dial direct or call person-to-person? Here are the costs for the calls:

Dialed	$1.35
Person-to-person	3.60

If Tom were there both times, which type of call would be cheaper? If Tom were gone both times, which would be cheaper?

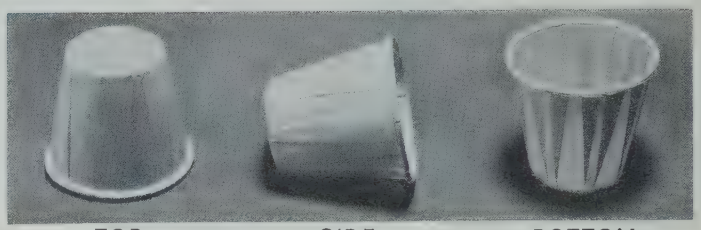
Suppose Jill calls and Tom isn't there. Whoever answers can tell Jill (or the operator) when Tom will be back. If Jill dials the call at that time, Tom will answer. Under these conditions, is it cheaper to make the first call by dialing or calling person-to-person?



In the games we've been playing you could figure out the probability of an outcome before playing the game. For example, one Wheel of Fortune had 5 sections, 2 of which were black; so the probability of stopping on black was $\frac{2}{5}$.

Sometimes you can know all the possible outcomes but not be able to figure out the probabilities ahead of time. Then you need some facts before you can estimate the probabilities.

1. Find a paper cup with a flat bottom. If you toss the cup onto the floor, it can end up in 3 different positions: on its top, on its side, or on its bottom. Without more information, can you tell what the probability is that it will land on its bottom? on its side? on its top?



TOP *SIDE* *BOTTOM*

2. Actually toss a cup 25 times and tally the results.

3. Repeat this experiment 25 more times. Are the results the same as before?

4. Out of the 50 flips, how many times did the cup land on its top? on its side? on its bottom? According to these results, what's the probability that a cup will land on its top? on its side? on its bottom?

5. Suppose you tossed the cup 50 more times. Do you think you'd get *exactly* the same results as for the first 50 tosses? About the same results?

6. Do you think your results would be true for any kind of cup, or do they depend on the size and shape of the cup?

You can often predict what's going to happen by looking at what happened before in a similar situation. You study the facts.

Do an experiment.

1. Open a book to any page. Look at the first 30 words on that page. Make a tally chart showing how many words have 1 letter, how many have 2 letters, how many have 3 letters, and so on.
2. Make a bar graph showing your results.
3. Subtract the shortest word length from the longest. This difference will show you the *range* of word length. Compare your results with others'. What is the greatest range of word length that anyone found?
4. Find the average length of the words in your sample. To do this, add the lengths of all the words. Divide their sum by 30. (You're considering a total of 30 words.) This average is called the *mean*. Since we are reporting the mean for a sample of words, we could call this average the *sample mean*.
5. If you were to look at another page in the same book, do you think the range of word lengths would be the same? What do you think the sample mean would be for the lengths of the first 30 words? Select a page and check your predictions. Were your answers close?
6. Turn to another page in the same book and find the sample mean of the first 30 words. Is it close to the two sample means you calculated previously?

CHECKOUT

1. Here is a tally chart of votes for the class election.

a Total the votes.

b Complete a bar graph to show the results.

CLASS ELECTION		
Candidates	Votes	Total
Ann	𝍸 ||||	?
Jim	𝍸 |||	?
Sam	𝍸 ||	?
Sue	𝍸 𝍸	?

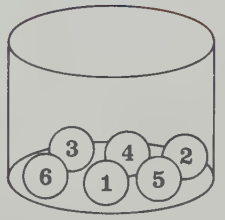
2. **a** Probability of stopping on **A**? on **B**?
Bet costs 10¢.
Stop on black—win 25¢.
Stop on color—win 0¢.

b Outcomes that win?

c Cost of 12 bets?

You win 4 times.

d Did you earn any money?



3. **a** What is the probability of drawing marble 2?

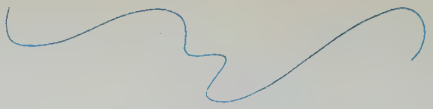
b What is the probability of drawing marble 5?

4. A coin is flipped to decide which team starts the game. What is the probability your team will start?

5. Six students apply for summer jobs. Four will be chosen. What is the probability of being chosen?

9 ANGLES

A piece of string or yarn lying on a table would probably look like this.

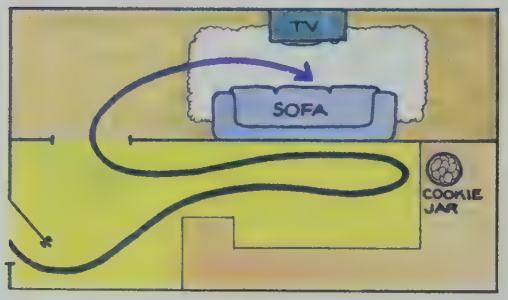


If you traced the location of the string, would you draw a curved line or a straight line?

You know some geometric shapes are made up of curved lines.

If you look at a map, you could see curved lines used to represent rivers and some roads or streets.

You would probably draw a curve if you traced the path you take after you go in the door at home.



If you looked at the edge of a step into your house or the frame of the door or a window, you would see straight lines. If you were to draw a straight line, would you use something more than pencil and paper?

As you pick up objects in the kitchen, you can feel straight and curved lines. You can feel a curve as your fingers go around the top of a can of vegetables or the edge of a bowl, plate, or drinking glass.

You can feel straight lines as you let your fingers touch the edge of a milk carton, a box of cereal, or the edge of a pancake turner.

The angles on these pages are math types. Your goal is to explore what an angle is and what it isn't.

1. Which of the drawings below represent straight lines?

a

b

c

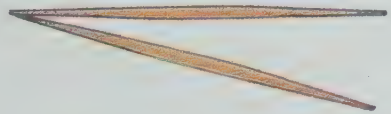
d

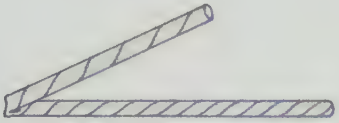
e

f

2. The focus of this chapter is on angles. An angle must have two straight sides that come together at one point.

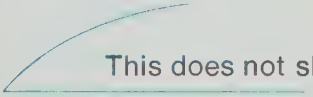
Two toothpicks can represent an angle or a bent soda straw can represent an angle.





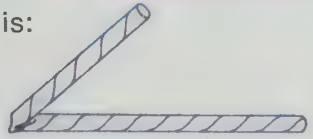
The kind of angles you will be looking at are formed by straight sides only.

This shows an angle.



This does not show an angle.

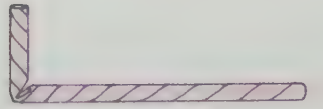
a Bend a soda straw so that it looks like this:
The straw takes the shape of an angle.
Does the angle have straight sides?



b Push the sides of your soda straw closer together.
This action makes the angle smaller.

c Let the straw take the shape of a square corner.

A square-corner angle is called a *right angle*.



d Let the straw take the shape of a straight line.
Can you still see the two sides?

This kind of angle is called a *straight angle*.

For this project you will need masking tape and a door that can be opened.

Open the door a little way. Put down a piece of tape on the floor that shows the location of the open door.

Close the door. Put another piece of tape on the floor to show the location of the door now. Open the door all the way. Do you see an angle formed by the tape? Does the angle have two straight sides?

Now use the tape and the door to make a right angle.

1. Can you use the door to mark a straight angle?

2. Can you use the door to mark an angle larger than a straight angle?

3. What kind of door do you need in order to mark an angle larger than a straight angle?

4. Think; then draw—

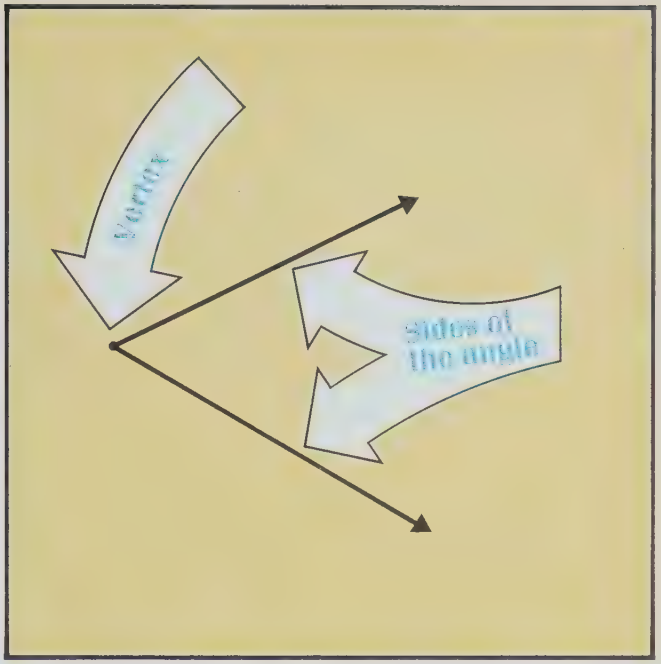
   a angles that can be formed by a door that opens into a room;

   b angles that can be formed by a swinging door;

   c angles that can be formed by a revolving door.

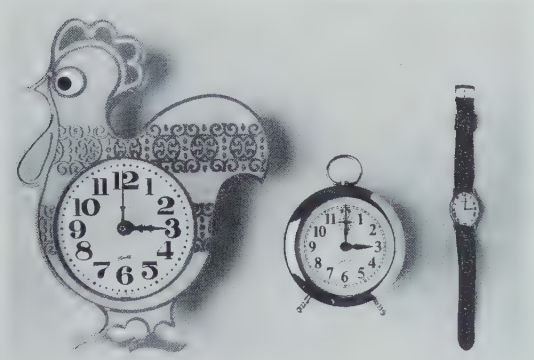
1. An angle is formed by two rays joined at their endpoints. The point where they are joined is called the *vertex* of the angle.
Remember—a ray has one endpoint and continues in a straight line forever.

   The length of the side of an angle in a drawing can be made longer without changing the size of the angle.



2. Look at the hands on your room clock. They show an angle. Each hand forms a side of an angle, and the pin in the center is located at the vertex of the angle.

   The hands of the clocks shown below all show the same angle. The hands are different lengths, but the angles are the same size.



3. Could the same-size angle be shown if the hands pointed to another time? Back up your answer with examples. Be careful. This is tricky.

What makes one angle larger than another?

1. Look at these clocks. Each shows a different hour.

a Is the angle formed by the clock hands in a smaller or larger than the angle in b?

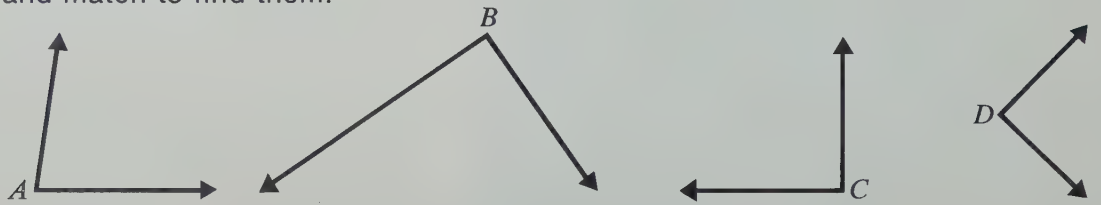
b Is the angle in c larger than the angle in b?

c Which of the four angles is largest?

2. Draw ten angles of your own. Make some less than the size of a right angle. Make some greater.

Do any of your angles look as if they might be the same size? One way to find out if they are the same size is to trace one angle and put the tracing on top of another angle.

3. There are three same-size angles below. Trace and match to find them.



The length of the sides didn't make any difference, did it? The size is determined by the opening of the sides. Angles that have the same opening are congruent.

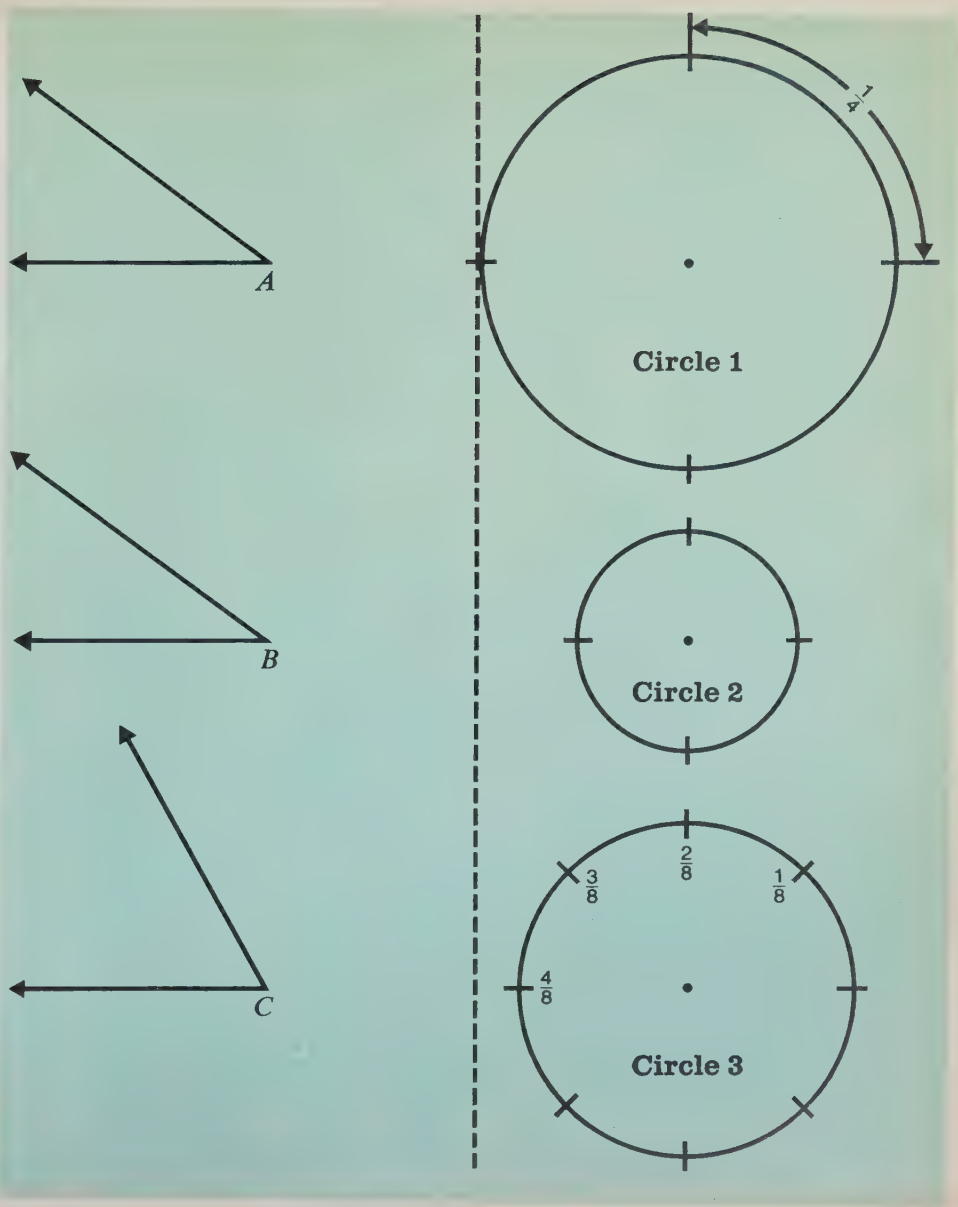
We can see how big an angle is by measuring it with a circle. Circle 1 has been divided into 4 equal parts. We can measure an angle and tell whether it is less than, equal to, or greater than $\frac{1}{4}$ of a circle. Trace all the angles and circles and the broken line, exactly as shown. Be sure your paper doesn't slip.

1. Fold your tracing on the broken line, from right to left. The vertex of each angle will be at the center of a circle if you folded carefully. Look at angle A. Is it less than $\frac{1}{4}$ of circle 1?

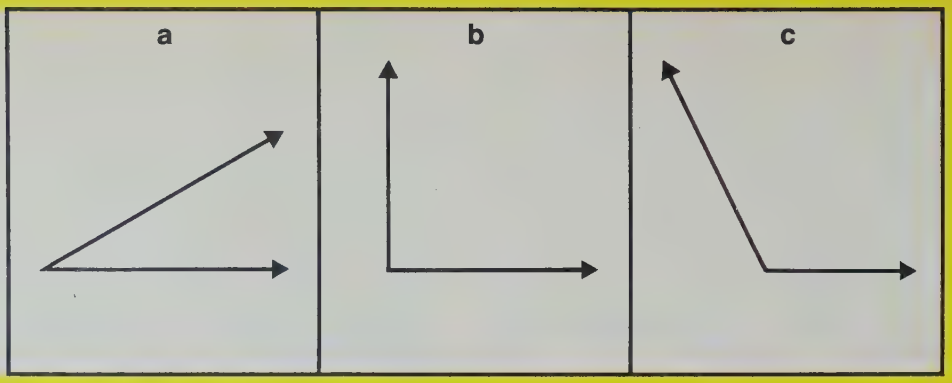
2. Now look at angle B. Measure it with circle 2. Is it less than $\frac{1}{4}$ of a circle? It should be; it's the same size as angle A.

3. Circle 3 is divided in 8 equal parts. Is angle C greater than, less than, or equal to $\frac{1}{8}$ of a circle?

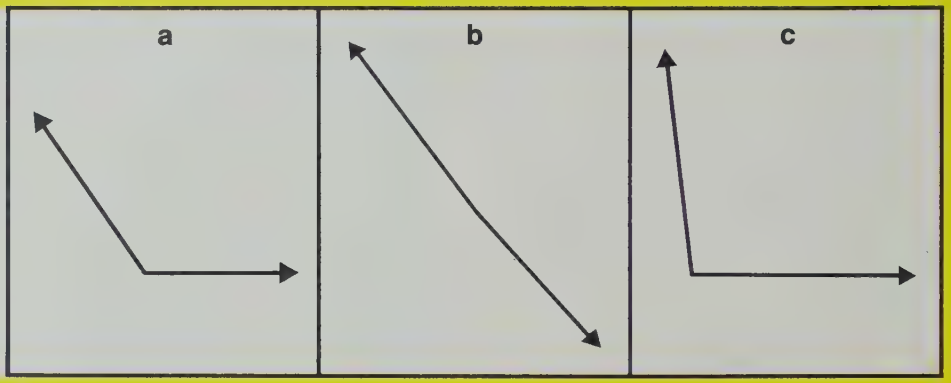
SAVE YOUR TRACING



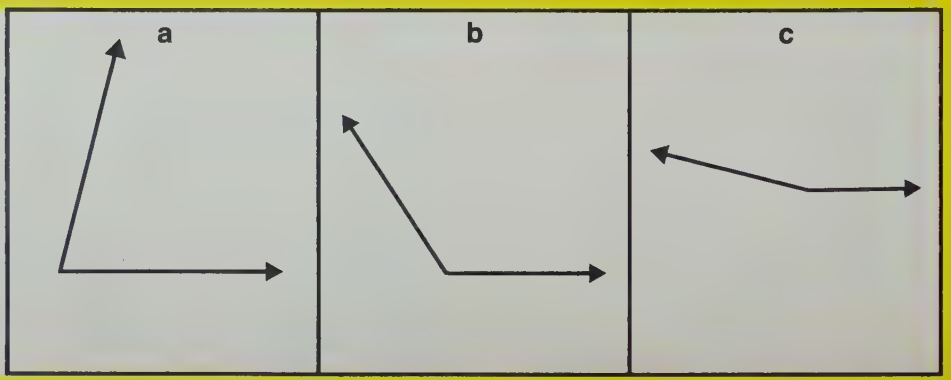
1. Use your tracing of circle 2 from the preceding page to measure each of these angles. Tell whether they are greater than, less than, or equal to $\frac{1}{4}$ of a circle. Be sure the center of the circle is at the vertex of the angle.



2. Use your tracing of circle 2 and tell whether these angles are greater than, less than, or equal to $\frac{2}{4}$ of a circle.



3. Use your tracing of circle 3 to measure these angles. Write the name of the fraction of a circle their size comes closest to.



Many, many years ago it was decided to divide a circle into 360 equal parts. An angle that has the same size as one part of the 360 parts looks like this:

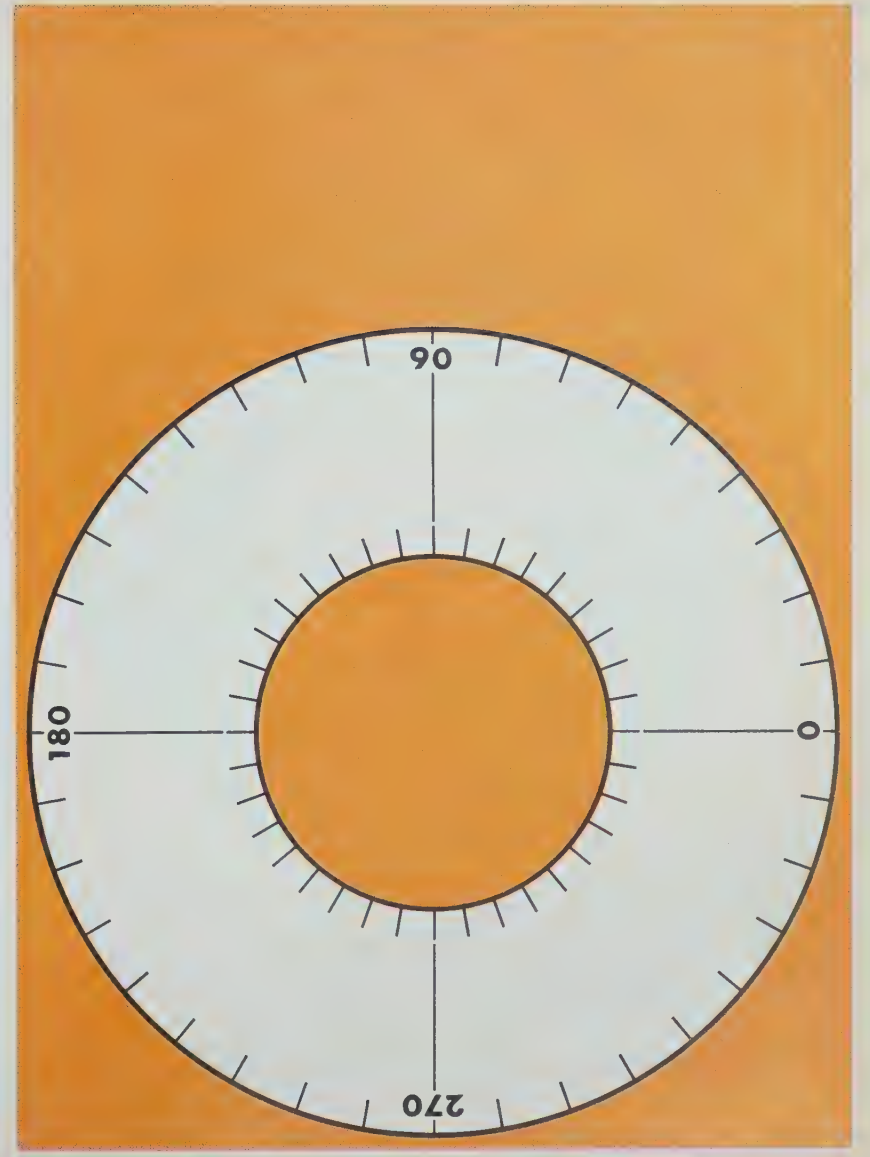
It was agreed that the size of this angle would be called 1 degree. (That name does not mean the same as the temperature measure called degree. It's too bad somebody didn't pick another name.)

Writing the word *degree* takes time. There is a symbol for the word. It is a small raised circle:

90° means "ninety degrees."

The instrument used to measure an angle opening is called a protractor. Make one of your own. Trace this.

Cut out your tracing and paste it on thin cardboard. Then cut the cardboard around the outside and center of your tracing.



1.

If you put your protractor on top of an angle now, you would not have a way to match the center of the protractor and the vertex of the angle.

a Stretch a piece of string across the hole. Tape it down so that one end of the string is on the 0° mark and the other is on the 180° mark.

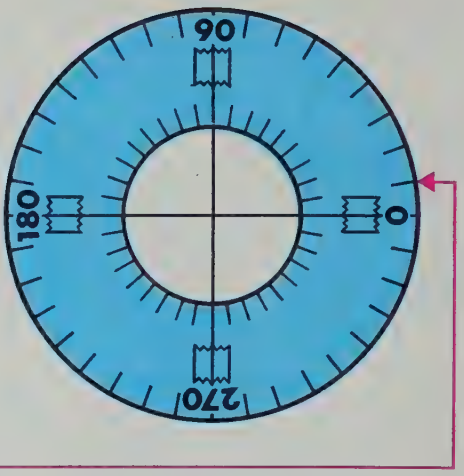
b Tape another piece of string so that one end is on the 90° mark and the other end is on the 270° mark.

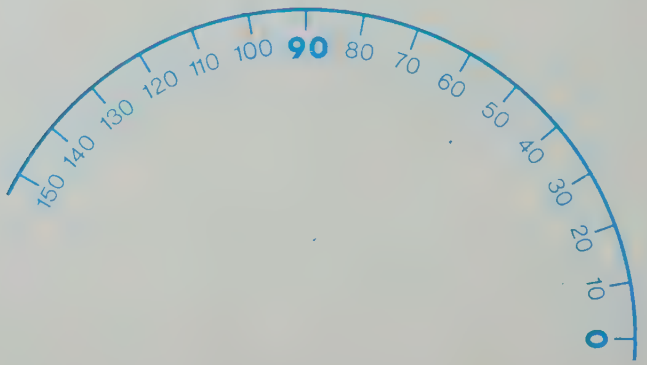
Your measuring instrument is nearly finished.

c Look at the hatch marks around the edge. How many between 0 and 90?

d What does each mark stand for?

e Label each on your protractor.





2.

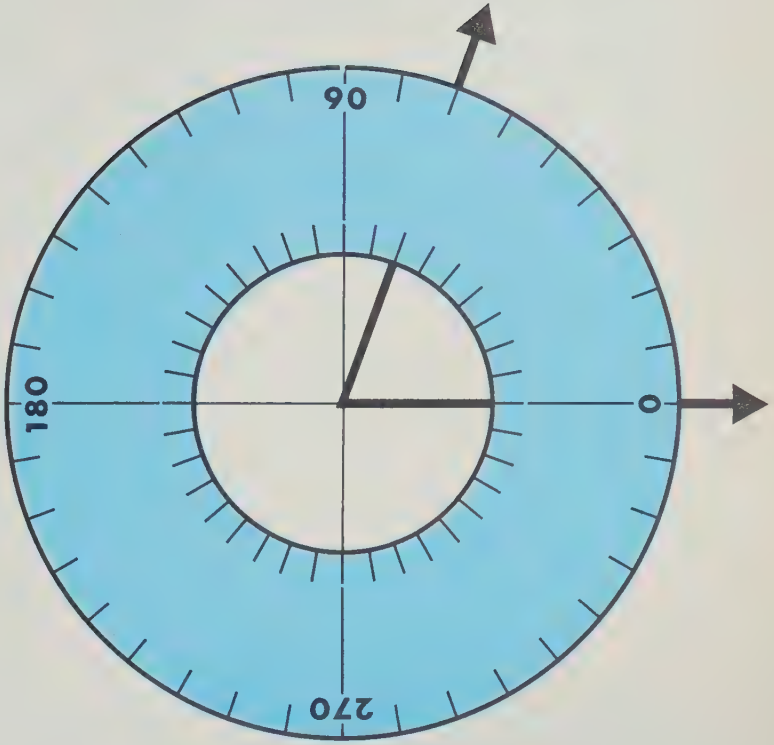
How many marks should be between 0 and 10?

a Try to put these on your protractor too. There isn't enough room to label them.

b If you made the 1° marks around the entire edge, how many marks would you have?

Now you are ready to do some measuring!

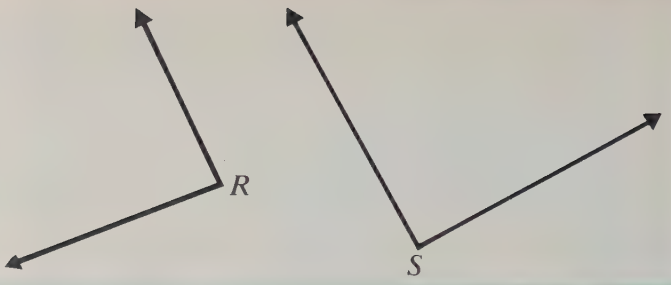
Get ready to measure an angle with a protractor. Place the protractor over the angle so that the vertex of the angle is at the center and one side of the angle is at 0. Then, read up from zero to where the other side of the angle crosses the edge of the protractor. That number tells you the number of degrees in the angle. In the diagram at the right, the angle measures about 70°.



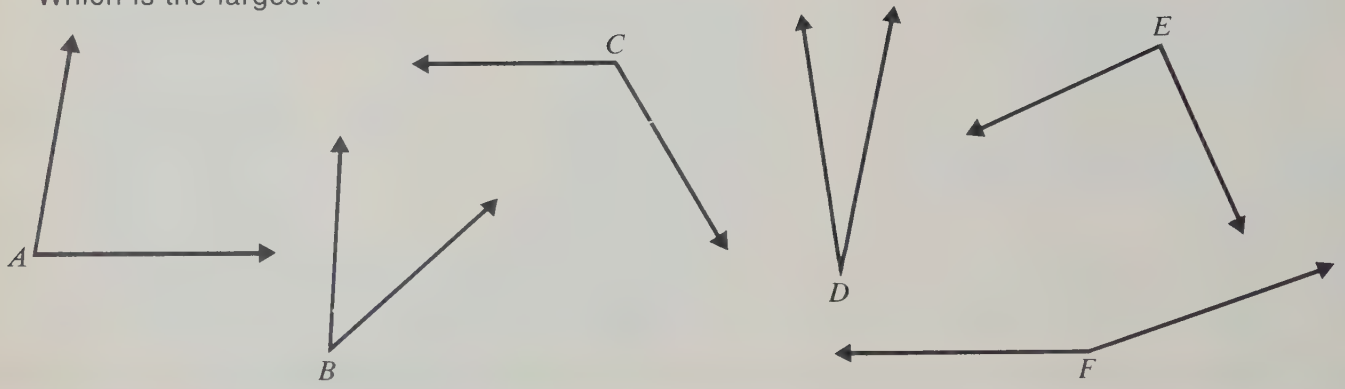
MEASURE THIS ONE

Did you get 25°?

1. Measure these two angles.
Which is larger?
How many degrees larger?



2. Measure these angles.
Which is the largest?



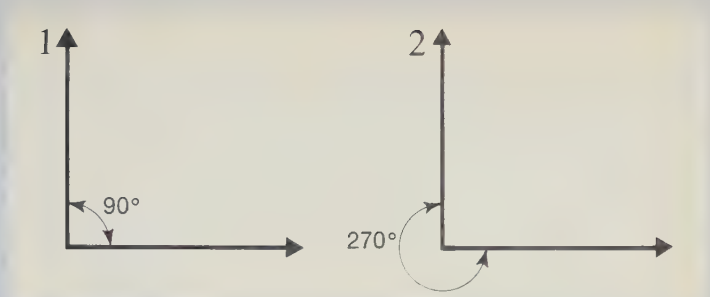
3. Can you find the right angle above?
How many degrees in a right angle?

4. Complete. Use the relation symbols $>$ and $<$.

a angle A ⍰ angle B **b** angle C ⍰ angle F **c** angle B ⍰ angle E

d angle D ⍰ angle B **e** angle C ⍰ angle E **f** angle E ⍰ angle A

Have you noticed that there are actually two angles in one? Look at the angles at the right. The arrows show the two ways that you can measure an angle.



In this chapter measure the smaller angle unless an arrow is pointing to the larger angle.

Name the angles that measure—

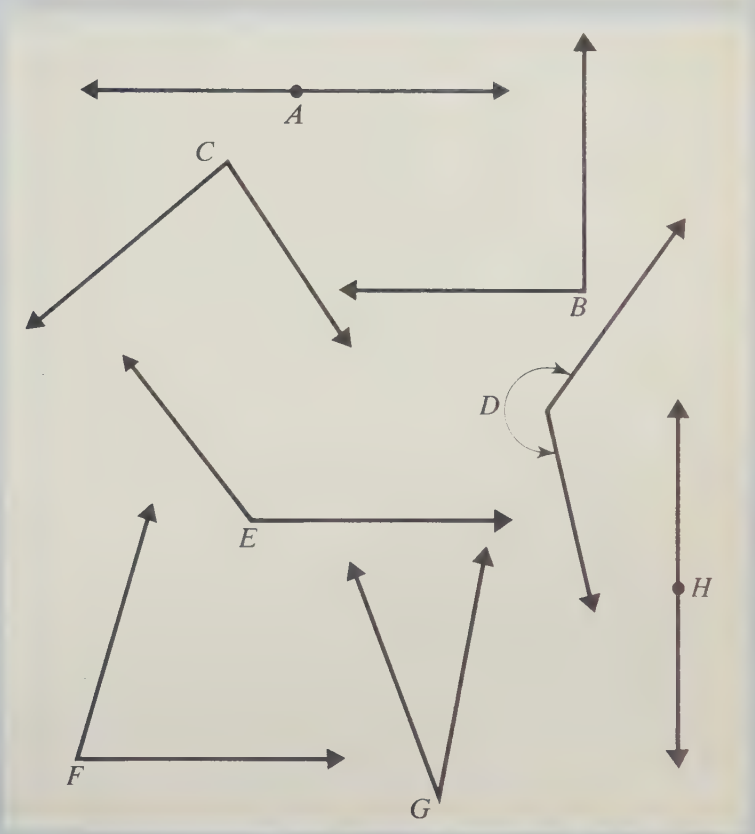
1. less than 90°.
   These angles are called *acute angles*.

2. 90°.
   These angles are called *right angles*.

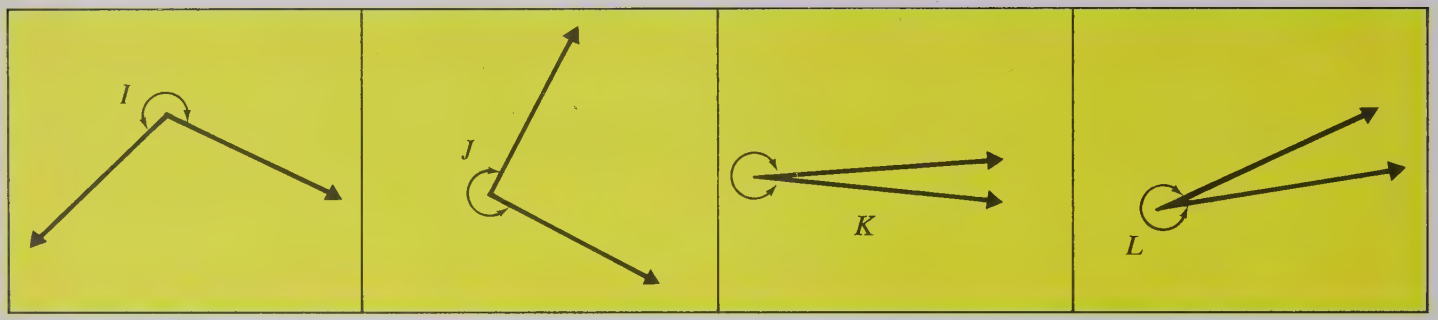
3. 180°.
   These angles are called *straight angles*.

4. more than 90°
   BUT less than 180°.
   These angles are called *obtuse angles*.

5. more than 180°.
   These angles are called *reflex angles*.



1 Measure these angles.
Then tell what kind of angle each is.

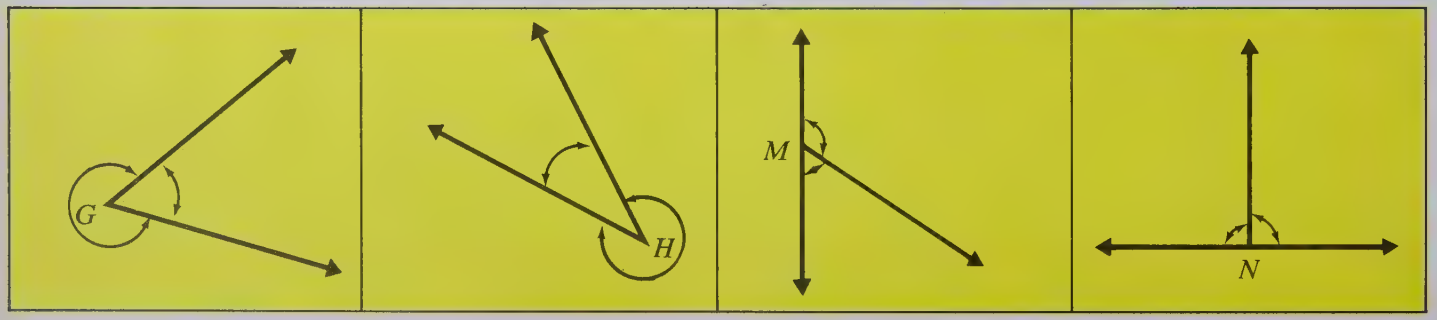


2 Measure both angle G and angle H.

a Measure the acute angle.
b Measure the reflex angle.
c How many degrees in the sum of the two angles?

3 Measure and find how many degrees in the sum of these two angles.

4 How many degrees in the sum of these two angles?

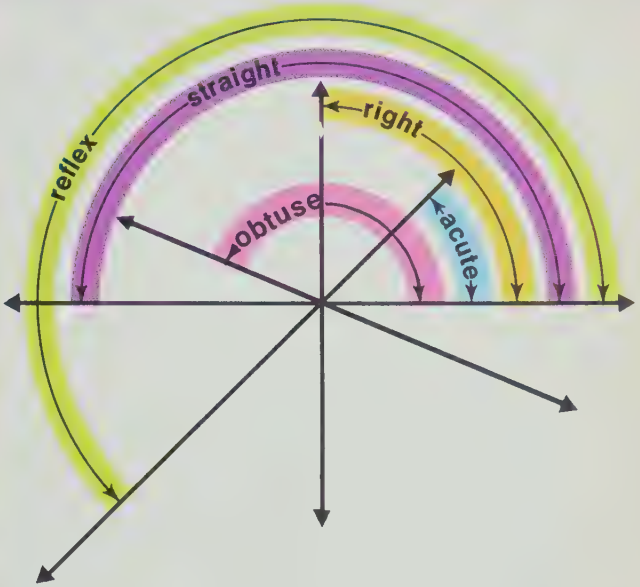


Use your protractor. Measure and label angles you see around you. Are any two the same size?

Which are the largest angles you found?
Which are the smallest?

Use the clock in your room to help you answer these questions.

1. How large an angle are the hands forming?
2. What angle is formed at two o'clock?
   at ten o'clock?
3. At what hours do the hands form
   right angles?
4. At what hours do the hands form two angles
   greater than 90°?



*5. Look for angles formed by these intersecting lines.

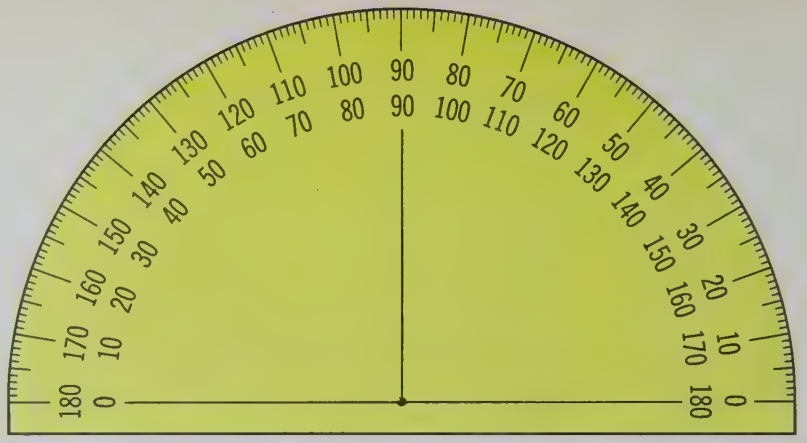
   a How many acute angles?

   b How many right angles?

   c How many obtuse angles?

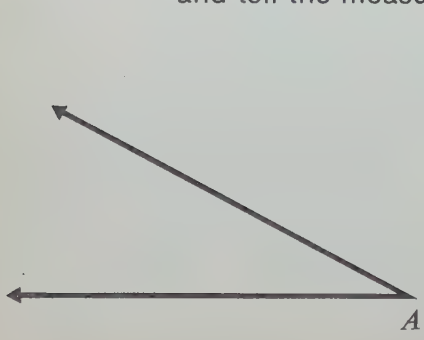
   d How many reflex angles?

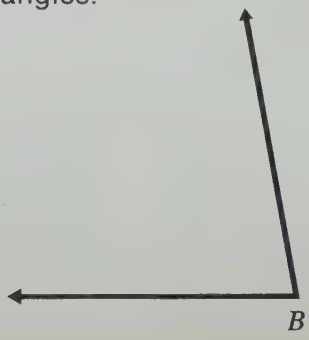
The protractor you made shows 360° in all. Most protractors that you buy in a store show only 180°. They are shaped like a half circle.

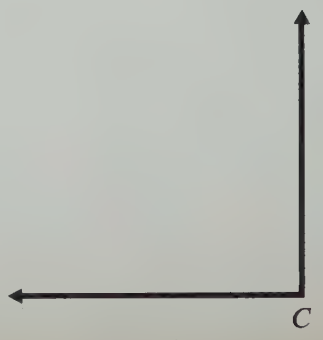


They might be larger or smaller than the one pictured above. There is one difference between the way the numerals are marked on the ones purchased from a store and the way they are marked on your handmade one. What is the difference? Mark your protractor the same way.

1. Use your new 0 location, read the inside scales, and tell the measures of these angles.



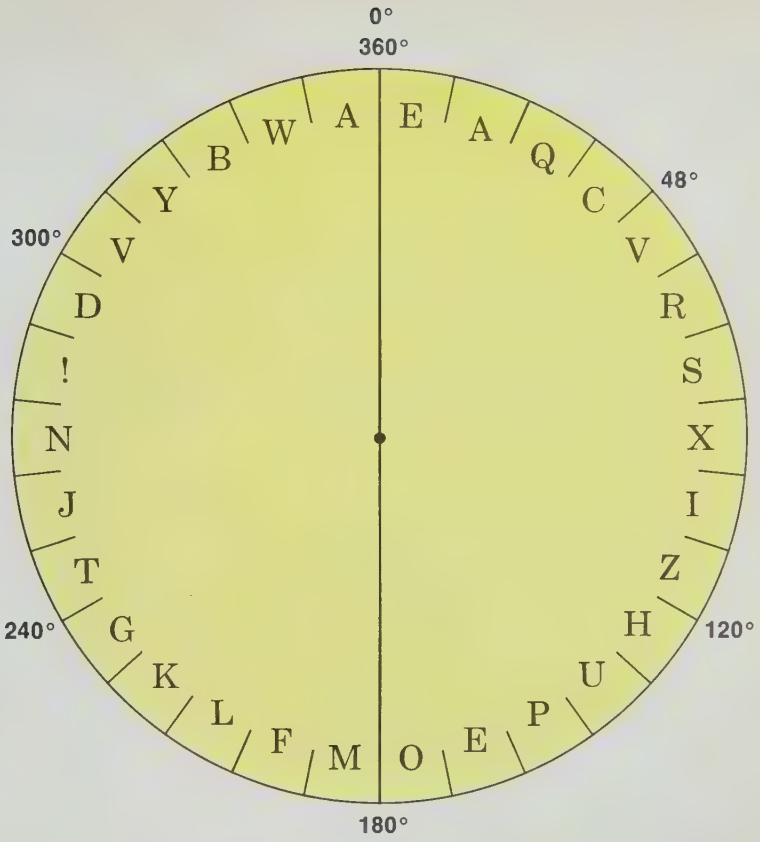




You've stumbled across a secret code. And you have finally found the key to it. It is a circle divided into 30 equal sections. Each section measures 12°. And each section has a special meaning.

First trace this special protractor. Then write the degree measure for each mark. You may use your protractor for this.

Now try to crack the code!
Each degree measure can be replaced by a letter. Can you figure out the message?



323° 177° 143° * 130° 15° 310° 8° *

242° 128° 166° * 100° 293° 6° 350°

Get it?

238° 173° * 243° 169° * 98° 242° 284°

Now make up some coded messages of your own.
Can your friends figure them out?

IT'S A LONG, LONG TRAIL TO ANGLE

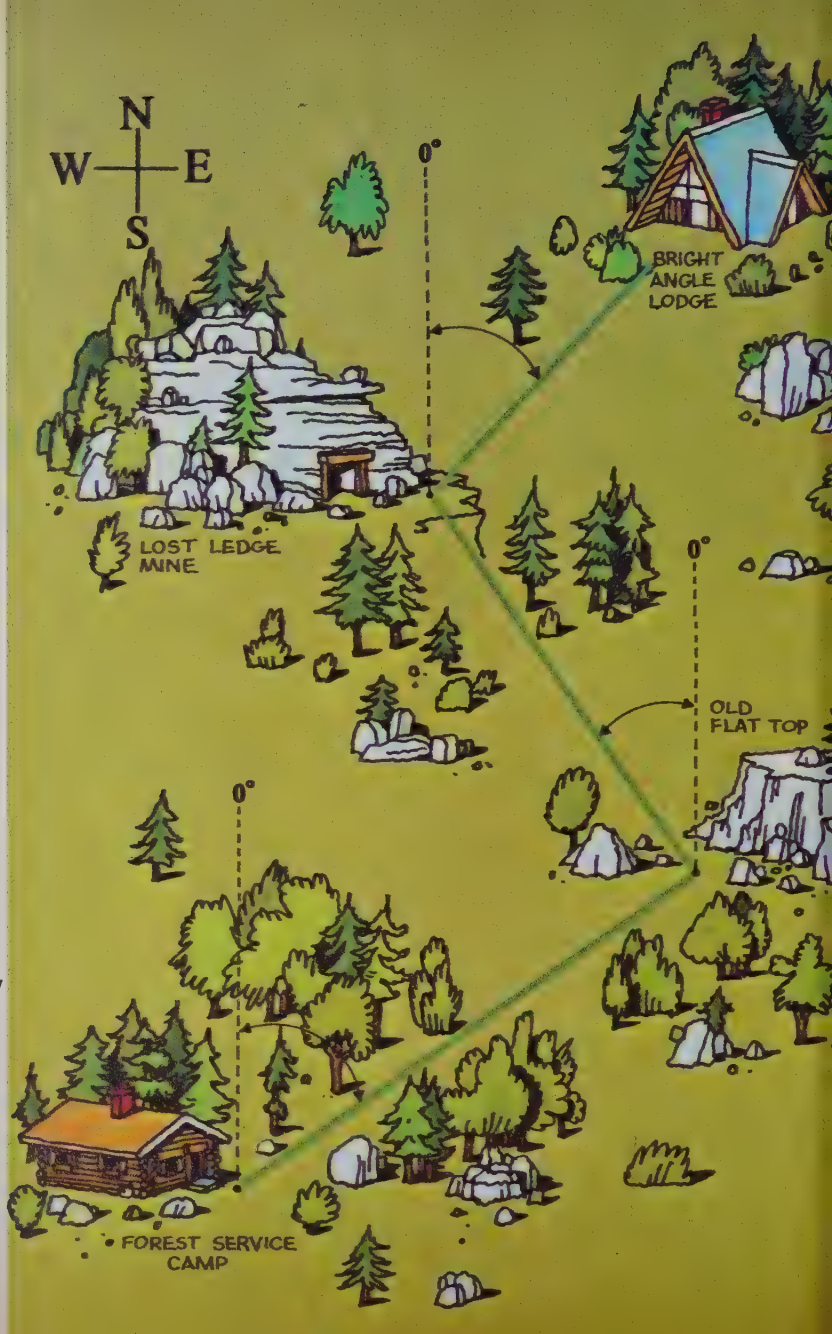
Tommy Trailblazer gave the Explorers Club directions to get from Forest Service Camp to Bright Angle Lodge. He told them what headings they should follow. A *heading* is a degree measure that tells how far a given direction is from north.

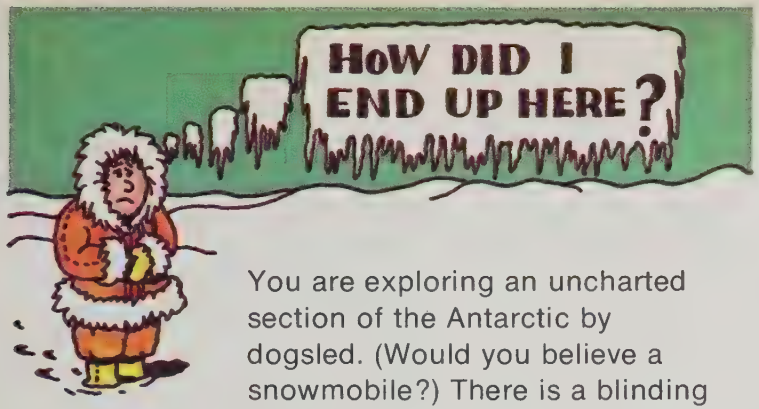
He wanted to make sure everyone knew exactly where they were going, so he gave them headings to checkpoints. He said, "The heading to follow from the Camp to Old Flat Top is ? degrees east of north."

1. Use your protractor. Find the heading Tommy gave to get from the camp to Old Flat Top.

2. Then Tommy told them to go on the heading ? degrees west of north to get to Lost Ledge Mine. What heading did they follow?

3. On the last leg of the trip Tommy told them to follow a heading of ? degrees east of north. What was their heading from Lost Ledge Mine to Bright Angle Lodge?





You are exploring an uncharted section of the Antarctic by dogsled. (Would you believe a snowmobile?) There is a blinding snowstorm. You head straight south towards your camp, but your compass is off by 10°.

1. You travel 2 km . Notice on the map that you are already more than 0.3 km off course.

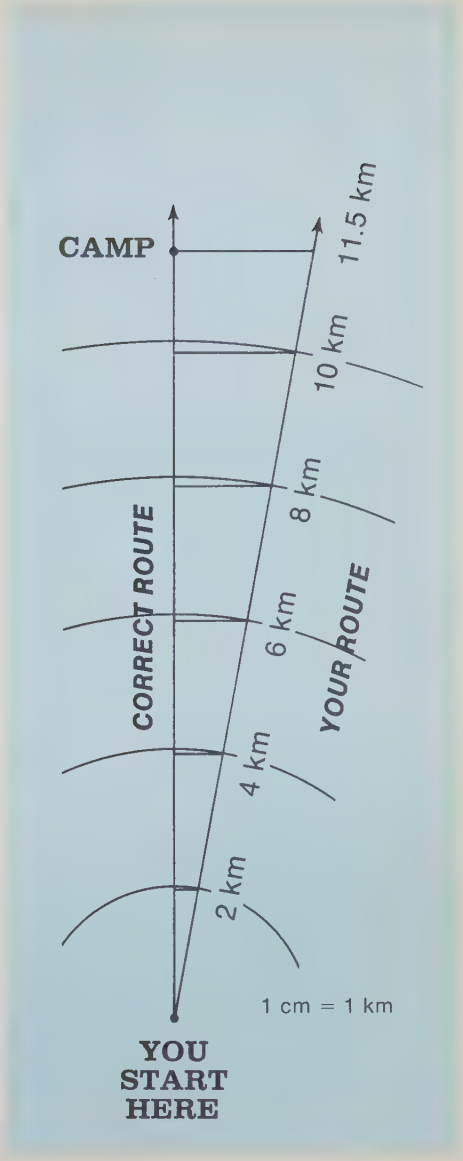
2. You travel 2 km farther. You are farther off course than before. About how far?

3. Are you farther off course after 6 km? At least a kilometre off?

4. How far off course are you after travelling 8 km? After 10 km?

5. After travelling 11.5 km on the wrong path, you are straight across from your camp, but quite far from it. Measure and record this distance.

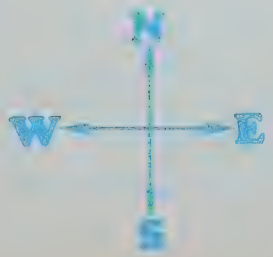
A 10° mistake in the Antarctic is serious, isn't it?



I WON'T RIDE WITH YOU AGAIN

Planes use very complicated electronic equipment to keep on course. Imagine what would happen if the equipment were not so accurate. Suppose a plane were *only* 1° off course. If that plane were flying from Montreal to Vancouver, it might end up in the Coast Mountains some 55 km away.

Use your protractor to see if the incorrect course shown on the map is indeed 1° off.



Do you see how important a 1° error can be? This plane flew only about 3235 km, and it was almost 55 km off course. Can you guess what would happen if a plane flew 1° off course for many thousands of kilometres.

SO NEAR, AND YET SO FAR

Suppose you were planning a trip to Venus in a spaceship. Venus is 38 640 000 km away. At that distance it would take you almost 5 months to get there. If your flight path were 1° off course, you would miss the planet. Can you imagine by how many kilometres you would miss Venus?

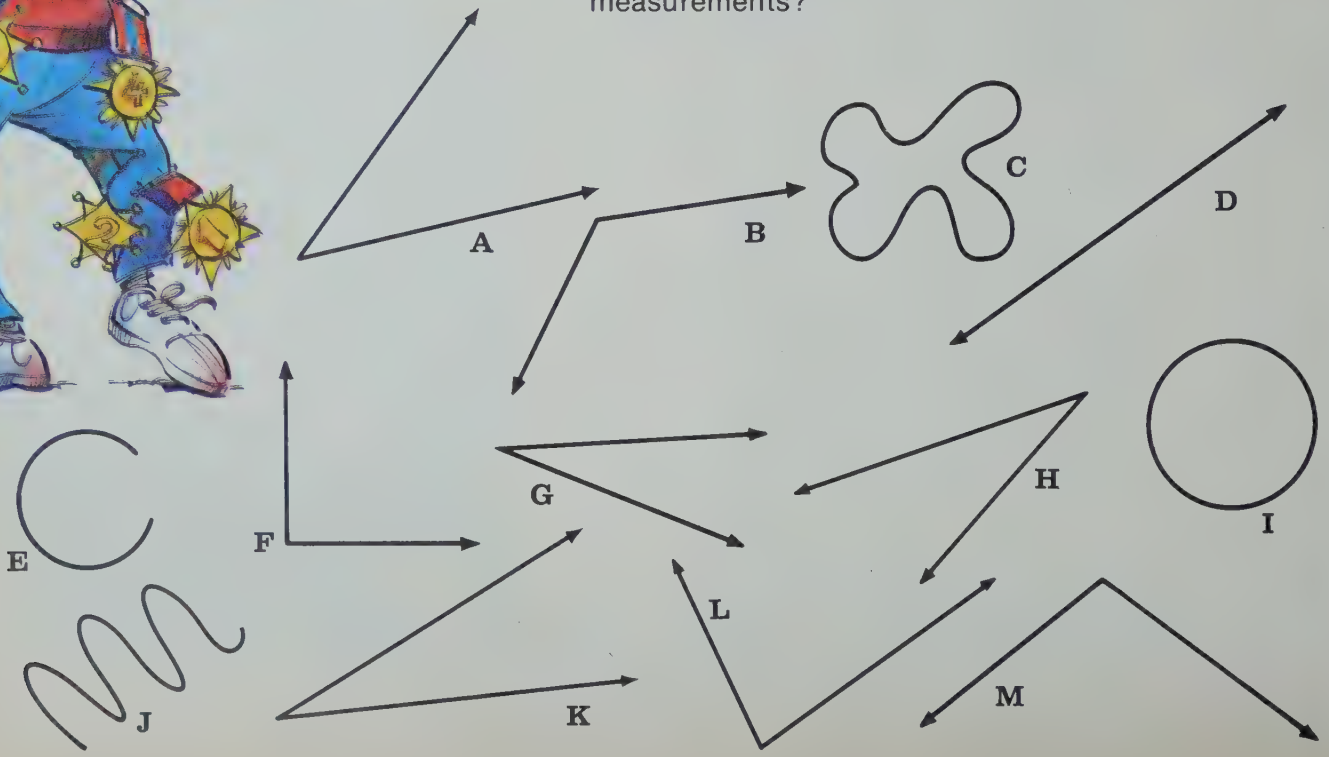
What if you were flying to Pluto and you were 1° off course? Can you imagine by how many kilometres you would miss that planet? Even if you were off course by only 0.5°, you would still miss it by thousands of kilometres. Pluto is 4 266 500 000 km away. It would take you 30 years to travel that distance! Can you imagine travelling for 30 years and then missing the planet? You couldn't let that happen, so you would need equipment that measures angles smaller than 1°.

If you would like to know more about very small angles, you may want to read a book on astronomy or space travel.

CHECKOUT

Look at the figures below.
Use them to answer these questions.

1. Which figures show angles?
2. Which angles show right angles?
3. Which angles are obtuse?
4. Which angles are acute?
5. Measure the angles with your protractor. Tell how many degrees each angle measures. Do your answers to questions 2, 3, and 4 agree with your measurements?

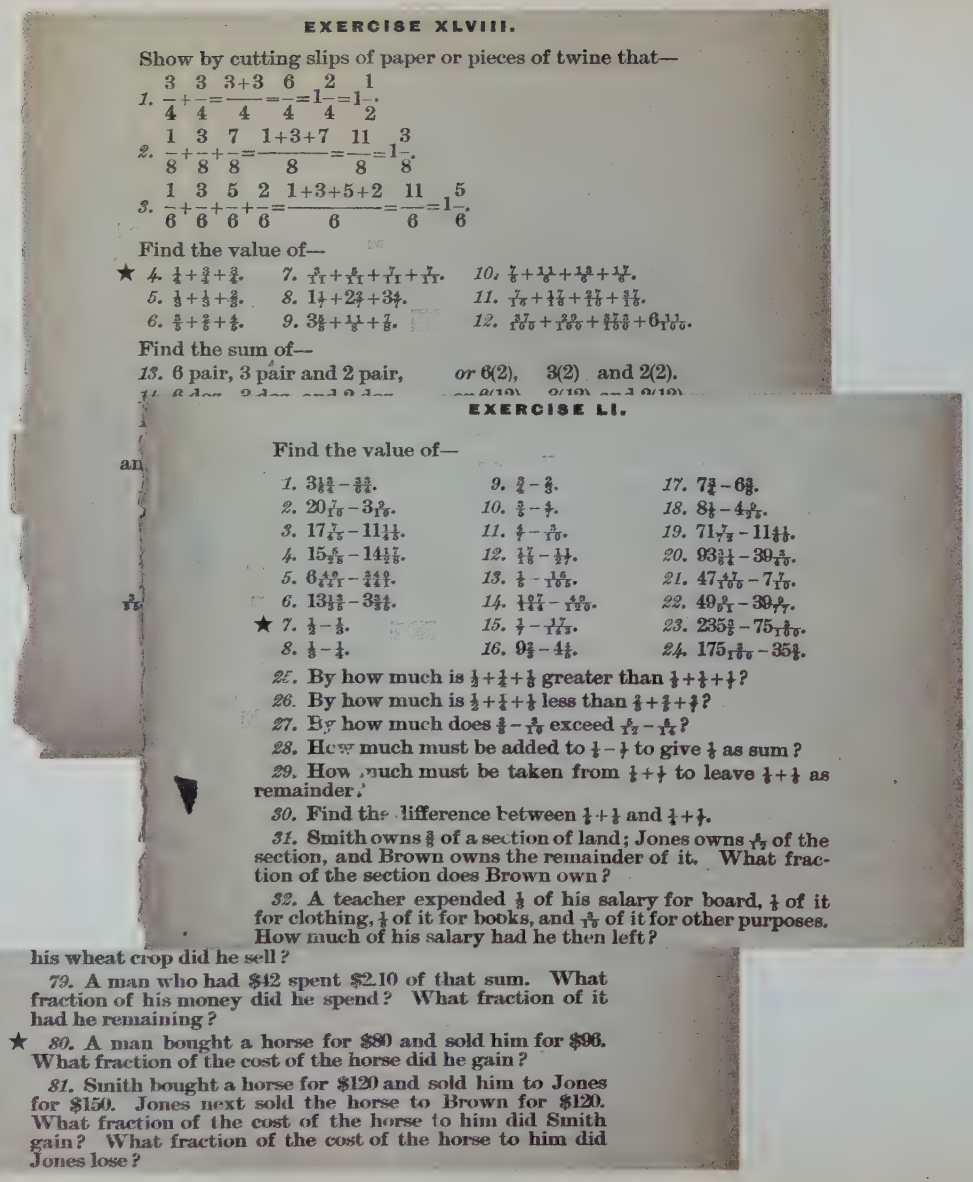


10

FRACTIONS +, −, AND ÷

These pages come from an old arithmetic book used around 1895. As you can see, your great-great-grandparents had to work very hard. Today we don't have to worry so much about difficult fractions. See if you can do the problems. Your great-great-grandparents could.

Do the problems that are marked with a ★.



EXERCISE XLVIII.

Show by cutting slips of paper or pieces of twine that—

1. $\frac{3}{4}+\frac{3}{4}=\frac{3+3}{4}=\frac{6}{4}=1\frac{2}{4}=1\frac{1}{2}$.
2. $\frac{1}{8}+\frac{3}{8}+\frac{7}{8}=\frac{1+3+7}{8}=\frac{11}{8}=1\frac{3}{8}$.
3. $\frac{1}{6}+\frac{3}{6}+\frac{5}{6}+\frac{2}{6}=\frac{1+3+5+2}{6}=\frac{11}{6}=1\frac{5}{6}$.

Find the value of—

★ 4. $\frac{1}{4}+\frac{3}{4}+\frac{3}{4}$.
5. $\frac{1}{3}+\frac{1}{3}+\frac{2}{3}$.
6. $\frac{2}{5}+\frac{2}{5}+\frac{4}{5}$.
7. $\frac{3}{11}+\frac{8}{11}+\frac{7}{11}+\frac{7}{11}$.
8. $1\frac{1}{7}+2\frac{2}{7}+3\frac{4}{7}$.
9. $3\frac{5}{8}+\frac{11}{8}+\frac{7}{8}$.
10. $\frac{7}{6}+\frac{11}{6}+\frac{13}{6}+\frac{17}{6}$.
11. $\frac{7}{18}+\frac{17}{18}+\frac{27}{18}+\frac{37}{18}$.
12. $\frac{37}{100}+\frac{20}{100}+\frac{37}{100}+6\frac{11}{100}$.

Find the sum of—

13. 6 pair, 3 pair and 2 pair, *or* 6(2), 3(2) and 2(2).

EXERCISE LI.

Find the value of—

1. $3\frac{13}{16}-3\frac{3}{16}$.
2. $20\frac{7}{10}-3\frac{2}{10}$.
3. $17\frac{7}{45}-11\frac{11}{45}$.
4. $15\frac{5}{24}-14\frac{17}{24}$.
5. $6\frac{40}{441}-\frac{340}{441}$.
6. $13\frac{13}{35}-3\frac{34}{35}$.
★ 7. $\frac{1}{2}-\frac{1}{8}$.
8. $\frac{1}{8}-\frac{1}{4}$.
9. $\frac{3}{4}-\frac{3}{8}$.
10. $\frac{3}{5}-\frac{1}{2}$.
11. $\frac{1}{7}-\frac{3}{10}$.
12. $\frac{11}{16}-\frac{11}{17}$.
13. $\frac{1}{6}-\frac{16}{105}$.
14. $\frac{127}{144}-\frac{40}{120}$.
15. $\frac{1}{7}-\frac{17}{143}$.
16. $9\frac{3}{5}-4\frac{4}{5}$.
17. $7\frac{3}{4}-6\frac{3}{8}$.
18. $8\frac{1}{5}-4\frac{9}{25}$.
19. $71\frac{7}{72}-11\frac{11}{88}$.
20. $93\frac{31}{64}-39\frac{3}{40}$.
21. $47\frac{47}{100}-7\frac{7}{10}$.
22. $49\frac{9}{91}-39\frac{9}{77}$.
23. $235\frac{3}{8}-75\frac{1}{180}$.
24. $175\frac{3}{10}-35\frac{3}{5}$.

25. By how much is $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ greater than $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$?
26. By how much is $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ less than $\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$?
27. By how much does $\frac{3}{8}-\frac{3}{16}$ exceed $\frac{5}{12}-\frac{5}{14}$?
28. How much must be added to $\frac{1}{4}-\frac{1}{7}$ to give $\frac{1}{2}$ as sum?
29. How much must be taken from $\frac{1}{4}+\frac{1}{7}$ to leave $\frac{1}{4}+\frac{1}{8}$ as remainder?
30. Find the difference between $\frac{1}{4}+\frac{1}{8}$ and $\frac{1}{4}+\frac{1}{7}$.
31. Smith owns $\frac{3}{8}$ of a section of land; Jones owns $\frac{4}{7}$ of the section, and Brown owns the remainder of it. What fraction of the section does Brown own?
32. A teacher expended $\frac{1}{3}$ of his salary for board, $\frac{1}{5}$ of it for clothing, $\frac{1}{7}$ of it for books, and $\frac{2}{15}$ of it for other purposes. How much of his salary had he then left?

his wheat crop did he sell?

79. A man who had $42 spent $2.10 of that sum. What fraction of his money did he spend? What fraction of it had he remaining?

★ 80. A man bought a horse for $80 and sold him for $96. What fraction of the cost of the horse did he gain?

81. Smith bought a horse for $120 and sold him to Jones for $150. Jones next sold the horse to Brown for $120. What fraction of the cost of the horse to him did Smith gain? What fraction of the cost of the horse to him did Jones lose?

Your goal is to learn about the addition, subtraction, and division of fractions.

Complete these addition problems.

Set E (easy)

1. Start with an easy one: $\frac{1}{7} + \frac{5}{7} = \underline{?}$ There is no renaming.

2. Here is a rather easy one: $\frac{1}{8} + \frac{3}{8}$ You will have to rename the sum.

Set A (average)

3. Try an average type: $\frac{2}{3} + \frac{1}{6}$ You will rename to get a common denominator.

4. Here is another average type: $\frac{2}{3} + \frac{5}{6}$ You will rename to get a common denominator. And the sum will need renaming too.

5. This is average: $\frac{3}{4} + \frac{4}{5}$. Rename both fractions to get a common denominator. And rename the sum.

Set H (hard)

6. This one isn't hard unless you don't know how to do it. Try it.

$$\begin{array}{r} 3\frac{2}{3} \\ +\,2\frac{1}{2} \\ \hline ? \end{array}$$

There is renaming all over the place.

Need help? You'll find it on practice pages that follow. But check out subtraction first, please. Complete the following.

1. Easy type: $\frac{5}{9} - \frac{1}{9} = \underline{?}$

2. Rather easy type: $\frac{5}{6} - \frac{1}{6} = \underline{?}$

3. Average type: $\frac{1}{2} - \frac{1}{4} = \underline{?}$

4. Another average type: $\frac{3}{4} - \frac{2}{8} = \underline{?}$

There is more renaming involved in these.

5. $\frac{2}{3} - \frac{1}{4}$ 6. $2 - \frac{3}{4}$

7. And these are your challenges.

a $3\frac{1}{2} - 1\frac{1}{8}$ **b** $4\frac{2}{3} - 2\frac{1}{5}$

c $5\frac{1}{4} - 2\frac{3}{4}$ **d** $4\frac{1}{3} - 2\frac{1}{2}$

The easy type of problem had a common denominator
(**Set E** on page 241).

$\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2}=\frac{2}{2}$ $\qquad$ $\frac{1}{5}+\frac{3}{5}=\frac{1+3}{5}=\frac{?}{5}$

$\frac{3}{8}+\frac{5}{8}=\frac{8}{8}$ ← You know that's another name for 1.

Pay attention here. $\frac{7}{10}+\frac{9}{10}=\frac{7+9}{10}=\frac{16}{10}$ ← That needs to be renamed.

Divide the numerator and the denominator by the same number.
What is the greatest common factor of 16 and 10? 2? Maybe 4? No! 5? No! 8? No!

$\frac{16}{10}=\frac{16\div 2}{10\div 2}=\frac{8}{5}$ ← What's the simplest name for $\frac{8}{5}$?

You probably don't ever make a mistake in addition.
But renaming can cause trouble.

Renaming is needed in the average-type addition problems too.
This might help you with problems in **Set A** on page 241.
Focus on this problem. If you know how to do this one, you
know how to do all problems of the average type.

$\frac{2}{3}+\frac{5}{8}=\frac{?}{}$

THINK

No common denominator.
Get that job done first.
Thirds can't be renamed as eighths.
You need a denominator that has a
factor of 3 and a factor of 8.
So multiply: 3×8. Use 24 as the denominator.

rename $\frac{16}{24}+\frac{15}{24}$. Then add: $\frac{16+15}{24}=\frac{31}{24}$ AND *rename* again! You finish it.

Here is a QUICK review of renaming if you need it.

1. When you need a simplest name, divide the numerator and the denominator by the same number.

Pick the number that is the greatest common factor of the numerator and the denominator. It will save you a lot of time and work.

$\frac{12}{16}$ needs renaming. What is the greatest common factor?

2 is a factor of both 12 and 16. Is 3? No. Is 4? Yes.
But is it the greatest common factor?
Try 6. Try 8. $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

Here's another. No new thinking is needed.

$\frac{15}{12}$ needs renaming. What is the greatest common factor? 2? 3? 4? 5? 6?

$\frac{15}{12} = \frac{15 \div 3}{12 \div 3} = \underline{?}$ You finish it. There is still another renaming.

2. This is the rest of the renaming story.
When you need to find a common denominator, you rename.

$\frac{3}{8} + \frac{5}{6} = \underline{?}$ You can multiply 8×6 to get a common denominator.
Or you can find a common multiple of the denominators.

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, and 48. Why can you stop at 48?

Multiples of 8 are 8, 16, 24. You can stop here, because 24 is a common multiple.

$\frac{3}{8} + \frac{5}{6}$ *Rename* $\frac{?}{24} + \frac{?}{24}$. You take it from there. Complete the addition.

1. The easy type of subtraction had a common denominator.

$\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8}$ ← This is easy to rename. Do it.

$\frac{3}{5} - \frac{1}{6}$ ← You must find a common denominator.

What is a common multiple of 5 and 6?
Multiples of 5 are 5, 10, 15, 20, 25, and 30. ← Why can you stop here?
Multiples of 6 are 6, 12, 18, 24, 30. Stop!
30 is the common multiple.

$\frac{3}{5} - \frac{1}{6}$

Rename $\frac{18}{30} - \frac{5}{30} = \frac{18-5}{30} = \underline{?}$ You finish it.

Once again, renaming is the important skill.

2. Nothing was done about mixed numbers in addition or subtraction. Take care of that now.

$2\frac{3}{5} + 1\frac{1}{5}$ or $3\frac{6}{7} - 2\frac{3}{7}$ are easier to work if you stack them.

$$\begin{array}{r} 2\frac{3}{5} \\ +\,1\frac{1}{5} \\ \hline 3\frac{4}{5} \end{array}$$

There is a common denominator.
Add the fractions first.
Then add the whole numbers.

$$\begin{array}{r} 3\frac{6}{7} \\ -\,2\frac{3}{7} \\ \hline \blacksquare\frac{\blacksquare}{\blacksquare} \end{array}$$

There is a common denominator.
Subtract fractions first.
Then subtract whole numbers.
You finish it.

You know all problems will not be that easy.
Roll up your sleeves and get ready to add and subtract mixed numbers. It won't be too bad. That is a promise.

1. John worked for $2\frac{1}{4}$ hours on Friday. He worked $5\frac{1}{2}$ hours on Saturday. He had to know how much time he worked before he could get paid.

$$\begin{array}{r} 2\frac{1}{4} \\ +\,5\frac{1}{2} \\ \hline \end{array}$$

You need a common denominator.

$$\begin{array}{r} 2\frac{1}{4} \\ +\,5\frac{2}{4} \\ \hline \blacksquare\blacksquare \end{array}$$

Where did $\frac{2}{4}$ come from?

Now you can add. Fractions first.
Then add the whole numbers.
Do you have to rename the fraction?
This time you're lucky.

There are no new rules for subtraction.

2. John worked $10\frac{1}{2}$ hours one week. He worked $7\frac{1}{4}$ hours the second week. How many more hours did he work in the first week?

$$\begin{array}{r} 10\frac{1}{2} \\ -\ 7\frac{1}{4} \\ \hline \end{array}$$

Rename

$$\begin{array}{r} 10\frac{2}{4} \\ -\ 7\frac{1}{4} \\ \hline \blacksquare\blacksquare \end{array}$$

Now you can subtract.

Do you have to *rename?*

Try these for practice. Watch the operation symbols.

LOOK OUT

Get your whole numbers all together.

	a	b	c	d	e	f	g	h
3.	$2\frac{1}{2}$	$9\frac{5}{6}$	$5\frac{5}{6}$	$6\frac{1}{5}$	$6\frac{7}{8}$	$8\frac{4}{5}$	$2\frac{1}{3}$	$3\frac{5}{6}$
	$+\,5\frac{1}{4}$	$-\,2\frac{1}{3}$	$-\,2\frac{1}{2}$	$+\,2\frac{2}{3}$	$-\,2\frac{1}{3}$	$-\,2\frac{1}{2}$	$+\,5\frac{3}{4}$	$+\,8\frac{5}{8}$

1 Subtraction of mixed numbers can be tricky.

$5\frac{1}{4}$ > *Rename* $5\frac{1}{4}$
$-2\frac{1}{2}$ $-2\frac{2}{4}$
? ?

Rename again. $4\frac{4}{4}+\frac{1}{4}$ or $4\frac{5}{4}$

This is subtraction. $-2\frac{2}{4}$

■■

2 Look at one more example before you practise.

$7\frac{1}{3}$ > *Rename* $7\frac{8}{24}$
$-2\frac{5}{8}$ $-2\frac{15}{24}$
? ?

Rename again. $6\frac{24}{24}+\frac{8}{24}$ or $6\frac{32}{24}$

$-2\frac{15}{24}$

You subtract this.

3 Take your time on this practice. Think carefully.

a	b	c	d	e	f
$3\frac{2}{5}$	$8\frac{1}{2}$	$4\frac{1}{6}$	$7\frac{2}{3}$	$2\frac{3}{4}$	$3\frac{1}{3}$
$-1\frac{2}{3}$	$-2\frac{7}{8}$	$-2\frac{4}{5}$	$-2\frac{7}{8}$	$-1\frac{9}{10}$	$-2\frac{5}{6}$

4

4	5	$6\frac{1}{4}$	$8\frac{1}{6}$	3	$5\frac{1}{8}$
$-2\frac{1}{8}$	$-2\frac{3}{5}$	$-2\frac{1}{8}$	$-2\frac{1}{8}$	$-1\frac{3}{10}$	$-2\frac{3}{4}$

Yes, you have to rename 4 as $3\frac{8}{8}$.

If you got through these in good shape, you are an expert.
Don't feel bad if you need more practice.
They aren't the easiest problems in the world.

PROGRESS CHECK

Compute. Watch the operation symbols.

(1.) $\frac{2}{7} + \frac{3}{7}$ (2.) $\frac{5}{9} - \frac{4}{9}$ (3.) $\frac{3}{8} + \frac{3}{8}$ (4.) $\frac{9}{10} - \frac{3}{10}$

(5.) $\frac{1}{2} + \frac{3}{4}$ (6.) $\frac{5}{6} - \frac{3}{4}$ (7.) $1\frac{1}{8} + 2\frac{3}{8}$ (8.) $4\frac{3}{5} - 1\frac{1}{5}$

(9.) $2\frac{2}{3} + 1\frac{5}{6}$ (10.) $5\frac{1}{4} - 2\frac{3}{4}$ (11.) $4 - \frac{1}{6}$ (12.) $1\frac{5}{6} + 2\frac{1}{6}$

(13.) Joan worked $1\frac{3}{4}$ hours in her garden Saturday morning. She worked the same amount of time in the afternoon. How much time has she spent in her garden so far?

On Sunday she worked for another $2\frac{1}{2}$ hours. How many hours did she spend in her garden during the weekend?

(14.) In one way Pauline is fortunate. She has homework only 3 days a week. But on those days she works very hard. On Monday and Tuesday she works $1\frac{3}{4}$ hours each day. How many hours has she worked so far? Wednesday is free, but her homework on Thursday totals $2\frac{1}{2}$ hours. How many hours of homework a week does Pauline have?

It's hard to find situations in everyday life to show that you need to know how to divide fractions.

THEN WHY LEARN HOW TO DO IT?

There are lots of reasons, people might say. But think about this reason:

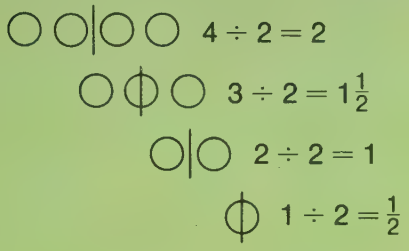
History books about mathematics don't tell much about when operations on fractions were invented. But they do give us an idea of when people needed these numbers.

Clay tablets dating back to 2400 B.C. show that the Babylonians had a way to represent fractions. They probably wouldn't have thought of it if they hadn't needed the idea of a fraction. The Egyptians got into the act hundreds of years later and carried the fraction idea further. The Romans gave the things a name, but they didn't like to use them unless they had to. Finally, the Hindus figured out an easy way to write them.

From 2400 B.C. to this year is a long time. It's hard to tell when people started to compute with fractions. But you can guess why they needed to add, subtract, and multiply.

Maybe somebody decided that since they knew how to add, subtract, multiply, and divide whole numbers, it would be great to do all those operations with fractions, too. You know the operations of addition, subtraction, and multiplication with fractions. Why not join the rest of the world and take a look at division of fractions? It won't hurt at all.

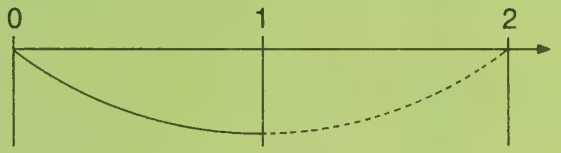
1. Look at some patterns in the answers to division problems.



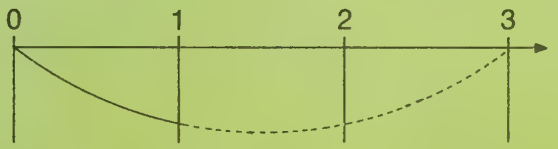
How many 2s in 1?

2. A number line would be helpful to see the action of division.

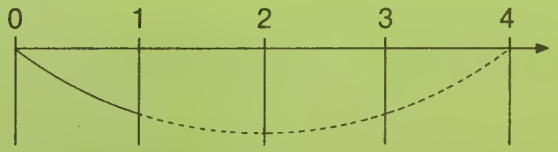
The loop shows 2.
Is there a two in 1?
How much of a two is in 1?
$1 \div 2 = \frac{1}{2}$



3. The loop shows 3.
Is there a three in 1?
How much of a three is in 1?
$1 \div 3 =$?

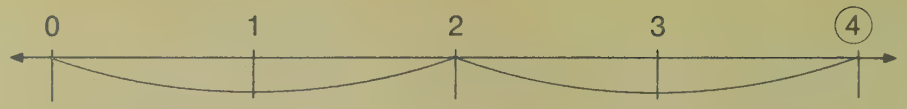


4. The loop shows 4.
Is there a four in 1?
How much of a four is in 1?
$1 \div 4 =$?

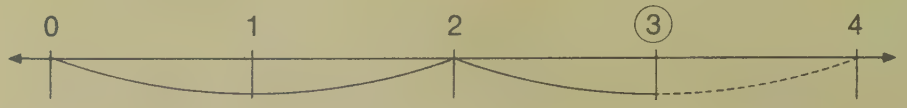


Make sure you understand. Backtrack. Start with what you know about division of whole numbers.

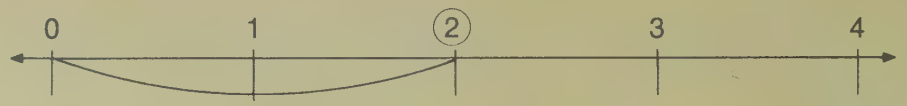
How many 2s in 4?
$4 \div 2 = \underline{?}$



How many 2s in 3?
$3 \div 2 = \underline{?}$



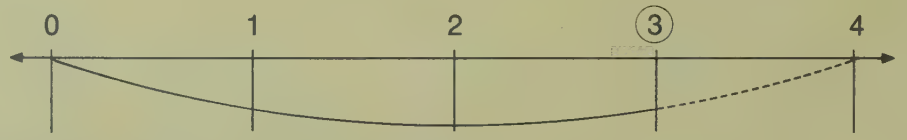
How many 2s in 2?
$2 \div 2 = \underline{?}$



How many 2s in 1?
$1 \div 2 = \underline{?}$

Does the number line show $\frac{1}{2}$ of two in 1?

Look at one more example.
How many 4s in 3?
$3 \div 4 = \frac{?}{4}$



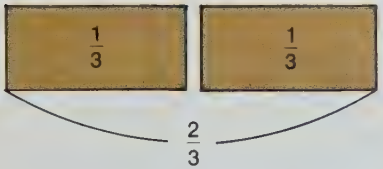
Complete these sentences. Each quotient will be a fraction.

1. $2 \div 3 = \underline{?}$ **2.** $4 \div 5 = \underline{?}$ **3.** $3 \div 5 = \underline{?}$ **4.** $8 \div 9 = \underline{?}$

5. $6 \div 11 = \underline{?}$ **6.** $4 \div 7 = \underline{?}$ **7.** $5 \div 7 = \underline{?}$ **8.** $4 \div 9 = \underline{?}$

Extra practice on page 267—set I

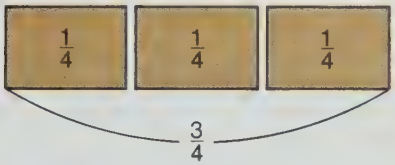
1. After dinner, he had $\frac{2}{3}$ of a meat loaf left. How many $\frac{1}{3}$-pieces did he have?



How many $\frac{1}{3}$s in $\frac{2}{3}$?

$\frac{2}{3} \div \frac{1}{3} = 2$

2. She had $\frac{3}{4}$ of a chocolate bar. How many $\frac{1}{4}$-pieces did she have?

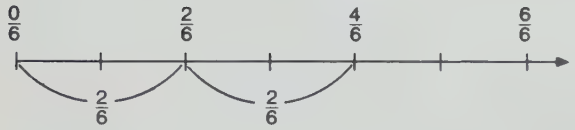


How many $\frac{1}{4}$s in $\frac{3}{4}$?

$\frac{3}{4} \div \frac{1}{4} = 3$

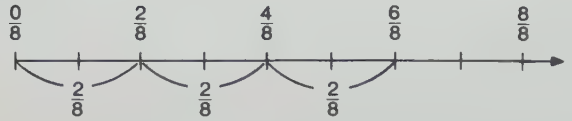
3. How many $\frac{2}{6}$s in $\frac{4}{6}$?

$\frac{4}{6} \div \frac{2}{6} = \underline{?}$



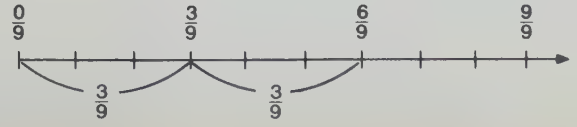
4. How many $\frac{2}{8}$s in $\frac{6}{8}$?

$\frac{6}{8} \div \frac{2}{8} = \underline{?}$

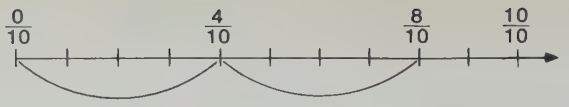


5. How many $\frac{3}{9}$s in $\frac{6}{9}$?

$\frac{6}{9} \div \frac{3}{9} = \underline{?}$



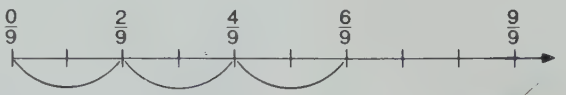
How many $\frac{4}{10}$s in $\frac{8}{10}$?



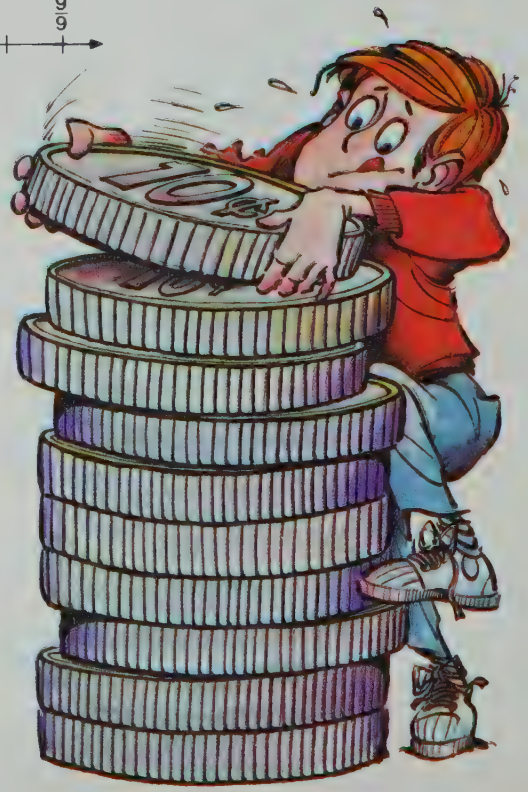
$\frac{8}{10} \div \frac{4}{10} = \underline{?}$

As you work, pay special attention to the denominators.

How many $\frac{2}{9}$s in $\frac{6}{9}$?



$\frac{6}{9} \div \frac{2}{9} = \underline{?}$



Use a number line to help when necessary.

1. How many $\frac{3}{8}$s in $\frac{9}{8}$? $\frac{9}{8} \div \frac{3}{8} = \underline{?}$
2. How many $\frac{2}{5}$s in $\frac{4}{5}$? $\frac{4}{5} \div \frac{2}{5} = \underline{?}$
3. How many $\frac{3}{9}$s in $\frac{9}{9}$? $\frac{9}{9} \div \frac{3}{9} = \underline{?}$
4. How many $\frac{2}{3}$s in $\frac{4}{3}$? $\frac{4}{3} \div \frac{2}{3} = \underline{?}$
5. How many $\frac{3}{4}$s in $\frac{9}{4}$? $\frac{9}{4} \div \frac{3}{4} = \underline{?}$
6. How many $\frac{2}{5}$s in $\frac{6}{5}$? $\frac{6}{5} \div \frac{2}{5} = \underline{?}$
7. How many $\frac{1}{10}$s in $\frac{7}{10}$? $\frac{7}{10} \div \frac{1}{10} = \underline{?}$
8. How many $\frac{3}{12}$s in $\frac{9}{12}$? $\frac{9}{12} \div \frac{3}{12} = \underline{?}$

What did you notice about the denominators in every problem?

Some people think of estimating as a guessing game. The notion of estimation is so important that you should at least be able to make an "educated guess." (That means you have a *reason* for your guess.)

Can you know whether an answer will be greater than 1 or less than 1? Here's one way.

1. $\frac{5}{3} \div \frac{2}{3} \;?\; 1$

Is there one $\frac{1}{3}$ in $\frac{5}{3}$?

Is there one $\frac{2}{3}$ in $\frac{5}{3}$?

More than one $\frac{2}{3}$ in $\frac{5}{3}$?

$\frac{5}{3} \div \frac{2}{3} \;?\; 1$

2. $\frac{5}{7} \div \frac{3}{7} \;?\; 1$

Is there one $\frac{1}{7}$ in $\frac{5}{7}$?

Is there one $\frac{2}{7}$ in $\frac{5}{7}$?

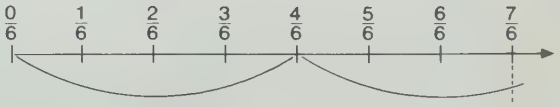
Is there one $\frac{3}{7}$ in $\frac{5}{7}$?

More than one $\frac{3}{7}$ in $\frac{5}{7}$?

$\frac{5}{7} \div \frac{3}{7} \;?\; 1$

3. Here is another way.

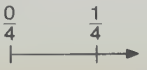
$\frac{7}{6} \div \frac{4}{6} \;?\; 1$



How many $\frac{4}{6}$s in $\frac{7}{6}$? $\frac{7}{6} \div \frac{4}{6} \;?\; 1$

4. This is a special problem.

$\frac{1}{4} \div \frac{3}{4} \;?\; 1$



How many $\frac{3}{4}$s in $\frac{1}{4}$? $\frac{1}{4} \div \frac{3}{4} \;?\; 1$

Use whatever way you want to determine whether the answer is greater than or less than 1.

$\frac{3}{8} \div \frac{2}{8}$ ⓘ 1

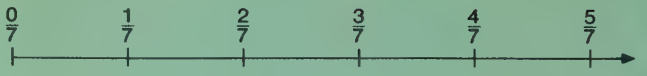
Is there one $\frac{1}{8}$ in $\frac{3}{8}$?

Is there one $\frac{2}{8}$ in $\frac{3}{8}$?

Is there more than one $\frac{2}{8}$ in $\frac{3}{8}$?

$\frac{3}{8} \div \frac{2}{8}$ ⓘ 1

$\frac{4}{7} \div \frac{5}{7}$ ⓘ 1



How many $\frac{5}{7}$s in $\frac{4}{7}$?

$\frac{4}{7} \div \frac{5}{7}$ ⓘ 1

Replace the ⓘ with $<$ or $>$.

1. $\frac{1}{2} \div \frac{3}{2}$ ⓘ 1 **2.** $\frac{5}{6} \div \frac{4}{6}$ ⓘ 1 **3.** $\frac{7}{3} \div \frac{9}{3}$ ⓘ 1

4. $\frac{7}{8} \div \frac{2}{8}$ ⓘ 1 **5.** $\frac{4}{5} \div \frac{2}{5}$ ⓘ 1 **6.** $\frac{8}{4} \div \frac{9}{4}$ ⓘ 1

7. Compare the denominators in each problem.

8. Compare the numerators in each problem.

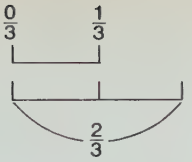
9. Look at the problems in which you used $>$.
Compare the fractions.
Is the first fraction less than the second fraction?

10. Look at the problems in which you used $<$.
Compare the fractions. What is the pattern here?

1. $\frac{1}{3} \div \frac{2}{3} < 1$

How many $\frac{2}{3}$s in $\frac{1}{3}$?

What part of $\frac{2}{3}$ is in $\frac{1}{3}$?



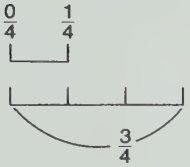
Only half of $\frac{2}{3}$ is in $\frac{1}{3}$.

$\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$

2. $\frac{1}{4} \div \frac{3}{4} < 1$

How many $\frac{3}{4}$s in $\frac{1}{4}$?

What part of $\frac{3}{4}$ is in $\frac{1}{4}$?

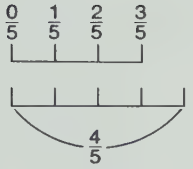


Only $\frac{1}{3}$ of $\frac{3}{4}$ is in $\frac{1}{4}$.

$\frac{1}{4} \div \frac{3}{4} = \frac{1}{3}$

3. $\frac{3}{5} \div \frac{4}{5} < 1$

What part of $\frac{4}{5}$ is in $\frac{3}{5}$?



Only $\frac{3}{4}$ of $\frac{4}{5}$ is in $\frac{3}{5}$.

$\frac{3}{5} \div \frac{4}{5} = \frac{3}{4}$

Compare the answers with the numerators of the fractions in the problem. There's a pattern. Do you see it?

Ask yourself, will the answers be less than 1?

4. $\frac{1}{2} \div \frac{3}{2} = \underline{?}$

5. $\frac{1}{4} \div \frac{2}{4} = \underline{?}$

6. $\frac{2}{5} \div \frac{3}{5} = \underline{?}$

7. $\frac{2}{6} \div \frac{4}{6} = \underline{?}$

8. $\frac{1}{3} \div \frac{4}{3} = \underline{?}$

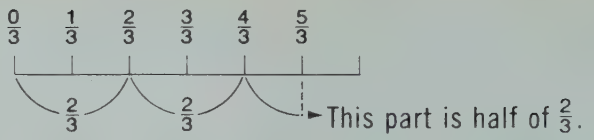
9. $\frac{3}{8} \div \frac{6}{8} = \underline{?}$

10. $\frac{3}{10} \div \frac{4}{10} = \underline{?}$

11. $\frac{3}{12} \div \frac{5}{12} = \underline{?}$

1. $\frac{5}{3} \div \frac{2}{3} > 1$ How many $\frac{2}{3}$s in $\frac{5}{3}$?

$\frac{5}{3} \div \frac{2}{3} = \underline{?}$



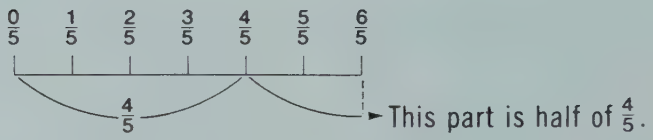
From $\frac{0}{3}$ to $\frac{2}{3}$ is one $\frac{2}{3}$.

From $\frac{2}{3}$ to $\frac{4}{3}$ is one more $\frac{2}{3}$.

From $\frac{4}{3}$ to $\frac{5}{3}$ is $\frac{1}{2}$ of a $\frac{2}{3}$. So $\frac{5}{3} \div \frac{2}{3} = 2\frac{1}{2}$

2. $\frac{6}{5} \div \frac{4}{5} > 1$ How many $\frac{4}{5}$s in $\frac{6}{5}$?

$\frac{6}{5} \div \frac{4}{5} = \underline{?}$

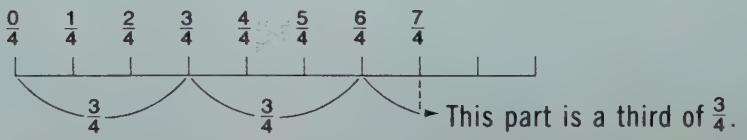


From $\frac{0}{5}$ to $\frac{4}{5}$ is one $\frac{4}{5}$.

From $\frac{4}{5}$ to $\frac{6}{5}$ is $\frac{1}{2}$ of a $\frac{4}{5}$. So $\frac{6}{5} \div \frac{4}{5} = \underline{?}$

3. $\frac{7}{4} \div \frac{3}{4} > 1$ How many $\frac{3}{4}$s in $\frac{7}{4}$?

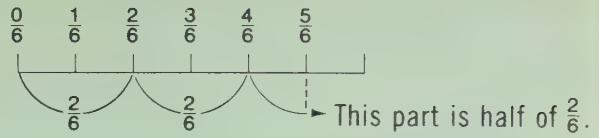
$\frac{7}{4} \div \frac{3}{4} = \underline{?}$



So $\frac{7}{4} \div \frac{3}{4} = \underline{?}$

Draw a number line for each problem. Then divide.

Example: $\frac{5}{6} \div \frac{2}{6} = \underline{?}$



$\frac{5}{6} \div \frac{2}{6} = 2\frac{1}{2}$

YOUR TURN

1. $\frac{6}{8} \div \frac{4}{8} = \underline{?}$ How many $\frac{4}{8}$s in $\frac{6}{8}$?

2. $\frac{5}{4} \div \frac{2}{4} = \underline{?}$ How many $\frac{2}{4}$s in $\frac{5}{4}$?

3. $\frac{9}{10} \div \frac{4}{10} = \underline{?}$ How many $\frac{4}{10}$s in $\frac{9}{10}$?

4. $\frac{7}{3} \div \frac{2}{3} = \underline{?}$ How many $\frac{2}{3}$s in $\frac{7}{3}$?

You can also use this idea on problems written as $1\frac{2}{3} \div \frac{2}{3}$.
All you have to do is rename $1\frac{2}{3}$ as $\frac{5}{3}$ and then—

5. $\frac{5}{3} \div \frac{2}{3} = \underline{?}$ How many $\frac{2}{3}$s in $\frac{5}{3}$?

6. $1\frac{1}{4} \div \frac{3}{4} = \underline{?} = \frac{5}{4} \div \frac{3}{4} = \underline{?}$ How many $\frac{3}{4}$s in $\frac{5}{4}$?

Look back at the example at the top of the page.
The answer is $2\frac{1}{2}$. Think of $2\frac{1}{2}$ expressed as the fraction $\frac{5}{2}$.

Do you see a relationship between the answer $\frac{5}{2}$ and
the two numerators of the fractions in the problem?

Can you find this pattern in the other problems?

1. June was helping her father, a fruit farmer, pick cherries. She worked for 2 hours. It took her $\frac{1}{3}$ of an hour to fill a basket. How many baskets of cherries did she pick?

The math sentence for this story is $2 \div \frac{1}{3} = \underline{?}$
You think, How many $\frac{1}{3}$s in 2? You can picture it on a number line.

Each arrow on the number line shows $\frac{1}{3}$ of an hour.
The arrowhead shows the end of a period worth $\frac{1}{3}$ h .
Count to find how many periods of $\frac{1}{3}$ h there are in 2 h .

Does $2 \div \frac{1}{3} = 6$?

You could have computed rather than counted.

You are dividing by $\frac{1}{3}$.
Can 2 also be named as some number of thirds?

$2 \div \frac{1}{3} = \frac{?}{3} \div \frac{1}{3}$ Is $\frac{6}{3}$ another name for 2?

$\frac{6}{3} \div \frac{1}{3}$ How many $\frac{1}{3}$s in $\frac{6}{3}$?

$\frac{6}{3} \div \frac{1}{3} = 6$

2. What if June had worked for 3 h? How many baskets would she have filled?

$3 \div \frac{1}{3}$ How many $\frac{1}{3}$s in 3?

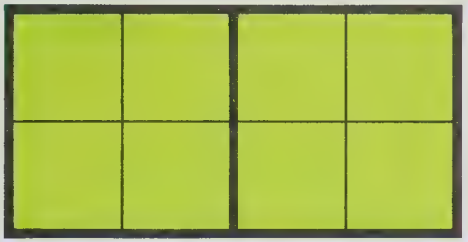
3. Can 3 be renamed as some number of thirds?

$\frac{?}{3} \div \frac{1}{3}$ Is $\frac{9}{3}$ another name for 3?

$\frac{9}{3} \div \frac{1}{3}$ How many $\frac{1}{3}$s in $\frac{9}{3}$?

THE SMALL DEAL LAND CO. is dividing 2 large lots of land into $\frac{1}{4}$-sized lots on which to build houses. How many lots will there be?

The math sentence for this story is $2 \div \frac{1}{4} = \underline{?}$
You think, How many $\frac{1}{4}$s in 2? You can picture it.



There are $\frac{4}{4}$ in 1 large lot.
And another $\frac{4}{4}$ in the other large lot.
Now you can count to find how many $\frac{1}{4}$s in 2.

But see if you know how to compute the answer.

$2 \div \frac{1}{4}$ You're dividing by fourths, so rename 2 as fourths.

Renaming whole numbers is the first step to computing problems like this. Show that you know how.

1. $1 = \frac{?}{2}$ **2.** $1 = \frac{?}{3}$ **3.** $1 = \frac{?}{4}$ **4.** $1 = \frac{?}{5}$

5. $2 = \frac{?}{2}$ **6.** $3 = \frac{?}{2}$ **7.** $4 = \frac{?}{2}$ **8.** $5 = \frac{?}{2}$

9. $4 = \frac{?}{3}$ If you're having trouble, draw a picture.

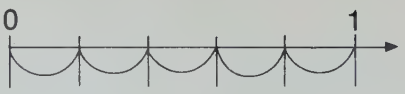
10. $2 = \frac{?}{5}$ Still in trouble? Then try a number line.

11. $5 = \frac{?}{4}$ **12.** $3 = \frac{?}{6}$ **13.** $2 = \frac{?}{8}$ **14.** $6 = \frac{?}{2}$

15. $2 = \frac{?}{3}$ **16.** $4 = \frac{?}{4}$ **17.** $3 = \frac{?}{3}$ **18.** $10 = \frac{?}{10}$

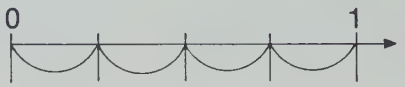
Complete each sentence to describe the division shown on the number line.

1. $1 \div \frac{1}{5} = \frac{?}{5} \div \frac{1}{5} = \underline{?}$



2. $1 \div \frac{1}{8} = \frac{?}{8} \div \frac{1}{8} = \underline{?}$

3. $1 \div \frac{1}{4} = \frac{?}{4} \div \frac{1}{4} = \underline{?}$



4. $2 \div \frac{1}{5} = \frac{?}{5} \div \frac{1}{5} = \underline{?}$

5. $3 \div \frac{1}{2} = \frac{?}{2} \div \frac{1}{2} = \underline{?}$

6. You draw a number line to show $2 \div \frac{1}{2}$. Then compute the answer.

You don't need the number line for these.
Rename each whole number as a fraction and compute.

7. $4 \div \frac{1}{3} = \underline{?}$

8. $3 \div \frac{1}{6} = \underline{?}$

9. $1 \div \frac{1}{7} = \underline{?}$

10. $2 \div \frac{1}{8} = \underline{?}$

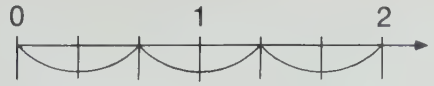
11. $5 \div \frac{1}{5} = \underline{?}$

12. $1 \div \frac{1}{10} = \underline{?}$

Extra practice on page 267– set II

$2 \div \frac{2}{3} = \underline{?}$ How many $\frac{2}{3}$s in 2? Rename 2 as thirds and compute.

$2 \div \frac{2}{3} = \frac{?}{3} \div \frac{2}{3} = \underline{?}$



1. Complete each sentence to describe the division shown on the number line.

a $3 \div \frac{3}{4} = \frac{?}{4} \div \frac{3}{4}$

b $2 \div \frac{2}{5} = \frac{?}{5} \div \frac{2}{5}$

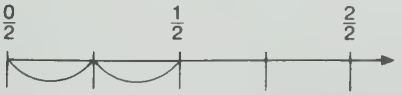
Renaming the whole number so that it has the same denominator as the fraction divisor is a very important first step.

2. Sometimes denominators have to be renamed too.

a $\frac{1}{2} \div \frac{1}{4} = \underline{?}$

Each loop shows $\frac{1}{4}$.

How many $\frac{1}{4}$s in $\frac{1}{2}$?



$\frac{1}{2}$ has to be named as fourths. A common denominator is needed.

$\frac{1}{2} \div \frac{1}{4}$

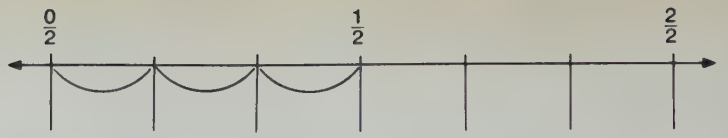
Rename $\frac{2}{4}$

Rewrite $\frac{2}{4} \div \frac{1}{4} = \underline{?}$ Now you can divide without any trouble.

1 Look at another one.
How many $\frac{1}{6}$s in $\frac{1}{2}$?

$$\frac{1}{2} \div \frac{1}{6}$$

Rename $\frac{3}{6} \div \frac{1}{6} = \underline{?}$



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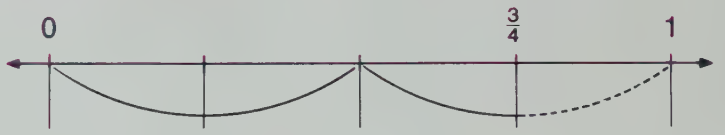
2 How many $\frac{1}{2}$s in $\frac{3}{4}$?
Find a common denominator.

$$\frac{3}{4} \div \frac{1}{2}$$

$$\frac{3}{4} \div \frac{2}{4} = \underline{?}$$



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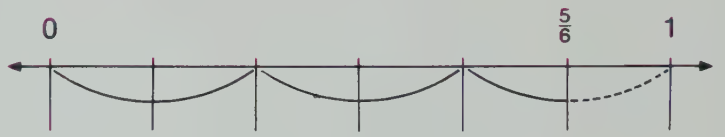
3 How many $\frac{1}{3}$s in $\frac{5}{6}$?
Find a common denominator.

$$\frac{5}{6} \div \frac{1}{3}$$

$$\frac{5}{6} \div \frac{2}{6} = \underline{?}$$



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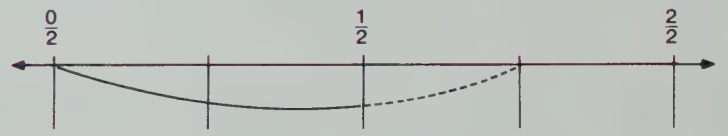
4 $\frac{1}{2} \div \frac{3}{4} = \underline{?}$ The loop shows $\frac{3}{4}$.
How many $\frac{3}{4}$s in $\frac{1}{2}$?
How much of a $\frac{3}{4}$ in $\frac{1}{2}$?

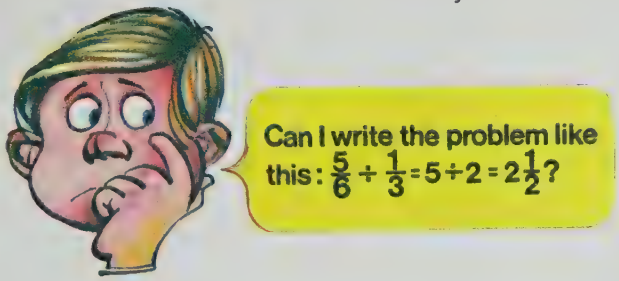


Once again, $\frac{1}{2}$ has to be renamed as fourths.
Find a common denominator. $\frac{1}{2} \div \frac{3}{4}$

Rename $\frac{2}{4}$

Rewrite $\frac{2}{4} \div \frac{3}{4} = \underline{?}$

Watch the denominators!
Make sure you have a common denominator.
Then all you have to do is divide the numerators.



Can you answer Dan's question? What steps did he leave out? Where did the second step, $5 \div 2$, come from? Look back at the problems you have done on other pages. After you have a common denominator, can you just divide the numerators and find the quotient? Think carefully!

In this example the number line shows the need for a common denominator.

How many $\frac{1}{2}$s in $\frac{2}{3}$?

$\frac{2}{3} \div \frac{1}{2}$

$\frac{4}{6} \div \frac{3}{6} = 4 \div 3 = 1\frac{1}{3}$

You can write this step or just think it. It's up to you.

You complete these problems.

1. $\frac{4}{5} \div \frac{1}{2}$

$\frac{8}{10} \div \frac{5}{10} = 8 \div 5 =$?

2. $\frac{7}{10} \div \frac{1}{5}$

$\frac{7}{10} \div \frac{2}{10} = 7 \div 2 =$?

3. $\frac{8}{9} \div \frac{1}{3}$

$\frac{?}{9} \div \frac{?}{9} =$?

4. $\frac{3}{4} \div \frac{1}{3}$

$\frac{?}{12} \div \frac{?}{12} =$?

5. $\frac{7}{6} \div \frac{1}{3}$

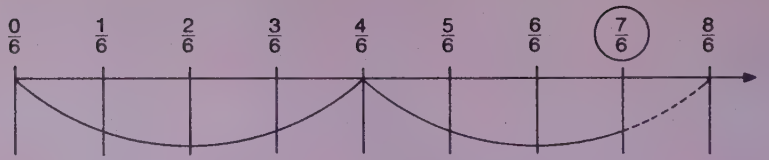
$\frac{?}{6} \div \frac{?}{6} =$?

6. $\frac{9}{10} \div \frac{2}{5}$

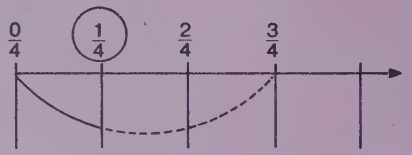
$\frac{?}{10} \div \frac{?}{10} =$?

1. Here's a special review.

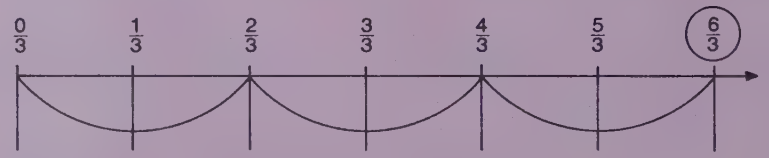
a How many $\frac{4}{6}$s in $\frac{7}{6}$?
More than 1?
More than 2?
$\frac{7}{6} \div \frac{4}{6} = \underline{?}$



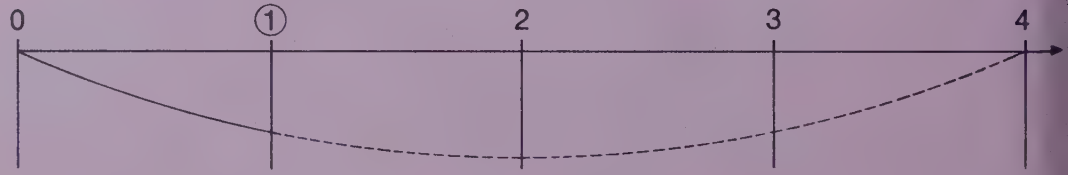
b How many $\frac{3}{4}$s in $\frac{1}{4}$?
More than 1?
Less than 1?
$\frac{1}{4} \div \frac{3}{4} = \underline{?}$



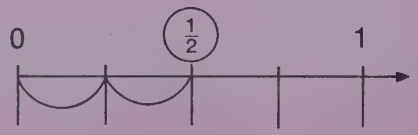
c How many $\frac{2}{3}$s in 2?
More than 1?
More than 2?
$2 \div \frac{2}{3} = \underline{?}$



d How many 4s in 1?
More than 1?
Less than 1?
$1 \div 4 = \underline{?}$



e How many $\frac{1}{4}$s in $\frac{1}{2}$?
More than 1?
Less than 1?
$\frac{1}{2} \div \frac{1}{4} = \underline{?}$



Extra practice on page 267 – set III

Estimation can help you decide whether your answer is reasonable.

1. Is $\frac{1}{2}$ greater than or less than $\frac{1}{3}$?

Is $\frac{1}{2} \div \frac{1}{3}$ greater than or less than 1?

Is $\frac{1}{3} \div \frac{1}{2}$ greater than or less than 1?

2. Is $\frac{1}{5}$ greater than or less than $\frac{1}{3}$?

Is $\frac{1}{5} \div \frac{1}{3}$ greater than or less than 1?

Is $\frac{1}{3} \div \frac{1}{5}$ greater than or less than 1?

3. Complete each sentence with $>$ or $<$.

a $\frac{3}{5} \div \frac{1}{2}$ ⓘ 1 **b** $\frac{1}{3} \div \frac{1}{4}$ ⓘ 1 **c** $\frac{5}{6} \div \frac{1}{4}$ ⓘ 1

d $\frac{2}{3} \div \frac{5}{6}$ ⓘ 1 **e** $\frac{1}{5} \div \frac{1}{2}$ ⓘ 1 **f** $\frac{5}{8} \div \frac{1}{3}$ ⓘ 1

g $\frac{2}{3} \div \frac{1}{6}$ ⓘ 1 **h** $\frac{1}{2} \div \frac{5}{6}$ ⓘ 1 **i** $\frac{5}{12} \div \frac{1}{2}$ ⓘ 1

PROGRESS CHECK

DIVIDE

(1.) $1 \div \frac{1}{8} = \underline{?}$ (2.) $1 \div \frac{3}{5} = \underline{?}$ (3.) $2 \div \frac{2}{3} = \underline{?}$

(4.) $3 \div \frac{4}{5} = \underline{?}$ (5.) $\frac{3}{4} \div \frac{1}{3} = \underline{?}$ (6.) $\frac{4}{5} \div \frac{1}{2} = \underline{?}$

$4 \div 1\frac{1}{3}$ should be thought of as $4 \div \frac{4}{3}$.
Where did the $\frac{4}{3}$ come from?
The whole number must be renamed as thirds.

$$4 \div \frac{4}{3} = \frac{12}{3} \div \frac{4}{3} = 12 \div 4 = \underline{?}$$

$3 \div 1\frac{1}{2}$ should be thought of as $3 \div \frac{3}{2}$.
Then get ready to compute. Rename the whole number.

$$3 \div \frac{3}{2} = \frac{?}{2} \div \frac{3}{2} = \underline{?}$$

Find these quotients.

1. $5 \div 1\frac{1}{4} = \underline{?}$ **2.** $5 \div 1\frac{2}{3} = \underline{?}$ **3.** $8 \div 1\frac{1}{3} = \underline{?}$

4. $7 \div 1\frac{1}{6} = \underline{?}$ **5.** $4 \div 2\frac{1}{2} = \underline{?}$ **6.** $6 \div 1\frac{3}{8} = \underline{?}$

7. $8 \div 2\frac{1}{2} = \underline{?}$ **8.** $2 \div 1\frac{3}{4} = \underline{?}$ **9.** $3 \div 1\frac{1}{2} = \underline{?}$

What would you do with a problem like $2\frac{1}{2} \div 1\frac{1}{3}$?
Think of it as $\frac{5}{2} \div \frac{4}{3}$. Then find a common denominator.

$\frac{5}{2} \div \frac{4}{3} = \frac{?}{6} \div \frac{?}{6} = \underline{?}$ There are really no new rules.

Find these quotients.

10. $3\frac{1}{2} \div 1\frac{1}{5} = \underline{?}$ **11.** $3\frac{2}{3} \div 1\frac{1}{8} = \underline{?}$ **12.** $1\frac{3}{4} \div 2\frac{1}{3} = \underline{?}$

13. $1\frac{4}{5} \div 4\frac{1}{2} = \underline{?}$ **14.** $1\frac{1}{2} \div 2\frac{1}{2} = \underline{?}$ **15.** $2\frac{1}{2} \div 1\frac{1}{4} = \underline{?}$

Extra practice on page 267—set IV

Extra Practice and Review

Set I

Compute these sentences.

a	b	c
$7 \div 10 = \underline{?}$	$10 \div 7 = \underline{?}$	$8 \div 6 = \underline{?}$
$6 \div 8 = \underline{?}$	$6 \div 4 = \underline{?}$	$4 \div 6 = \underline{?}$
$3 \div 7 = \underline{?}$	$8 \div 7 = \underline{?}$	$1 \div 10 = \underline{?}$

Set II

Rename each whole number as a fraction and compute.

a	b	c
$8 \div \frac{1}{2}$	$4 \div \frac{1}{4}$	$3 \div \frac{1}{8}$
$8 \div \frac{1}{3}$	$10 \div \frac{1}{5}$	$5 \div \frac{1}{10}$
$4 \div \frac{1}{6}$	$10 \div \frac{1}{10}$	$1 \div \frac{1}{10}$

Set III

Compute.

Watch the denominators!

a	b	c
$\frac{3}{5} \div \frac{1}{2}$	$\frac{3}{5} \div \frac{1}{10}$	$\frac{1}{3} \div \frac{5}{6}$
$\frac{1}{2} \div \frac{1}{5}$	$\frac{5}{6} \div \frac{2}{3}$	$\frac{1}{8} \div \frac{1}{2}$
$\frac{1}{2} \div \frac{1}{8}$	$\frac{1}{2} \div \frac{3}{5}$	$\frac{1}{2} \div \frac{4}{5}$

Set IV

Find the quotient.

a	b	c
$2\frac{1}{3} \div 1\frac{1}{2}$	$6 \div 2\frac{2}{3}$	$2\frac{1}{4} \div 3\frac{3}{8}$
$3\frac{3}{4} \div 1\frac{1}{2}$	$2\frac{1}{6} \div 1\frac{1}{4}$	$3 \div 3\frac{3}{4}$
$5 \div 2\frac{2}{5}$	$4 \div 3\frac{1}{3}$	$3\frac{1}{3} \div 4$

CHECKOUT

Add.

	a	b	c	d
1.	$1\frac{1}{2} + 2\frac{1}{3}$	$2\frac{1}{8} + 1\frac{3}{4}$	$\frac{1}{3} + \frac{2}{5}$	$\frac{3}{8} + \frac{7}{8}$

Subtract.

	a	b	c	d
2.	$2\frac{7}{8} - 1\frac{5}{8}$	$2 - \frac{1}{2}$	$3\frac{3}{8} - 1\frac{3}{4}$	$\frac{7}{10} - \frac{3}{10}$

Divide.

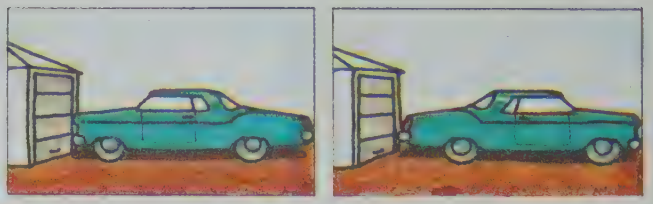
	a	b	c	d
3.	$4 \div \frac{1}{2}$	$3 \div \frac{1}{2}$	$3 \div \frac{3}{4}$	$2 \div \frac{2}{3}$
4.	$3 \div 1$	$2 \div 4$	$\frac{1}{2} \div \frac{1}{6}$	$\frac{1}{4} \div \frac{1}{8}$
5.	$\frac{1}{4} \div \frac{1}{3}$	$\frac{2}{3} \div \frac{1}{2}$	$\frac{5}{8} \div \frac{1}{4}$	$\frac{3}{10} \div \frac{1}{5}$
6.	$4 \div 1\frac{1}{3}$	$5 \div 1\frac{1}{4}$	$6 \div 1\frac{1}{2}$	$2\frac{1}{2} \div 1\frac{1}{2}$

11
GEOMETRY

1. John stood against the right wall of the room and Bill measured his height. He was 160 cm tall. If John walked over to the other side of the room, how tall would he be there? Would he measure the same around the waist? Would his arms be the same length? What about his legs? What about his feet?

2. If John walked two blocks away from Bill, he would look smaller. Did John's size change?

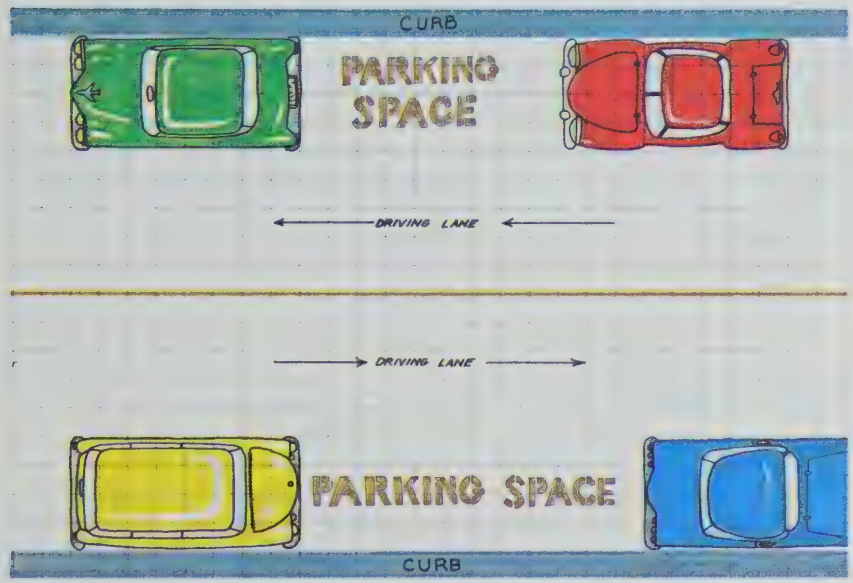
3. One day John's mother parked the family car in the driveway facing the garage. The next day she parked the car by backing into the driveway.



Is the car still the same car? Is its location the same? Is its position the same? Is anything else about the car different? Is it the same size? Is it the same shape?

4. When an object is moved, its location changes. It may also be in a different position and direction. But the size and shape of most objects don't change.
   - **a.** Name some objects in the room that do not change in size or shape when they are moved.
   - **b** Can you think of any objects whose size or shape does change when you move them?

Do you think most people know that solid shapes do not change size or shape when they are moved? If so, why do you think there are "fender bender" accidents when people try to park cars?



Cut out a piece of paper 1.5 cm wide and 4 cm long. Pretend it is a model of a car.

There are two typical parking situations shown. Will your car fit into both spaces? Try to park your car so that you don't hit any of the other cars. Don't forget you can't go over the curb.

What kind of motions do you have to think about as you try to get your car into position?

Believe it or not, geometry can help you solve problems like parallel parking. Your goal is to explore the motion of shapes.

1. Draw a closed figure with three straight sides. How many angles?

   a Draw another one. Is it the same size?

   b What is the name for all closed figures with three straight sides and three angles?

2. Draw a closed figure with four straight sides. How many angles?

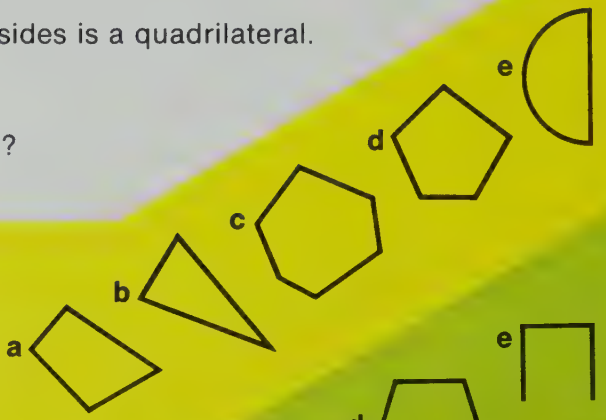
a Draw another one with opposite sides the same length and one with each side the same length.

b How are these two figures alike? How are they different?

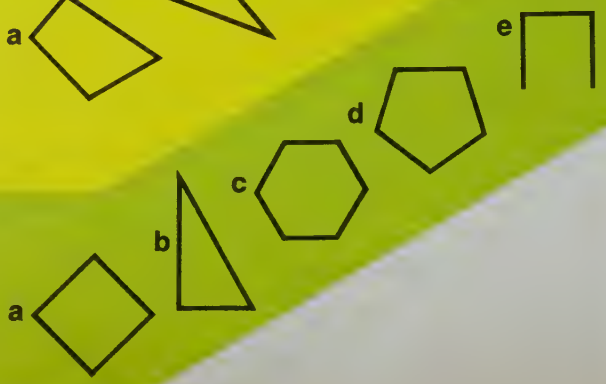
c Any closed figure with four straight sides is a quadrilateral.

3. Draw a closed figure with six straight sides. How many angles? Is this figure a quadrilateral? Why? Does this figure have a name?

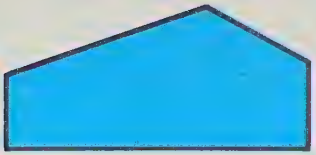
4. Wouldn't it be nice if all straight-sided closed figures had just one name? Many people thought so. They started to use the name *polygon* (*poly-* comes from a Greek word meaning "many" and *-gon* from a Greek word meaning "angles"). Which of these closed figures are polygons?



5. If every side of a polygon is the same length AND if every angle is the same size, the polygon is called a *regular* polygon. Which of these closed figures are regular polygons?



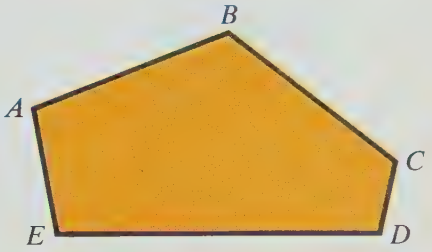
1. This is a polygon.



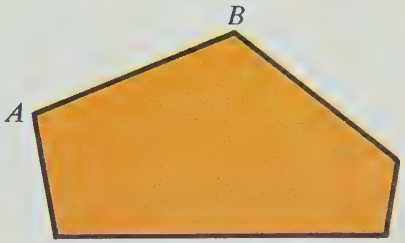
a How many sides does the polygon have?

b Could the front of a house have this shape?

2. You can label each line segment that makes the side of a polygon.

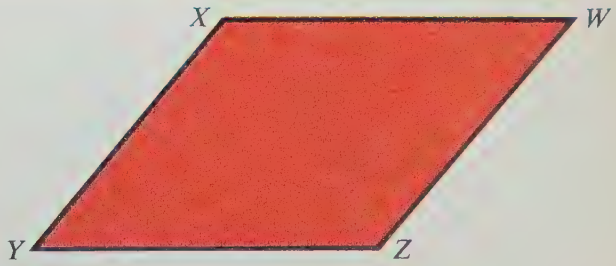


a The sides of this polygon are AB, BC, ?, ?, and ?.



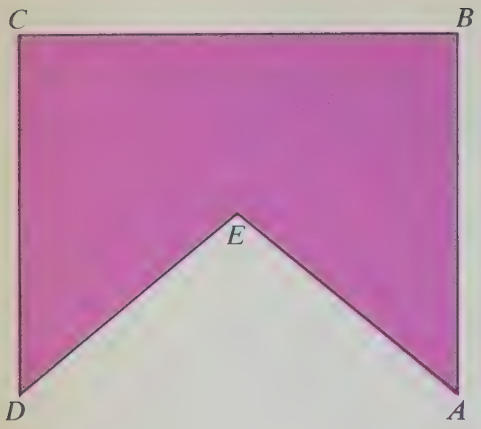
b This polygon is a tracing of the one above. Tell how you would label the corners of the tracing.

3. Trace this polygon on a separate sheet of paper. Label the corners of your tracing so that they match the original figure.

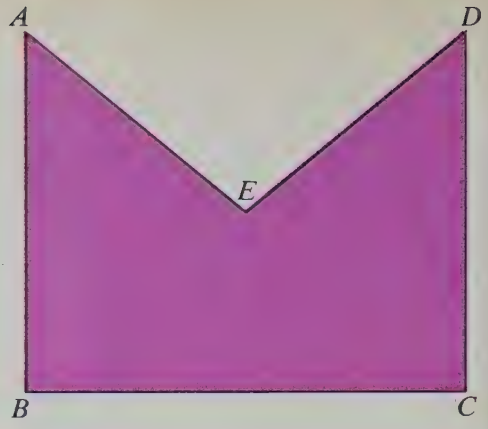


Is the figure you drew the same size and shape as the original one?

Figures that are the same size and shape are congruent.

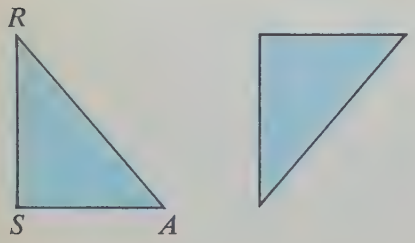


How many sides has the figure on the left? How many angles? Make a tracing of the polygon on the left. Is your tracing congruent to the original figure? Is your tracing congruent to the figure on the right?

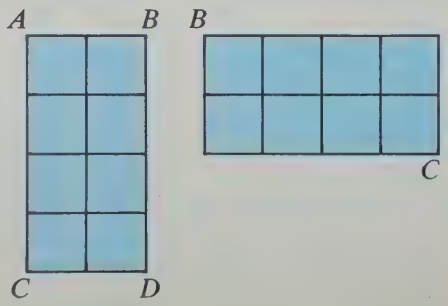


1.
Did you need to turn the tracing of the figure to make sure the tracing was congruent to the figure on the right?

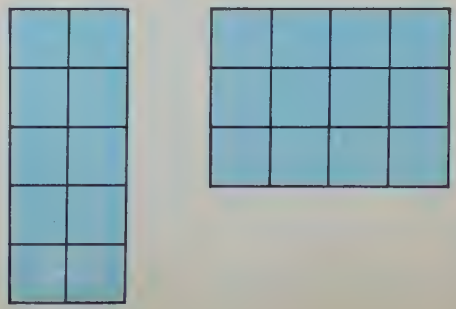
2.
Are the two triangles below congruent? Trace to make sure. Tell how you would label the second triangle. Label angles that are the same size with the same label.



3.
Are the two rectangles below congruent? How could you make sure? How would you finish labelling the second rectangle? Do both rectangles cover the same number of square units?



4.
What is the area of each rectangle below? Do they have the same area? Do they have the same shape? Are they congruent?

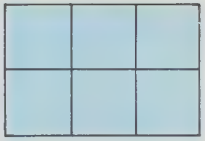


1. Two figures must have the same size and the same shape before they are congruent. Which of the following statements is true?

a If two polygons are congruent, they have the same area.

b If two polygons have the same area, they are congruent.

2. The area of this polygon is 6 square units. Draw a figure that is congruent to it. Call it figure S.

R 

a Do both figures have the same area?

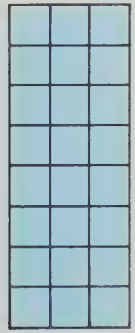
b Draw a figure that has the same area but is not congruent to figure R or figure S. Label your noncongruent figure T.

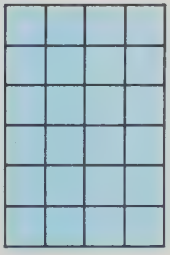
c What is the perimeter (distance around) of figure R?

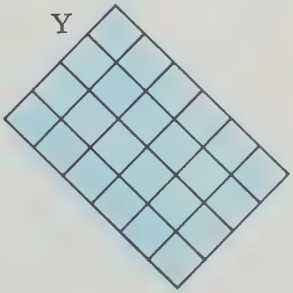
d What is the perimeter of figure S?

e What is the perimeter of figure T?

3. Which of the rectangles below have the same area?

W 

X 

Y 

Z

a Which have the same perimeter?

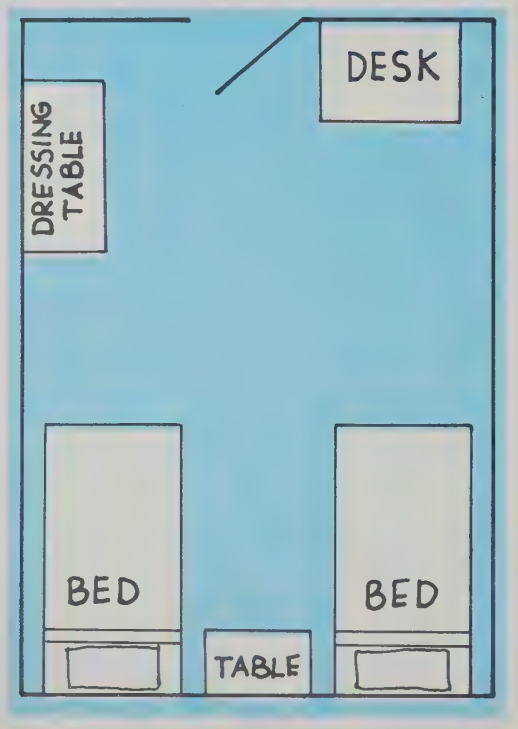
b Which are congruent?

c Do figures that are congruent have the same perimeter?

d Can figures that are *not* congruent have the same area?

e Can figures that are *not* congruent have the same perimeter?

Bette and Lil are planning how to rearrange the furniture in their room. This is the way the room is arranged now:

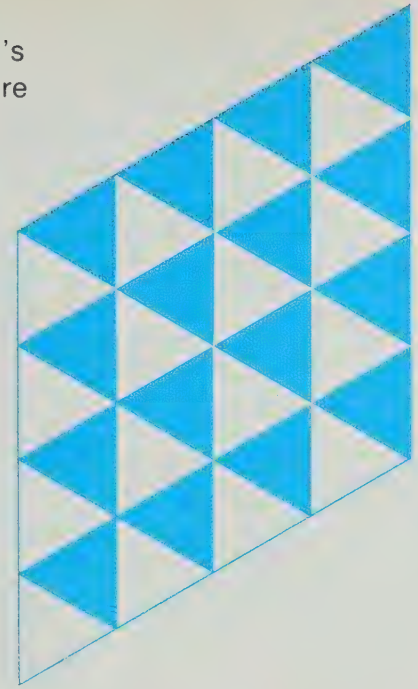


1. Bette wants one of the beds where the dressing table is now. Lil says it won't fit.
   - **a** Will moving the bed change its size?
   - **b** Will the space by the door change size?
   - **c** How can they check to see whether the bed will fit?

2. They can also use a drawing. The drawing is much smaller than the room and furniture. Each 2 cm of the drawing represents 1 m of the room. Such a drawing is called a *scale drawing*.
   - **a** Trace one of the beds in the drawing above. Use your tracing to see whether the bed will fit by the door where Bette wants it. Will it fit?
   - **b** Will the two beds fit end to end?
   - **c** What pieces of furniture can fit between the beds if the beds are not moved?

3. Make at least two sketches showing how the room could be rearranged. How would you arrange the room?

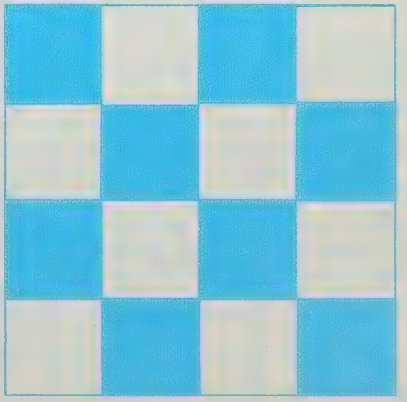
1.
The tiles on Fred's bathroom floor are arranged in a pattern like this:

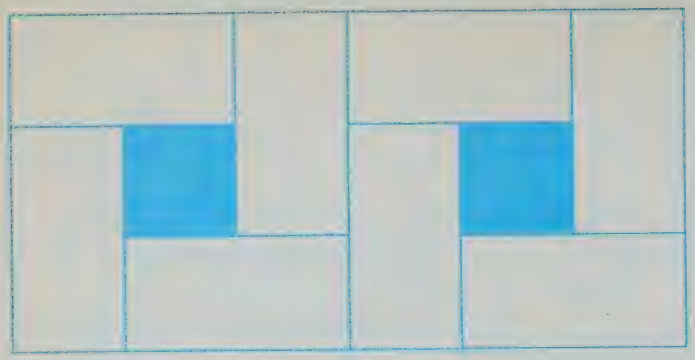


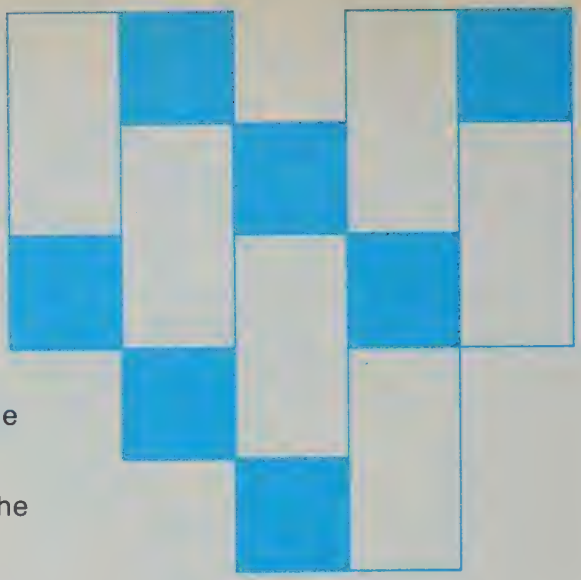
He imagines another design using the same size tiles:

Is the number of blue tiles in each design the same? the number of white tiles? Is there the same number of blue tiles as white?

2.
Find ways to rearrange each of these patterns.







3. These two patterns are made from tiles of the same shape.

   a Is the number of blue tiles the same as the number of white ones in both designs?

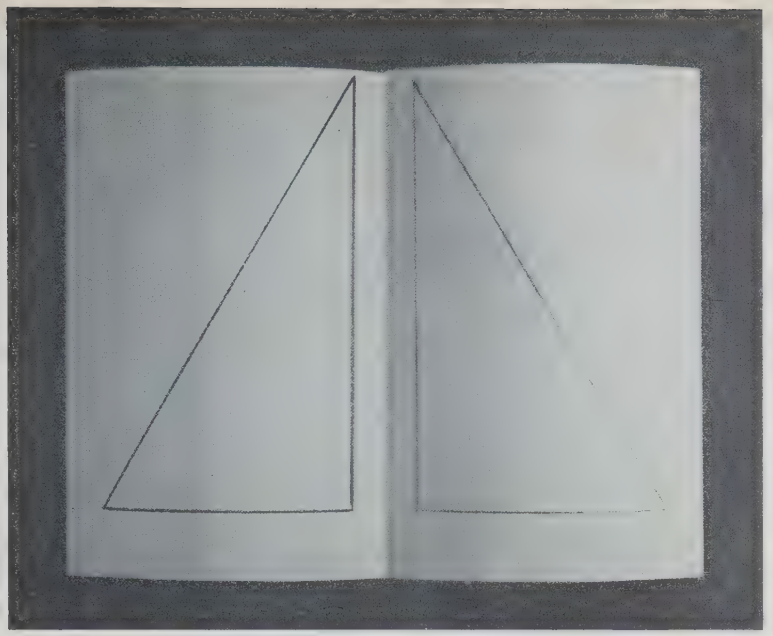
   b Use tiles of the same shape as shown here and make up a pattern of your own. See how many ways you can rearrange the shapes.

4. Look at your right hand and your left hand. How are the two hands alike? How are they different?

   a Put your left hand palm down on a piece of paper. Trace the outline of your hand.

   Make the lines dark enough to see from the other side of the paper. Now turn the paper over and put your right hand on the outline. Does it fit?

   There are small differences between the right and the left hand for most people. Except for these differences, the hands are the same.



1. Bill drew a triangle on a piece of paper. Then he folded the paper and traced the outline of the triangle on the other side of the fold. The two triangles look very much alike, but the tracing is reversed.

   Are the two triangles the same size and shape? Make a tracing of them, together with the fold line. Fold your tracing along the line so that the two triangles lie on top of each other. Do they match? Are they congruent?

2. Put a mirror on the fold line facing to the left. Do you see the image that looks like the triangle on the right of the fold line?

3. Sue parts her hair on the right side. On which side does the part appear when she looks in a mirror? Trace this sketch and draw her image in the mirror.

Things on the right look as if they are on the left in a mirror image. Things on the left look as if they are on the right.

4. Fred is winking his left eye. Trace the sketch and draw his image in the mirror. Which eye is the image winking?

1. Tom and Mike wanted to write messages to each other. But they didn't want anyone else to read what they wrote.

Tom planned to send this message:

MIKE,
I WILL MEET
YOU AT THE
CORNER AFTER
SCHOOL.
TOM

But he wrote it so that it looked like this:

MIKE,
I WILL MEET
YOU AT THE
CORNER AFTER
SCHOOL.
TOM

Can you read it?

2. Here is another mirror image. Write the original message. If you can't figure it out, hold the message up to a mirror.

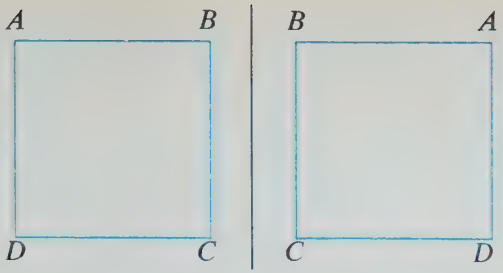
THIS SECRET
MESSAGE IS
A REFLECTION.

3. Now try writing a message of your own as it would look in a mirror.

4. What does the mirror image of the capital letter M look like? What does O look like? How many other capital letters look the same as their mirror image? List them.

5. There are some whole words that look the same as their image. MOM is one example. Find as many words as you can think of that look the same as their mirror image.

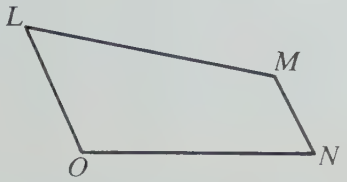
6. A pool of water can act like a mirror. But when you see the image of the bridge, the image is reversed *and* upside down.



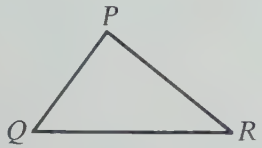
Look at the figures on the left.
The second rectangle is a mirror image of the first.

Trace the following figures and then draw each shape as it would look in a mirror. Be sure to label the three polygons in your tracing and on your drawing.

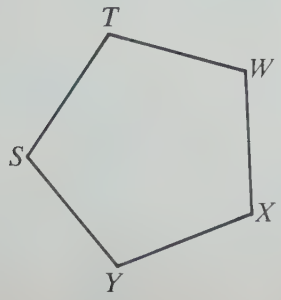
1.



2.



3.



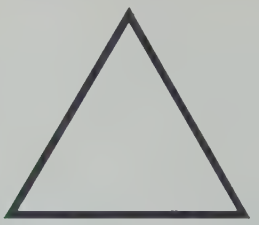
CHECK THESE OUT AT HOME. . .

4. Look in a mirror. Does your right hand appear to be your left hand?

5. Work with a friend. Develop a mirror-image pantomime. You be the person. Your friend will be the mirror.

1 A plane figure is symmetrical if it can be folded to form two halves that match. The idea of symmetry can be seen if you look at a human being, for example. The eye on the right matches the eye on the left. The arm on the right matches the arm on the left. Does a human being have a nose on the right that matches a nose on the left? But does the part of the nose on the left match the part on the right? We can't test a human being to see if the body is symmetrical, because we can't fold a human being. We could fold a picture of a human being, of course. But let's start with plane figures.

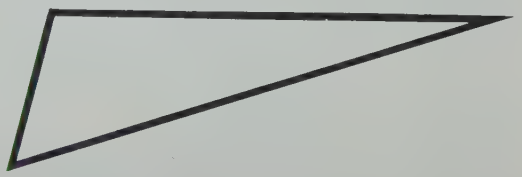
2 All regular polygons are symmetrical. Trace the regular polygon below. How many different ways can you fold your tracing so that two symmetrical parts are formed?



Mark each fold that divides the figure. These fold-mark lines are called the *axes of symmetry*. If a figure can be folded only one way, it will have only one line or *axis* of symmetry.

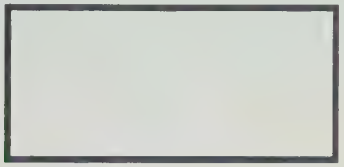
3 Is the figure below a regular polygon? Trace it. How many different ways can you fold it so that one half matches the other half? Is the figure symmetrical? Does it have more than one axis of symmetry?

4 Is the figure below a regular polygon? Trace it. Is it symmetrical? Each of the polygons on this page is a triangle. Think about how the lengths of the sides relate to the number of axes of symmetry.



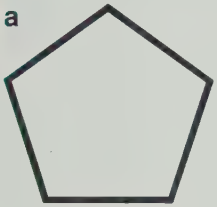
1 The polygon on the right is a square. What has to be true about the length of each side? Trace it. On the tracing draw its axes of symmetry.

2 Does a rectangle have four axes of symmetry? Trace the rectangle below. On the tracing draw its axes of symmetry.

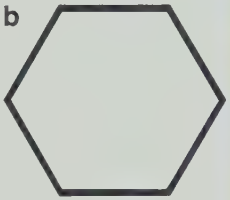


3 Trace each of the regular polygons below. On each tracing draw the axes of symmetry for each polygon.

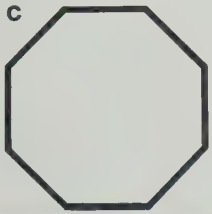
a



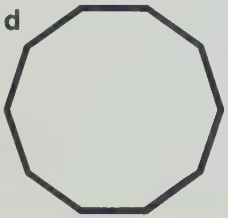
b



c



d



4 Review the tracings you have made for problem 3.

a How many sides does each polygon have?

b Is each of the sides the same length?

c How many angles does each polygon have?

d Is each angle the same size?

e How many axes of symmetry does each have?

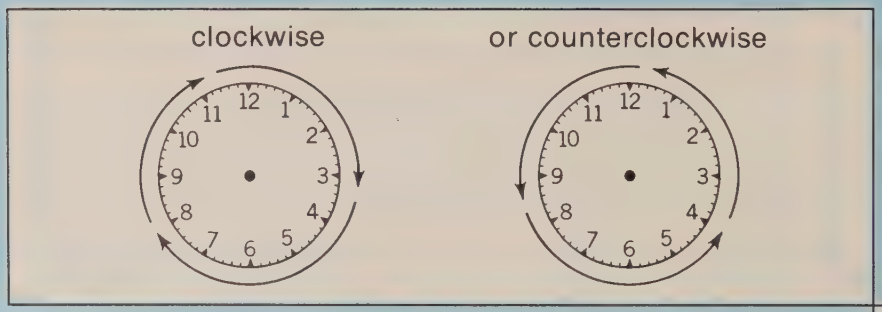
f Suppose a regular polygon has eleven sides. How many axes of symmetry do you think it has?

g How many axes of symmetry would a 50-sided regular polygon have? a 100-sided one?

5 The more sides a regular polygon has, the more it looks like a circle. How many axes of symmetry do you think a circle has?

1. Many things turn or rotate. You rotate a radio or TV dial; you see wheels rotate. What other things rotate?

Two-dimensional figures can turn or rotate in either of two directions,

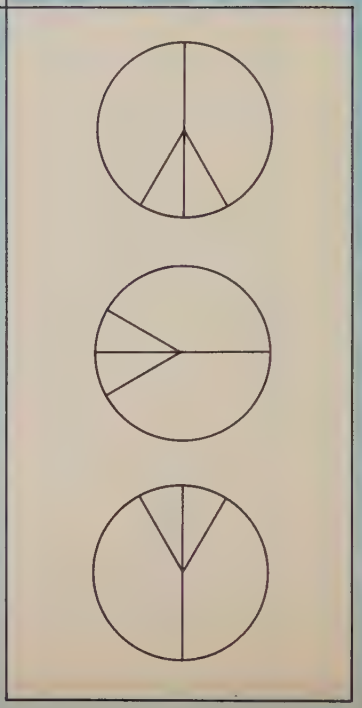


2. The figures at the right are congruent.

a The first symbol, when rotated 90° in a clockwise direction, is in the same position as the second symbol. How far must the second symbol be rotated to be in the same position as the third symbol?

b How far must the first symbol be rotated to be in the same position as the third figure?

c The first symbol can also be rotated 270° in a counterclockwise direction to be in the same position as the second symbol. How far must it be turned in a counterclockwise direction to be in the same position as the third figure?



1. Now draw a stick figure in each final position of the stick figures in this chart.

Original position	Degrees clockwise	*OR*	Degrees counter-clockwise	Final position
	90°		270°	?
	180°		180°	?
	270°		90°	?
	360°		0°	?

2. Look at this chart. The first chair has to be turned 90° clockwise *or* 270° counterclockwise to get into the final position. How many degrees must each be turned?

Original position	Degrees clockwise	*OR*	Degrees counter-clockwise	Final position
	90°		270°	
	?		?	
	?		?	
	?		?	

1. The following illustration has been rotated clockwise 120°. Trace the figure and show how it would look after it has been rotated 240° and 360°.

original position

rotated clockwise 120°

How many degrees must you rotate the figure until it is back in its original position?

2. This illustration is being rotated in a clockwise direction.

original position

90°

180°

270°

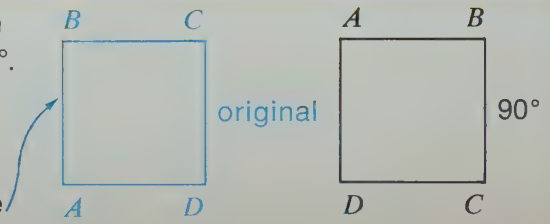
360°

When a figure is rotated 360°, is it always back in its original position? Are there any other rotations that return the illustration to its original position? Name some of them.

3. Draw a square. Use congruent squares to show its position after it has been rotated clockwise 90°, 180°, 270°, and 360°.

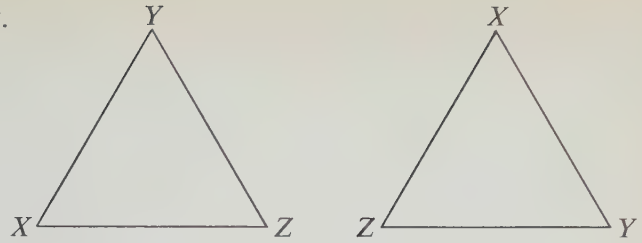
a Can you tell that each square shows a rotation?

b Label your original square. Then label the others to show the rotation. For example

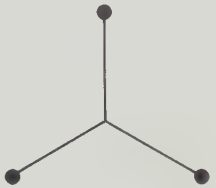


1. The second triangle below is a tracing of the first. How far has the tracing been rotated?

Rotate the tracing the same number of degrees again. Does it still look like the original?



2. Will this figure look the same after it has been rotated 120°? Trace and rotate to make sure of your answer.

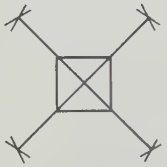


3. Draw at least two other shapes that will not be changed by a 120° rotation.

4. **a** Will this figure look the same after it has been rotated 120°? Trace and rotate to make sure.

b Will it look the same after it has been rotated 90°? Trace and make sure.

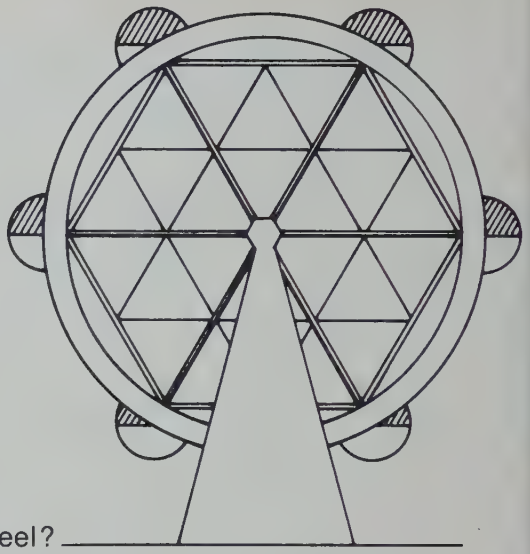
c Draw at least two other shapes that will not be changed by a 90° rotation.



5. Will this figure look the same after it has been rotated 60°? Trace and rotate to make sure.

CHECKOUT

1. A Ferris wheel is a beautiful thing to see and fun to ride on, too. If you got on a seat at the bottom and rode to the top, would you still be the same size and shape? Would you look the same size and shape to someone on the ground?



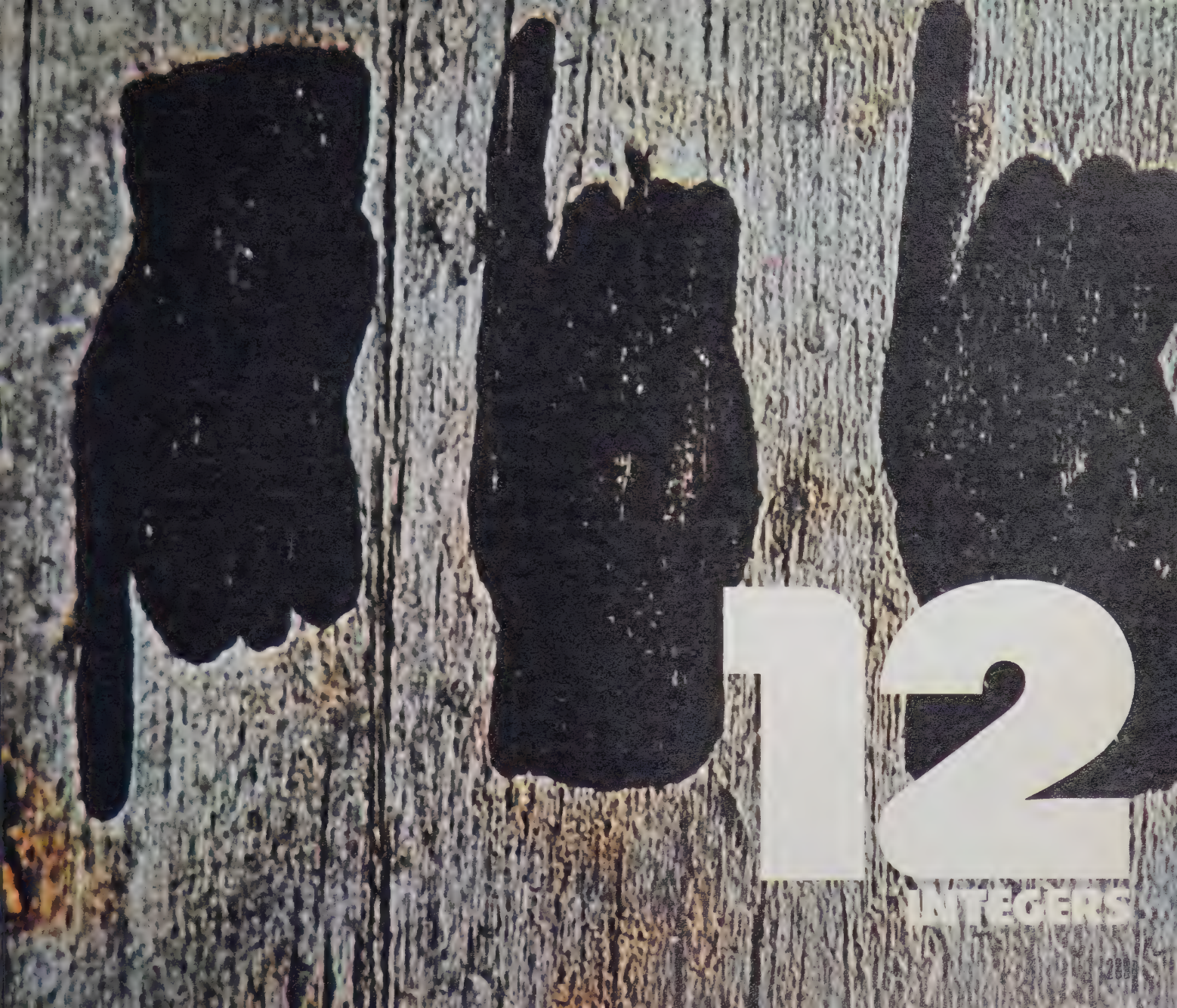
   a The Ferris wheel is constructed with triangular shapes to give strength to the huge wheel. Why are triangles used? How many triangles can you count in the picture of this Ferris wheel?

   b How many different sizes and shapes are there among the triangles you counted?

   c Are the smaller triangles congruent? Are the larger ones congruent? Make tracings to check your answers.

   d What figures do the triangles together form? Make sure you have the right names. How many sides does it have?

2. An amusement park or a carnival may be a good place to see congruent shapes. What congruent things can you think of that you can be sure to find there? Might you see any examples of reflections or rotations? Surely you can think of one thing, or two things, or . . .

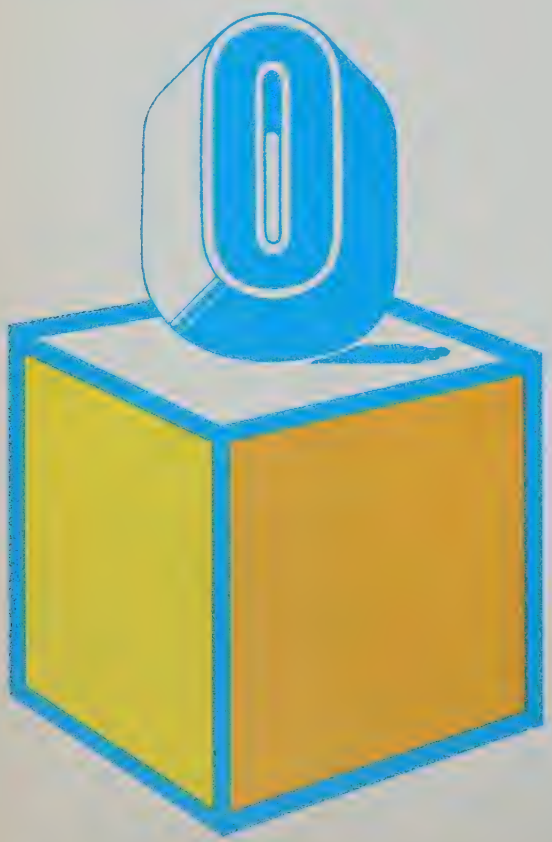

12
INTEGERS

YOUR

GOAL

is to get acquainted with *integers*.

***Integers* isn't the name of the new family that moved in. It isn't a new disease. It's another set of numbers.**



Every day we use numbers to count.

In the Lane family there are Mr. and Mrs. Lane, Lillian, Leonard, Louis, Lance, and Louise. How many children are there in the Lane family? How many girls? How many boys? How many people are there in the Lane family?

People invented counting numbers long ago to answer the question "How many are there?"

Centuries later, people invented a new number. It was a number that indicated that there are *no* objects in a set. What was this number?

The Jones family lives next door to the Lanes. The Jones children are named Jane and Janice. How many boys in the family?

Tim had a bag with 5 marbles in it. One day they all fell out through a hole in the bag. Tim lost all 5 marbles. How many did he have then? Which number answers this question?

Zero is especially useful in a numeration system that uses place value. Without it we couldn't tell the difference between 1 and 100, or 101 and 11.

Sometimes there is a need for numbers less than 0. One of these times is when cold weather is being discussed.

On Friday evening the local TV weatherman gave the weekend weather forecast:

> Today's high was 5 degrees, and tonight's low will be 2 degrees.
> Saturday the high will be 1 degree and the low will be 7 degrees below zero.
> On Sunday the high will be 2 degrees below zero and the low will be 10 degrees below zero.

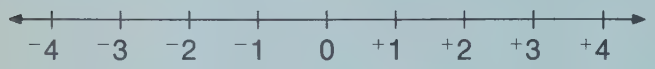
He wrote the temperatures on a chart. He used a small raised circle as a symbol for the word *degrees*. He used another symbol too. Instead of writing "7° below zero," he wrote "$^{-}7°$."

	Friday	Saturday	Sunday
High	5°	1°	$^{-}2°$
Low	2°	$^{-}7°$	$^{-}10°$

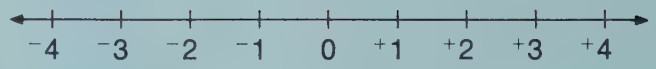
1. Does the temperature 5° mean 5 degrees above zero or 5 degrees below zero?
2. Is 5° warmer than 2°?
3. Is $^{-}7°$ warmer than $^{-}10°$?
4. Which is the lower temperature?
   a 2° or $^{-}2°$ **b** 1° or $^{-}7°$ **c** $^{-}10°$ or $^{-}2°$

1. The next weekend was even colder. Friday's high was 0°, and the low was 3° below zero. Saturday's high was 1° below zero, and the low was 6° below zero. Sunday's high was 5° below zero, and the low was 12° below zero. Make a chart to show these temperatures.

Think of the thermometer as a number line.



One less than zero is a negative number.
It is written $^{-}1$ and read as "negative one."
It is to the left of zero on the number line.
One more than zero is a positive number.
It can be written $^{+}1$ and read as "positive one."



Two less than zero is read "negative two" and is written $^{-}2$. Two more than zero is read "positive two" and can be written $^{+}2$.

The negative symbol is *never* left off any number to the left of zero, but sometimes the positive symbol is not written with the numbers to the right of zero.

All numbers to the left of zero are negative numbers.
All numbers to the right of zero are positive numbers.

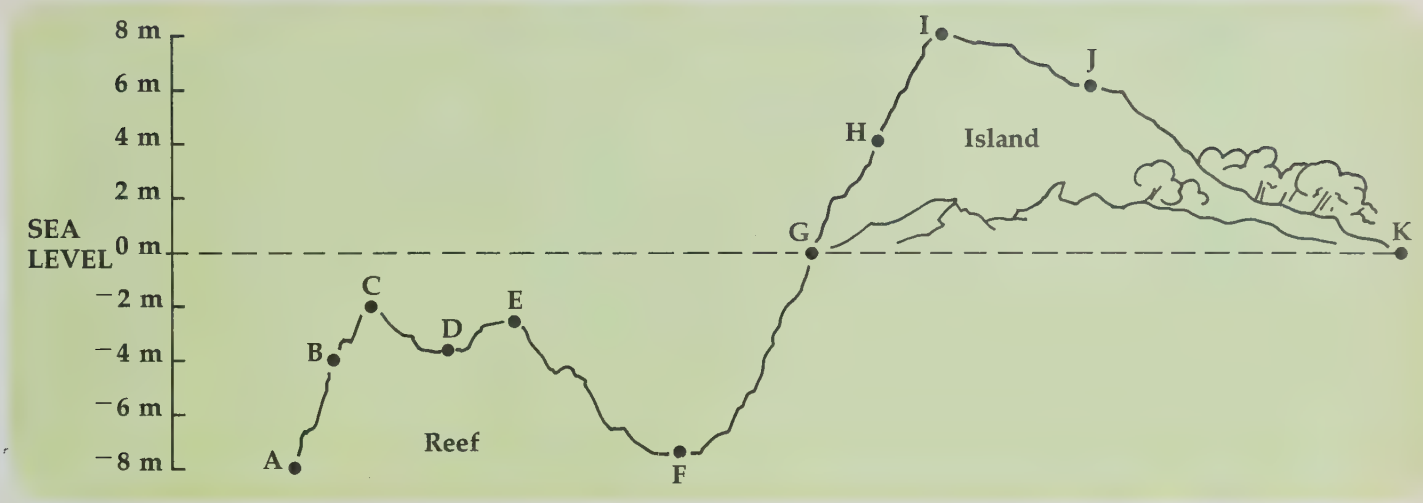
2. Tell whether each of these numbers is to the left or to the right of zero.

   a $^{-}5$ **b** $^{+}3$ **c** $^{-}3$

3. **THINK** Then describe each of these situations with a positive or negative number.

   a You lost 35¢ **b** You found 50¢

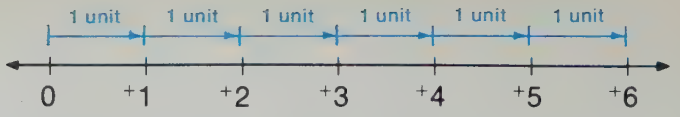
A coral reef lies near a small island. Several ships hit the reef. So divers were sent down to measure the depth. Here is the chart they drew.



1. What is the most dangerous point on the reef for a boat to cross?
2. How far below sea level is the reef at that point?
3. What number is used to show sea level?
4. The negative sign can be used to show metres below sea level. Using that sign, answer these questions. What is—
   - **a** the lowest charted point on the reef?
   - **b** the highest charted point on the reef?
5. Copy and complete this table, giving the depth or height of each point.

Point	A	B	C	D	E	F	G	H	I	J	K
Level	?	?	?	?	?	?	?	?	?	?	?

You know how to make a number line.
You start from a point marked zero.
You choose a unit of length, measure 1 unit
to the right of zero, mark the point, and label it $^{+}1$.
You keep the same unit length and label the
points $^{+}2$, $^{+}3$, $^{+}4$, and so on.



1. Where does the point for $^{+}5$ go?

2. How much farther can the line be extended to the right?

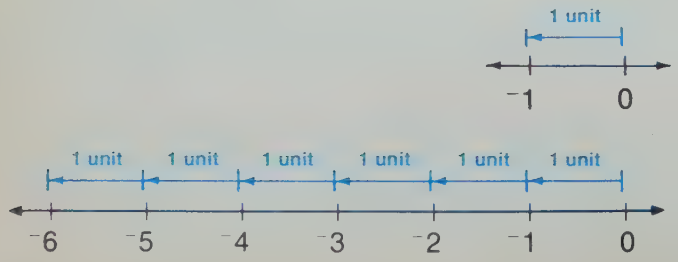
3. How far would the line extend if the number $^{+}43$ were marked on it?

4. In which direction from 0 are the whole numbers marked on the line?

5. Is $^{+}2 < {}^{+}3$? Is 2 placed on the left or the right of $^{+}3$ on the number line?

6. Is $0 < {}^{+}1$? Does 0 lie to the left or the right of $^{+}1$?

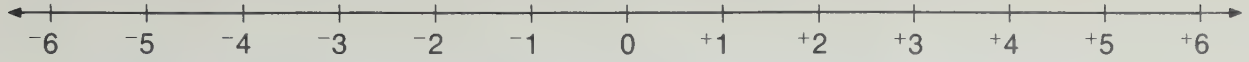
7. If one whole number is smaller than another whole number,
is the smaller one placed to the left or the right
of the larger one on the number line?



To show negative numbers, you use the same unit length you used for whole numbers. Measure one unit length to the *left* of 0, mark the point, and label it $^{-}1$.

Continue moving to the left. Mark and label the points for $^{-}2$, $^{-}3$, $^{-}4$, and so on.

Here is a number line that shows both positive and negative numbers.



The whole numbers, used for counting, are $^{+}1$, $^{+}2$, $^{+}3$, $^{+}4$, and so on.
These numbers are called *positive integers*.
They are always shown to the right of 0 on the number line.

The negative numbers, shown to the left of 0 on the number line, are called *negative integers*.

Zero is also an integer, but it is neither positive nor negative.

The positive integers, the negative integers, and zero all taken together are called the *integers*.

1. Can the number line for the integers be extended to the left? How far?

2. Is the negative integer $^{-}21$ to the right or the left of zero?

3. Is $^{-}7 < {}^{-}5$? Is $^{-}7$ placed to the left or the right of $^{-}5$ on the number line?

4. **a** How far from 0 is $^{-}5$? In which direction?

b How far from 0 is $^{+}5$? In which direction?

c How far is it between $^{-}5$ and $^{+}5$?

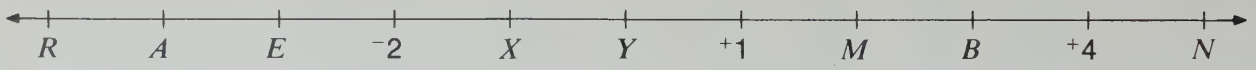
5. **a** How far from 0 to $^{-}4$? In which direction?

b How far from 0 to $^{+}1$? In which direction?

c How far from $^{-}4$ to $^{+}1$?

d How far from $^{-}4$ to $^{+}4$?

Some of the points on this number line are labelled with integers.
Others are labelled with letters.



1

a Name the integer represented by each of the following letters.

A E N X B M R Y

b Which of these are positive integers?

c Which are negative integers?

d Are any of these integers neither positive nor negative? Which?

2

Negative numbers can be used to show the loss of money.
For example, Jill lost 50¢ of her paper-route money.
This can be written as $^{-}50$. Positive numbers show a profit.
If Jill makes 25¢ in one day, it can be written simply as $^{+}25$.

3

Bill hasn't any money. In fact he owes his sister $3.00.
This debt can be shown by the integer $^{-}3$. Ellen doesn't
owe any money and she has $2.00. Her money can be shown
by the integer $^{+}2$.
What number shows a debt of $5.00? a sum of $8.00?

Show these profits and losses as positive or negative numbers:

a a loss of 10¢ **b** a profit of 10¢ **c** a loss of 40¢

d a profit of 40¢ **e** a loss of 99¢ **f** a loss of 75¢

On the number line, the lesser of two numbers is always to the left of the greater.
For example, $^{+}3 < {}^{+}4$, so $^{+}3$ is to the left of $^{+}4$.
The greater of two numbers is always to the right.
For example, $^{+}6 < {}^{+}7$, so $^{+}7$ is to the right of $^{+}6$.

1. Use the number line as a guide. Write $>$ or $<$ to make each of these statements true.

a $^{+}1 \,?\, {}^{+}2$ **b** $^{-}1 \,?\, {}^{+}1$ **c** $0 \,?\, {}^{-}1$ **d** $^{-}1 \,?\, {}^{+}2$ **e** $^{-}1 \,?\, {}^{-}2$ **f** $^{+}2 \,?\, {}^{-}2$

2. Study these number sentences. Which are true? Which are false?

a $^{-}1 < {}^{+}3$ **b** $^{-}1 < {}^{+}1$ **c** $^{-}1 < 0$ **d** $^{-}2 < {}^{+}3$ **e** $^{-}2 < {}^{+}1$

f $^{-}2 < 0$ **g** $^{-}3 < {}^{+}3$ **h** $^{-}3 < {}^{+}1$ **i** $^{-}3 < 0$

PROGRESS CHECK

True or false?

(1.) Every negative integer is less than $^{+}1$.

(2.) No negative integer is less than 0.

(3.) $^{-}1$ is the greatest negative integer.

(4.) $^{+}10\,000\,000$ is the greatest positive integer.

(5.) Every negative integer is less than any positive integer.

Arrange each set of integers in order from least to greatest.

(6.) $0, {}^{-}3, {}^{+}2$

(7.) $^{-}1, {}^{+}2, {}^{+}1$

(8.) $^{+}7, {}^{-}4, {}^{-}1, {}^{+}3$

(9.) $^{-}2, {}^{-}4, {}^{+}2, 0$

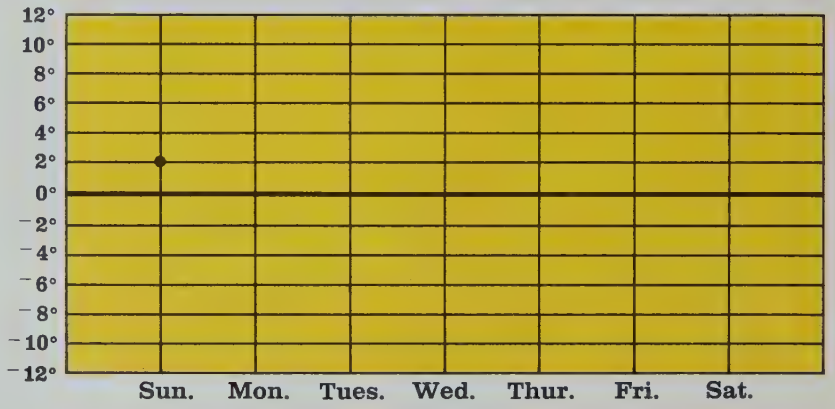
In Eis City, Kaltland, the high temperatures for one week were as follows:

Sunday 2°C	Monday 1°C	Tuesday $^{-}3$°C	Wednesday 0°C
Thursday $^{-}10$°C	Friday $^{-}5$°C	Saturday $^{-}12$°C	

1. Use this information to make a graph like this one.

On the line for each day, mark a point to represent the temperature. Sunday is shown as an example. When all the temperatures are marked, connect the points with straight lines.

Have you ever seen a graph like this on TV or in books? Does it make it easy to see what happened to the temperature during the week?



2. Did the temperature in Eis City mostly rise or fall during the week?

3. What day had the highest temperature? the lowest?

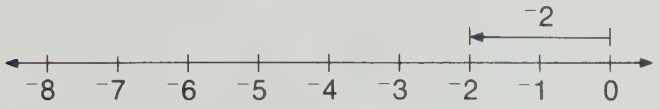
During the coming week, find the daily high temperatures for your area. Then make a graph of the temperatures like this one. Make sure you know where to find the information.

You already know how to show addition of positive integers on the number line. Now let's use the number line to show the addition of negative integers.

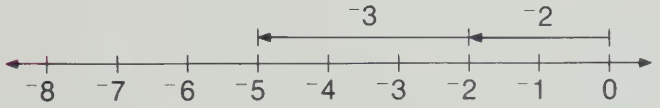
A sunken raft is 2 m below sea level. A sunken chest is 3 m deeper. How deep is the chest? $^{-}2 + {}^{-}3 = \underline{?}$

To show this addition on the number line, follow these steps.

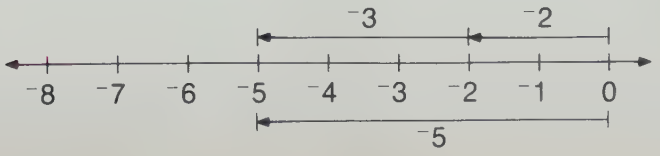
Step 1: Draw an arrow from zero for the first addend, $^{-}2$. Since the first addend is a negative integer, draw the arrow 2 units to the left from zero. Remember, negative integers are always to the left of zero.



Step 2: Draw a second arrow to show the second addend, $^{-}3$. It starts where the first arrow ends.

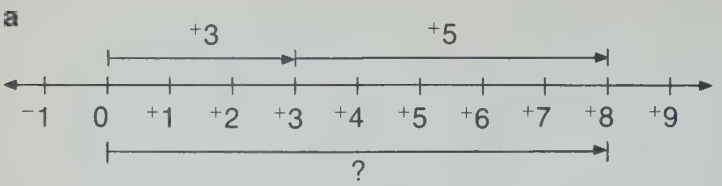


Step 3: Draw an arrow below the number line to show the sum. The starting point of this arrow is zero. It ends in line with the end of the second arrow.
This shows that $^{-}2 + {}^{-}3 = {}^{-}5$.

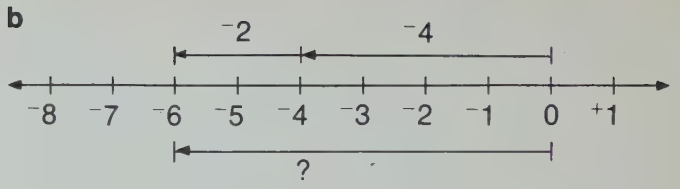


1. Using the number-line diagrams, write and complete an equation for each of the following.

a



b



2. Draw a number-line diagram for each of the following. Then complete the sentence.

a $^{-}1 + {}^{-}8 = \underline{?}$ **b** $^{-}4 + {}^{-}5 = \underline{?}$ **c** $^{+}6 + {}^{+}3 = \underline{?}$ **d** $^{-}3 + {}^{-}9 = \underline{?}$

3. Janet owes $2 to her brother and $4 to her father.
Write negative integers to show these debts.
Add them to find her total debt. Use a number line if you wish.

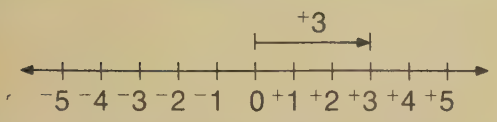
4. **a** Pegleg Pete is official treasure burier for Blackbeard the Pirate.
He dug a huge hole 3 m deep on Wednesday.
He recorded it in his book as $^{-}3$.
On Thursday he dug down 4 m more.
What numeral did he write to show the distance he dug that day?
How deep was the hole then?
Write an addition equation to show this.

b On Friday Pegleg finished the hole by digging down another 2 m .
What did he record in his book to show the distance he dug?
What integer did he write to show the total depth of the hole?
Write an addition equation to show this.

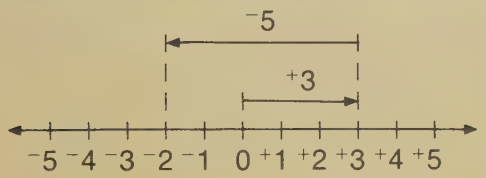
Suppose one of the addends is positive and the other is negative. Here are two problems. The problems are different, but you use the same **Think** process on both.

1

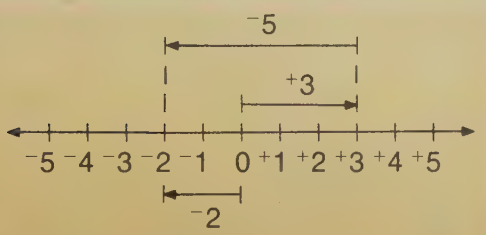
The temperature is $^{+}3°$.
It gets 5° colder.
Then what is the temperature?
$^{+}3 + {}^{-}5 = \underline{?}$



The first arrow always starts with zero. If it is positive, it goes to the right. If it is negative, it goes to the left.



The second arrow always starts where the first arrow ended. Once again—positive goes right; negative goes left.

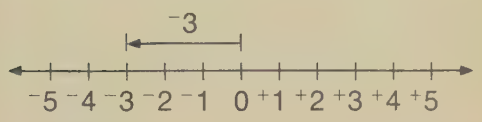


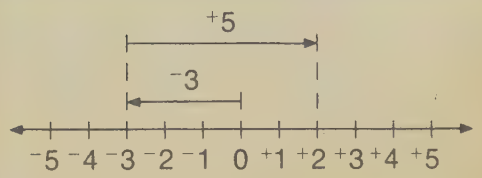
$^{+}3 + {}^{-}5 = {}^{-}2$

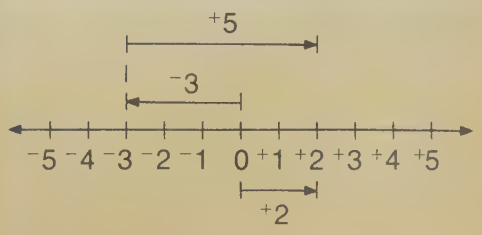
The sum is shown with an arrow below the number line. The arrow always starts at zero. It ends at the same point as the second arrow.

2

The temperature is $^{-}3°$.
It gets 5° warmer.
Then what is the temperature?
$^{-}3 + {}^{+}5 = \underline{?}$



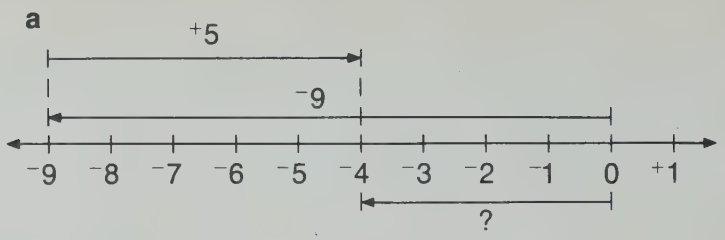




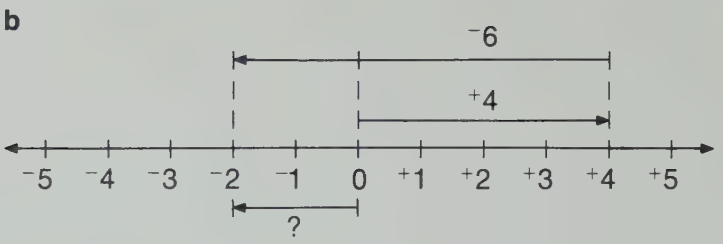
$^{-}3 + {}^{+}5 = {}^{+}2$

1. Write an equation for each of these number-line diagrams.

a



b



2. Draw a number-line diagram for each of the following. Then complete the sentence.

a $^{-}3 + {}^{+}1 = \underline{?}$ **b** $^{-}7 + {}^{+}1 = \underline{?}$ **c** $^{+}3 + {}^{-}3 = \underline{?}$

d $^{+}3 + {}^{-}6 = \underline{?}$ **e** $^{+}5 + {}^{-}8 = \underline{?}$ **f** $^{-}1 + 0 = \underline{?}$

3. Bob owed a friend $12, so he put a slip of paper in his bank that said he was in debt $12. He paid back $8. What should the slip of paper say now? Write an equation to show your answer.

***4.** A computer controls the inventory at Wilson's Warehouse. The records said there were 8 typewriters on hand. The next day 15 were shipped out. Now the records said there were $^{-}7$ on hand. But there were still 3 more on the shelf. Something was wrong. What number of typewriters should have been on the records to start with?

1.

Let a negative integer stand for a drop in price and let a positive integer stand for a rise in price. The price of meat drops 4¢/kg one week. It rises 9¢/kg the next week. Is the final price above or below the initial price? How much?

2.

Jeffery is on a diet to reduce. He loses 5 kg in March. He gains back 3 kg in April. How much does his mass change in the two months? Is this change positive or negative?

3.

Many manufacturing plants label parts that are too big " $+$." Parts that are too small are labelled " $-$."
If a part is $+2$ cm, it is 2 cm too long.
If a part is -3 cm, it is 3 cm too short.
If a part is supposed to be 9 cm long but a worker says it is -2, how long is the part?

4.

Mr. Jones's bank account had $6.00 in it. He wrote a cheque for $10.00. What balance should he have in his account?

Now suppose he puts $5.00 into his account. What balance should he have in his account this time?

#56	TO Super Mart	DATE	AMOUNT	7.00
	FOR Groceries	3-26	BALANCE	6.00
#57	TO Speedy Service	DATE	AMOUNT	10.00
	FOR Cash	3-28	BALANCE	?

CHECKOUT

1. Some new buildings are being built with many floors below ground level. Pretend there is a new building with 5 floors below ground level and 8 floors above ground level. Design an elevator control panel that will have a button that clearly shows each of the floors. Think carefully. There are some problems you'll have to solve.

2. You owe your dad $6.00. He says you cannot buy the blouse you want, which costs $4.00, until you pay him back. How much money do you have to have in order to buy the blouse?

3. The teacher wanted to discourage students from guessing the answers to a true-false test. The test was marked by giving 1 point for each right answer. Then 1 point was deducted for each question left blank and 2 points were deducted for each wrong answer. What were the final scores for each person on this true-false test?

	Student	Number left blank	Number wrong	Number right
a	Bill	0	2	8
b	Jim	5	0	5
c	Jan	2	2	6
d	Jay	2	3	5
e	Helen	4	0	6
f	Tim	0	5	5
g	Sam	3	3	4
h	Lil	6	0	4

13

MEASUREMENT COMPUTATION

You can't escape measurement. It's all over the place. Everybody gets involved with it sooner or later. Your goal is to explore measurement in the real world and review your computational skills at the same time.

Who can afford steak at $5.64/kg!
She's feverish, doctor. Her temperature is 39°C.
Joan has sprouted! She's grown about 10 cm in a year.
Yes, he's average for a one-year-old. He's 75 cm long and his mass is 10 kg.
Remember now-I want two 50 ml tubes of toothpaste.
The decorator says we need two 10 ℓ and one 4 ℓ tins to paint the outside.

1. They paid $2.69/kg for 4.5 kg of meat. How much did it cost?
2. Jan usually cycles to and fro from school. The journey takes 20 min . Because it was raining on Wednesday, she took the bus. How much time did she spend in cycling to school that week?
3. They bought 4 bags of rice. Each bag had a mass of 0.750 kg . What was the total mass of the 4 bags?
4. The Bereznickis drink 3 ℓ of milk per day. A litre costs 58¢. How much is their weekly milk bill?
5. Our living room is 21 m^2. How much will it cost to cover the room with carpet costing $12.95/$m^2$?

You have done a lot of work with numbers over the years. You should be able to put your knowledge to work for you. This chapter is about applications of numbers. All computation will be connected with a measurement situation.

How many types of measurement are there? Who knows? How many do you know? Find out. Make a chart. Put headings like these at the top of each column.

LENGTH	MASS	CAPACITY	OTHERS

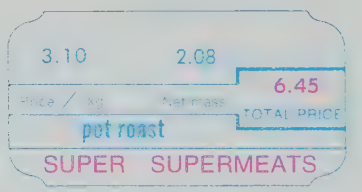
Now put all the units of measure you know in their proper column. What does it mean if a measure is called a *standard* measure?

Money is a standard unit of measure within any given country. There is no international system of currency. The coins of Canada are different from the coins of Australia. The coins of Mexico are different too. Here is a list of the coins that are minted (made) regularly in our country.

DENOMINATION	STANDARD MASS (grams)	DIAMETER (millimetres)
1-cent piece	3.20	19.05
5-cent piece	4.55	21.23
dime	2.10	17.91
quarter	5.03	23.82
half-dollar	7.78	26.45

1. Why must like coins have the same mass and diameter? Would a Canadian coin work in an Australian vending machine?
2. Does the largest coin also have the greatest value?
3. About how much mass would a bag of 100 quarters have?
4. About how many nickels would there be in a bag that has a mass of 25 g?
5. How many dollars would 100 quarters be worth?

Money is needed to buy food. Food comes in packages. But you pay for the amount that is inside the package. You pick up a package of meat and you see this label:



The scale automatically computes the price and mass of the meat. Some cuts of meat cost more than others. Pot roast costs $3.10/kg . It is cheaper than porterhouse steak which costs $6.55/kg, but more expensive than rolled roast which costs $2.99/kg .

1. Why do different cuts of meat vary in price?

2. Using the label above, how much would you pay for a pot roast with a mass of 3.08 kg? 3.64 kg? 2.56 kg?

3. If the pot roast costs exactly $3.00/kg, what would be the cost of 2 kg? 3.5 kg? 4.75 kg?

The machine in the meat market is letting labels slip before the total price is printed. The repair man has been called. But you have to get some prices on packages of meat so that people can buy them.

1.

2.60	1.00	?
Price / kg	Net mass	TOTAL PRICE
pork chops		
SUPER SUPERMEATS		

2.

2.99	4.27	?
Price / kg	Net Mass	TOTAL PRICE
rolled roast		
SUPER SUPERMEATS		

3.

6.55	1.07	?
Price / kg	Net mass	TOTAL PRICE
porterhouse steak		
SUPER SUPERMEATS		

4.

3.42	3.39	?
Price / kg	Net mass	TOTAL PRICE
half leg of lamb		
SUPER SUPERMEATS		

5.

2.26	0.89	?
Price / kg	Net mass	TOTAL PRICE
Italian sausage		
SUPER SUPERMEATS		

6.

2.54	1.19	?
Price / kg	Net mass	TOTAL PRICE
meatloaf mix		
SUPER SUPERMEATS		

PROGRESS CHECK

The store had a special sale on lamb chops and chicken. Compute the cost.

Lamb chops per kilogram $2.25	Chicken per kilogram $2.15
kg 1.00 (1.) ? cost	kg 1.00 (7.) ? cost
1.10 (2.) ?	1.30 (8.) ?
1.50 (3.) ?	1.80 (9.) ?
1.65 (4.) ?	2.23 (10.) ?
1.79 (5.) ?	2.55 (11.) ?
1.90 (6.) ?	2.63 (12.) ?

Remember the litre? It's the commonest unit for measuring capacity.
To measure larger or smaller amounts, we affix prefixes to the litre.

Before you answer the next two questions, check the prefixes that can be affixed to the litre.

smaller

0.001 ℓ is a millilitre.
0.01 ℓ is a centilitre.
0.1 ℓ is a decilitre.
☆ 1 ℓ
10 ℓ are a decalitre.
100 ℓ are a hectolitre.
1000 ℓ are a kilolitre.

larger

1. **a** How many decilitres in a litre?
b How many centilitres in a litre?
c How many millilitres in a litre?

2. **a** How many litres in a decalitre?
b How many litres in a hectolitre?
c How many litres in a kilolitre?

The symbol for litre is a script ℓ . When prefixes are affixed to the symbol, it is no longer necessary to use a script ℓ .
millilitre symbol = ml .
centilitre symbol = cl .
decilitre symbol = dl .
decalitre symbol = dal .
hectolitre symbol = hl .
kilolitre symbol = kl .

Using the chart, replace the question mark with the correct symbol.

3. **a** 40 ℓ can equal 4 ?
b 600 ℓ can equal 6 ?
c 9000 ℓ can equal 9 ?
d 0.8 ℓ can equal 8 ?
e 0.58 ℓ can equal 58 ?
f 0.006 ℓ can equal 6 ?

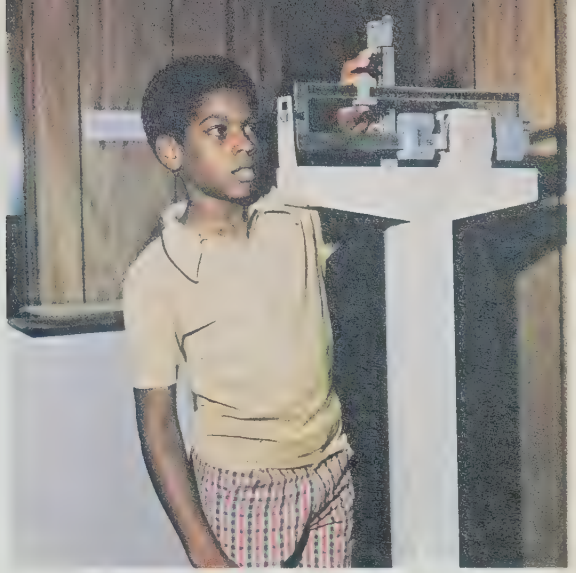
Think carefully.

ml	cl	dl	ℓ
80	?	?	?
?	468	?	?
?	?	5	?
?	?	?	2

ℓ	dal	hl	kl
50	?	?	?
?	50	?	?
?	?	50	?
?	?	?	50

What's a kilojoule?
It is a standard unit of measure that tells the energy-producing value of food. Usually people need to eat so that the kilojoules taken in through food equal the kilojoules used up through activities. (Extra kilojoules are turned into fat and stored in the body.)

Here's the number of kilojoules thought to be needed by one group of people.



	Age	Mass (kilograms)	Height (centimetres)	Kilojoules needed daily
For boys	10-12	35	140	10 500
	12-14	43	150	11 340
	14-18	60	170	12 600
For girls	10-12	35	142	9500
	12-14	44	155	9660
	14-16	52	158	10 080
	16-18	54	160	9660

1. Why do you think the listing for boys is different from that for girls?

2. If a boy is taller than the chart says for the age and mass, do you think the kilojoules needed would be more? Why?

3. If a girl's mass is more than the chart says for the age and height, do you think the kilojoules needed would be more? Why?

4. Do you know anyone who is on a diet? What does it mean to be on a diet? Why should you see a doctor before dieting?

Pretend you have a friend who is heavier than he should be. The doctor put him on a diet. He can have no more than 7560 kJ each day. Plan one day's meals. Pick foods from the list below. You'll need food from each of these four groups to keep him healthy.

1 Source of protein

meat, poultry, fish, eggs, dried beans, or peas

	Kilojoules
110 g roast beef	1260
half small chicken	1260
85 g ham	1424
85 g lamb chop	1530
85 g tuna	710
1 egg	323

2 Source of calcium

all foods from milk

	Kilojoules
1 cup milk	697
chocolate milkshake	2184
malted milk	2360
cocoa	987
1 tablespoon butter	420
1 cup cottage cheese	903

3 Source of vitamins and minerals

fruits and vegetables

	Kilojoules
apple	319
banana	370
1 cup orange juice	454
1 cup fried potatoes	1932
1 cup mashed potatoes	668
1 cup peas	466

Here are some sandwiches:

	Kilojoules
1 hamburger	1113
ham and cheese	1512
peanut butter/jelly	1134
bacon/tomato	1218
liverwurst	1470
corned beef	1701

4 Source of starch

food made from grain such as cereal or bread

	Kilojoules
1 doughnut	525
1 cup rice cereal	504
1 cup cereal	462
1 cup oatmeal	600
1 hard roll	672
1 slice bread	252

And you can't forget about the extras

	Kilojoules
1 olive	59
1 pickle	8
1 tablespoon sauce	84
25 g fudge	625
1 iced cake	609
1 piece apple pie	983

PROGRESS CHECK

1. The base unit of length is the metre. Without looking back, can you name the length that is 1000 m?

2. What is the length that is 0.01 m?

3. Use metric prefixes and make a list of three units of length smaller than 1 m .

4. Now list three units of length greater than 1 m .

SUPER SLEUTH

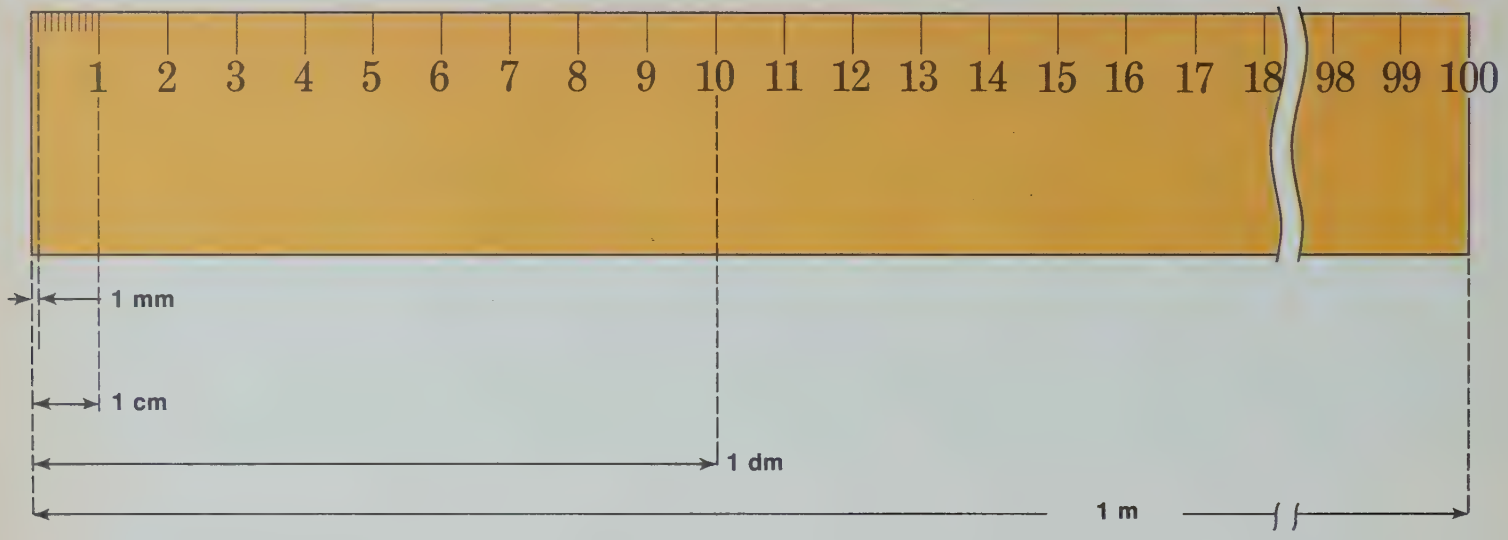
We use energy even when we are sleeping. The average person uses 5.5 kJ/min lying in bed resting or sleeping; 33.0 kJ/min bicycle riding; 47.0 kJ/min swimming; and 22.0 kJ/min walking.

1. Maybe you are a champion sleeper. How many kilojoules do you use in an eight-hour sleep?

2. What's the farthest you ever rode a bike? About how long did it take you? How many kilojoules did you use?

3. All athletes must practise. A champion swimmer practises at least two hours each day. How many kilojoules does he use practising?

4. A good walker can complete a 4 km race in less than 25 min . How many kilojoules would he use in one race?



If the stick weren't broken, it would show the length of 1 m . That is the base unit of length.

0.1 of 1 m is a decimetre.
You don't see that length used often.
It is easier to say 0.1 m .

0.01 of 1 m is a centimetre.
You will see that unit used a lot.

0.001 of 1 m is a millimetre.
That is a very short length. You have heard this unit used. Maybe you have 8 mm film in your classroom.

1. How many millimetres in 1 cm? in 1 m?
2. How many centimetres in 1 dm? in 1 m?
3. How many decimetres in 1 m?
4. Decimal fractions are used when referring to measurement. For example 1.5 cm would be the same length as 1 cm plus 5 mm, or the same as 15 mm . Name the following lengths in another way.

a 5 cm **b** 5 dm **c** 0.5 cm

d 0.5 m **e** 0.05 m **f** 0.005 m

1. Suppose you have a rule marked with centimetres only.

   Measure each of the following line segments.

   a Does this measure 1 cm?

   b Does this measure more than 1 cm but less than 2 cm? Which is closer?

   c Does this measure 2 cm?

   d Is this between 2 and 3 cm? Which is it closer to?

2. Now suppose you found a rule marked with centimetres and millimetres. Measure each of the segments again. Record the measurements.

3. Which set of measurements is more accurate?

   Could there be an even more accurate measurement taken?
   (You might have to use a magnifying glass to do it.)

   There is no such thing as a precise measurement. A measurement will be only as accurate as the measuring device you are using. That is why you will sometimes see the symbol $\approx$ used. The symbol stands for "*approximately equal to.*"

4. Measure each of these line segments to the nearest decimetre, to the nearest centimetre, and to the nearest millimetre.

   a

   b

   c

   d

ADD $\begin{array}{r} 1.345 \\ +1.46 \\ \hline \end{array}$ **NOW ADD THIS** $\begin{array}{r} 1.345\text{ m} \\ +1.46\text{ m} \\ \hline \end{array}$

Your answer should not be the same.
You must add like units of measure.
1.345 is a measurement to the nearest millimetre.
1.46 is to the nearest centimetre. You can't just annex a zero.
You must round 1.345 to the nearest centimetre. 1.345 rounds to 1.35

$\begin{array}{r} +1.46 \\ \hline \end{array}$

NOW ADD ?

PRACTICE

1. Round these millimetres to the nearest centimetre.

a 9.585 **b** 7.323 **c** 105.782 **d** 52.677

2. Round these centimetres to the nearest metre.

a 3.64 **b** 10.574 **c** 25.51 **d** 2.546

All the next problems will be in pairs.
One is not a measurement. The other is a measurement.
Compute both. Watch out! There are both addition and subtraction problems.

	a	b		a	b
3.	$\begin{array}{r} 4.68 \\ +3.759 \\ \hline \end{array}$	$\begin{array}{r} 4.68\text{ m} \\ +3.759\text{ m} \\ \hline \end{array}$	**4.**	$\begin{array}{r} 10.255 \\ -\ 4.68 \\ \hline \end{array}$	$\begin{array}{r} 10.255\text{ m} \\ -\ 4.68\text{ m} \\ \hline \end{array}$
5.	$\begin{array}{r} 15.246 \\ +\ 9.75 \\ \hline \end{array}$	$\begin{array}{r} 15.246\text{ m} \\ +\ 9.75\text{ m} \\ \hline \end{array}$	**6.**	$\begin{array}{r} 73.2 \\ -26.27 \\ \hline \end{array}$	$\begin{array}{r} 73.2\text{ m} \\ -26.27\text{ m} \\ \hline \end{array}$
7.	$\begin{array}{r} 35.28 \\ +64.7 \\ \hline \end{array}$	$\begin{array}{r} 35.28\text{ m} \\ +64.7\text{ m} \\ \hline \end{array}$	**8.**	$\begin{array}{r} 10.627 \\ -\ 9.62 \\ \hline \end{array}$	$\begin{array}{r} 10.627\text{ m} \\ -\ 9.62\text{ m} \\ \hline \end{array}$

new-born baby: 55 cm long
ribbon: 65 cm long

string: 125 cm long
carpet: 2 m long

Numbers attached to length don't mean a thing unless you can "see" the length in your mind. You should be able to draw a line segment about 1 cm long.

This line segment is 1 cm long.

This area is 1 cm^2. Area is a measure of a square unit. A square unit has two dimensions. It has both length and width.

Can you "see" the size of a square metre (m^2) in your mind? Your classroom can be measured in square metres by measuring its length and width.

Can you imagine the size of the field inside an athletic track? If you can, you can "see" the size of a hectare (ha). A hectare is 10 000 m^2.

Now imagine how much land is needed for 30 city blocks. That's a lot of land and is about the size of a square kilometre (km^2).

The square centimetre symbol is cm^2.
The square metre symbol is m^2.

The hectare symbol is ha .
The square kilometre symbol is km^2.

TRY THESE

Look at the symbols for area measurement and write the ones that would be used to describe the following areas:

a floor tiles	**b** farmland	**c** countries	**d** footprints
e carpet	**f** oceans	**g** parks	**h** doors
i the moon	**j** a bookcase	**k** graph paper	**l** houses

1. How many square centimetres in a tile 1 cm long and 1 cm wide?

   a How many square centimetres in a tile 100 cm long and 100 cm wide?

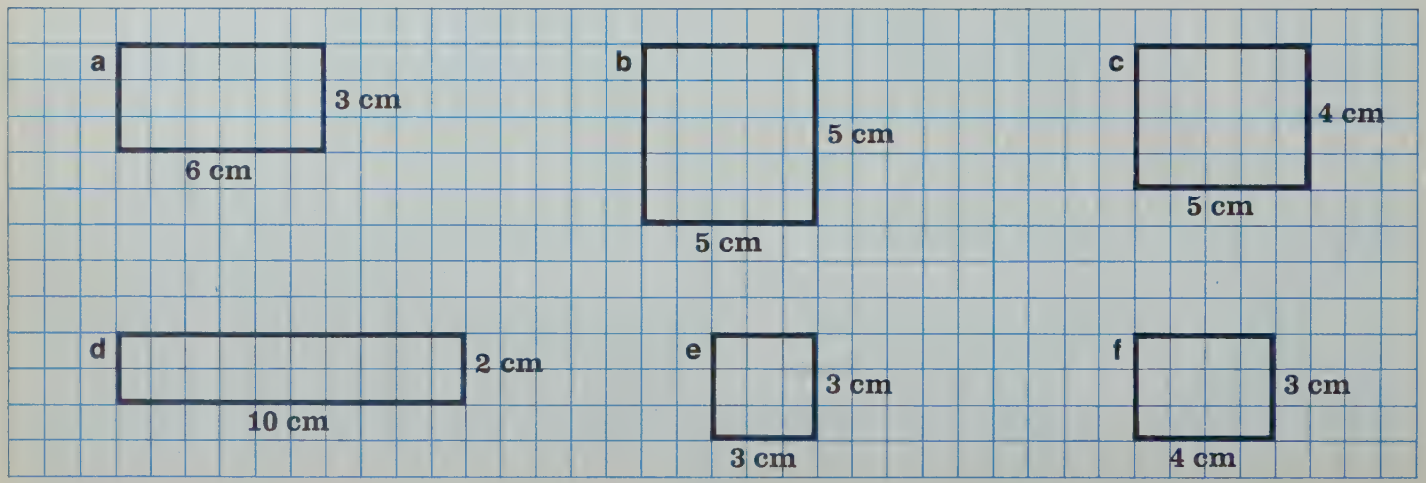
   b Then how many square centimetres in a square metre?

2. How many square metres in a piece of land 1 m long and 1 m wide?

   a How many square metres in a piece of land 1000 m long and 1000 m wide?

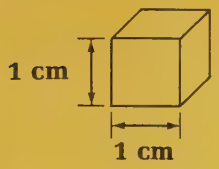
   b There are 1000 m in 1 km . How many square metres in a square kilometre?

3. Find the area. The graph paper will help you check your answers.
   ☐ Let this represent 1 cm^2. (It's not really one though.)



4. Go back. Find the perimeter of each of the figures above.

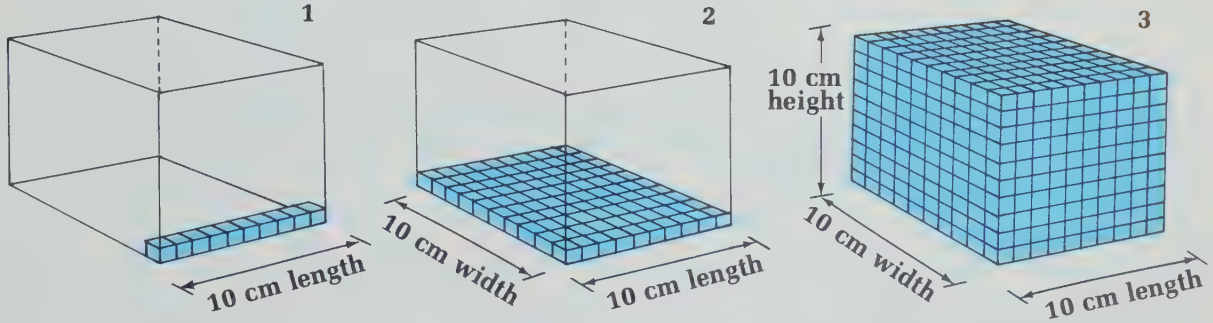
This is a picture of a cubic centimetre.



This is only a picture. A real cubic unit is 1 unit wide, 1 unit long, 1 unit thick. It has three dimensions. It takes up space. It has volume. Volume measurements are given in cubic units.

One unit of volume measurement is the cubic centimetre (cm^3).
Another unit of volume measurement is the cubic metre (m^3).

1. A block measures 10 cm high, 10 cm wide, and 10 cm long. How many cubic centimetres does it have?



a. How many cubic centimetres does the row have?

b. How many rows does the layer have? How many cubic centimetres in the layer?

c. How many layers does the block have? How many cubic centimetres does it have?

The volume of this block is $10 \times 10 \times 10$ cm^3, or 1000 cm^3. Each side is a decimetre long. Can you think of another unit to describe the volume of the block?

Finding the volume of blocks is easy.

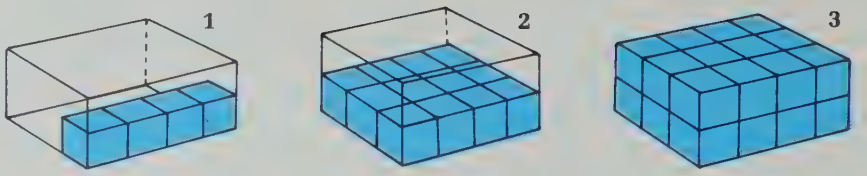
1. Length × width × height is the volume.

   a What's the length?

   b What's the width?

   c What's the height?

   d What's the volume?



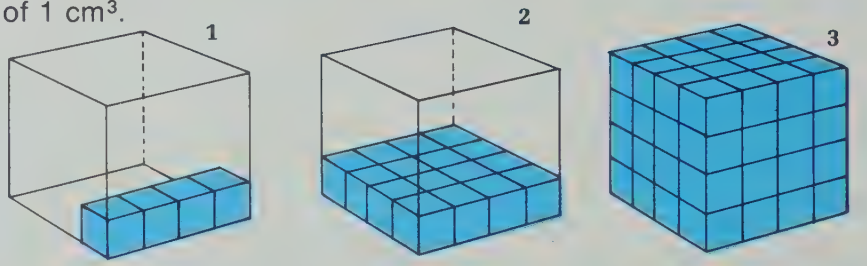
2. Each cube has a volume of 1 cm^3.

   a What's the length?

   b What's the width?

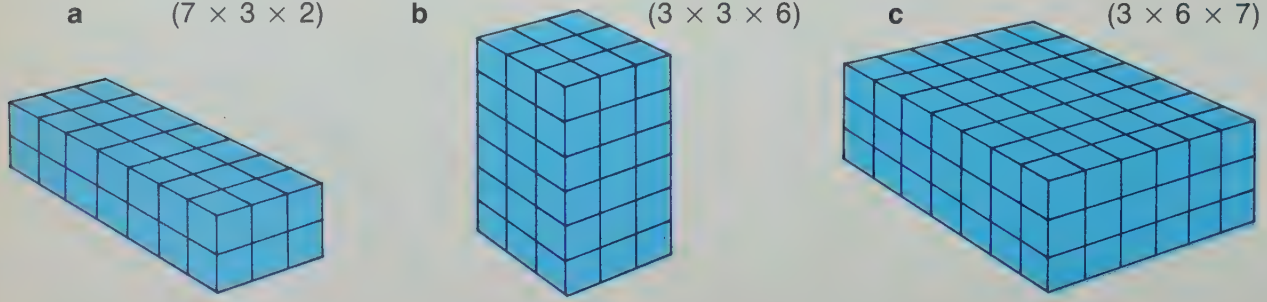
   c What's the height?

   d What's the volume?



3. Each unit cube measures 1 cm on each side. Find the volume of each block.

   a $(7 \times 3 \times 2)$ **b** $(3 \times 3 \times 6)$ **c** $(3 \times 6 \times 7)$



4. If each cube in problem 3 measured 1 m on each side, what would be the volume of each block?

1. What's the volume of a box that measures—

a 2 cm long, 3 cm wide, and 2 cm high?

b 5 m long, 3 m high, and 2 m wide?

c 3 cm high, 4 cm long, and 10 cm wide?

d 12 cm wide, 5 cm long, and 1 cm high?

e 10 m by 5 m by 2 m?

f 10 cm by 4 cm by 3 cm?

PROGRESS CHECK

What is the volume of a box that measures—

(1.) 1 m by 1 m by 1 m?

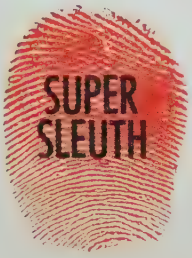
(2.) 3 cm by 2 cm by 1 cm?

(3.) 1 cm by 2 cm by 3 cm?

(4.) 6 cm by 4 cm by 4 cm?

(5.) 10 cm by 4 cm by 4 cm?

*(6.) 10 cm by 10 cm by 10 cm?

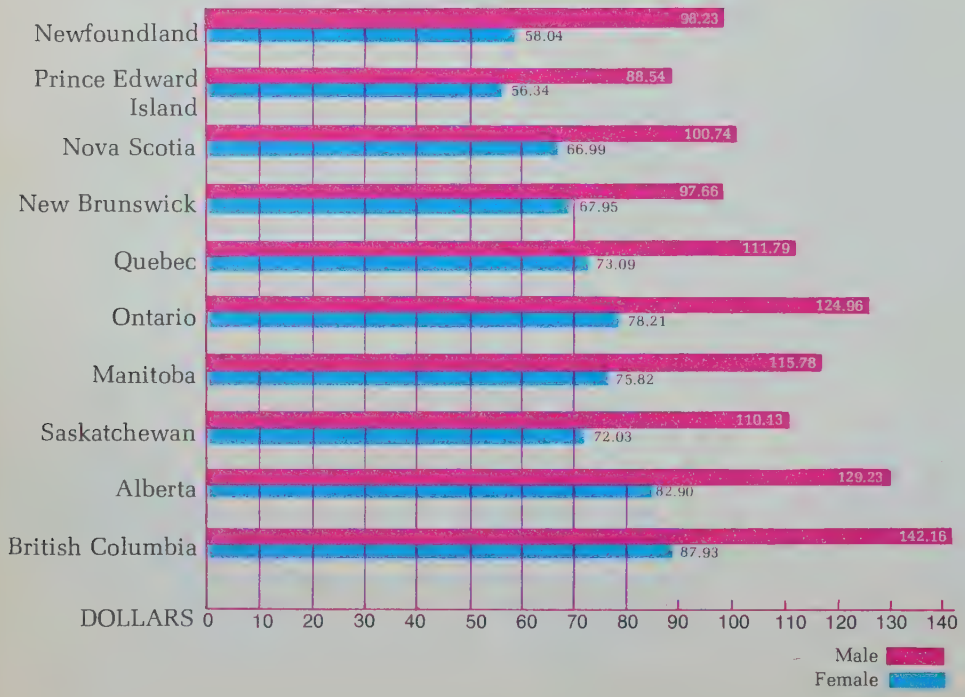


A power shovel was used to dig a hole 8.5 m wide, 1.5 m deep and 10 m long. How many cubic metres of earth were removed from the hole?

One dollar is a unit of measure too. But its value is not always the same. Its purchasing power is greater at one time than another. And it has been shown that the same number of dollars are not always paid to people for the same job.

Here is a chart that shows the average weekly pay men and women received in a recent year.

THE WAGE GAP: AVERAGE WEEKLY EARNINGS OF RETAIL-TRADE EMPLOYEES, 1970



1. What is the difference between the pay for men and the pay for women in each province?

2. Which province has the least inequality?

3. Which province has the greatest inequality?

4. Find this year's almanac. Get the same type of information. Has progress been made towards equal pay?

5. Would a person get to take home all the money earned?

Newspapers have pages that look like these:

Stock Exchange

1973 High	1973 Low	Stocks and Div. In Dollars	Sls. P/E	Sls. 100s	High	Low	Last	Net Chng
		A—B—C—D						
80⅞	61	AbbtLb 1.20	21	31	66½	66	66	−1
49¾	40½	ACF Ind 2.40	10	2	42¾	42¾	42¾	− ¼
17⅛	12¼	AcmeClv .88	7	1	13	13	13	+ ⅛
26	16½	AcmeMkt 1						

Stock Exchange

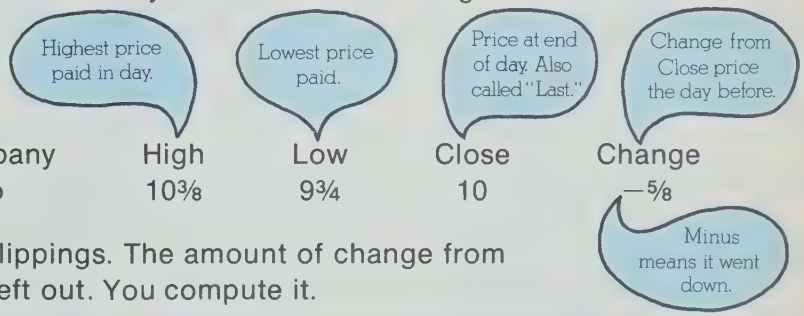
Day's Sales	Wednesday	Year Ago	Year to Date 1973	Year to Date 1972
1 933 620	1 879 730	3 176 443	488 239 340	816 669 530

Most Active Stocks

Company	Volume	Last	Net Chng.
Mansfd TR	**115 000**	5¼	— ¾
Kaiser Ind	**86 100**	7¼	+ ½

These are stock-market pages. They list a lot of companies. People can own parts of these companies by buying shares in them. A share in one company might cost less than a dollar. A share in another company might cost hundreds of dollars. The price depends on how much people will pay. Most stocks are bought and sold through stock exchanges such as the Montreal, Vancouver, and Toronto exchanges.

The value of a share is given in dollars and eighths of a dollar. There are usually these column headings:



Here are two very old clippings. The amount of change from Monday to Tuesday is left out. You compute it.

STOCK EXCHANGE

MONDAY, OCTOBER 28, 1929.

Day's Sales.	Saturday.	A Year Ago.
9,212,800	2,087,660	3,770,570

Stock	High	Low	Close	Ch'ge
Abitibi Pow & Paper	48½	43½	44¼	− 4¾
Adams Millis	27⅝	27½	27½	− ¼
Ahumada Lead	1¼	1	1⅛	....
Air Reduction	189¼	135	145	−49⅞
Air Way El Ap	28½	28½	28½	− 5⅜
Alleghany Corp	36	29	29⅛	− 7½

STOCK EXCHANGE

TUESDAY, OCTOBER 29, 1929.

Day's Sales.	Monday.	A Year Ago.
16,410,030	9,212,800	3,483,770

Stock	High	Low	Close	Ch'ge
Abitibi Pow & Paper	40	38	38	**a**
Adams Millis	26	24¾	24¾	**b**
Ahumada Lead	1	¾	¾	**c**
Air Reduction	128	100¼	120	**d**
Air Way El Ap	25	25	25	**e**
Alleghany Corp	24¾	18⅛	20⅝	**f**

Some very interesting history is connected with these clippings. If you have grandparents, they can tell you the story. You can find it in reference books, too.

CHECKOUT

Now it's time to measure yourself and your skills. This is a long Checkout. Don't try to do it all at once.

Part I–What do you know about numbers?

1. How many different digits are used in our number system?

2. Using each digit only once, write the largest whole number that can be written.

3. Write all the different 3-digit whole numbers that can be written with the digits 7, 8, and 9.

4. Write the place value of each digit in this number.
   9 182 736 450

5. Write the numbers in order from least to greatest.
   100 100 100 001 100 010 101 010 101 100

6. Are the following names for the same number?
   $\frac{1}{9}$, $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{5}$

   a Write them in order from least to greatest.

   b If the numerator is 1, which digit would be used in the denominator to show the fraction of greatest value?

7. Write these numbers in order from least to greatest.

   a 0.1, 0.001, 1.0, 0.01 1.1, 1.11, 1.01

   b $\frac{9}{2}$, $\frac{7}{2}$, $\frac{3}{2}$, $\frac{2}{2}$, $\frac{5}{2}$, $\frac{1}{2}$, $\frac{4}{2}$

Part II-What do you know about operations on whole numbers?

1. Pick any two whole numbers. Add them.
   Can you find a pair that does not have a whole-number answer?

2. Pick any whole number. Add zero.
   Is the answer always the same as the first number you picked?

3. Pick any two whole numbers. Add them.
   Change the order of the two numbers. Add again.
   Do you get the same answer?

4. Pick any three whole numbers. Add them.
   Write the three numbers, but group them differently.
   Add again. Do you get the same answer?

5. Go back to problems 1 through 4, but multiply this time.
   Do you get the same results? (If you say yes, you are WRONG!)
   Which problem has a different result?

6. Go back to problems 1 through 4. Subtract this time.
   Which problems have different results when you subtract?

Complete these.

	a	b	c	d	e
7.	$\begin{array}{r} 23\,756 \\ +\,16\,438 \\ \hline \end{array}$	$\begin{array}{r} 502 \\ \times\ 79 \\ \hline \end{array}$	$27\overline{)729}$	$\begin{array}{r} 6520 \\ -\,2001 \\ \hline \end{array}$	$14\overline{)1410}$
8.	$\begin{array}{r} 785 \\ 6215 \\ +\ \ 15 \\ \hline \end{array}$	$\begin{array}{r} 6704 \\ -\,1916 \\ \hline \end{array}$	$50\overline{)5390}$	$\begin{array}{r} 1628 \\ \times\ \ 25 \\ \hline \end{array}$	$\begin{array}{r} 6000 \\ -\,4201 \\ \hline \end{array}$

Part III-What do you know about operations with fractions?

There is a step-by-step thinking process you use for both addition and subtraction of fractions.

Review how you can find a common denominator.

$\frac{1}{4}+\frac{1}{6}$ 4 is not a factor of 6. What number has both 4 and 6 as factors? You can multiply 4×6. 24 will work as a common denominator. Or you can look again. You know 4 and 6 are factors of 12 also. 12 is the least common multiple you can use as a denominator.

Find a common denominator for each pair of fractions.

1. **a** $\frac{1}{2}, \frac{1}{8}$ **b** $\frac{1}{4}, \frac{1}{3}$ **c** $\frac{1}{10}, \frac{1}{4}$ **d** $\frac{1}{6}, \frac{1}{4}$

Now compute. Watch the operation symbols.
Write the simplest name for each answer.

2. **a** $3\frac{5}{9}+2\frac{2}{9}$ **b** $2\frac{3}{5}+1\frac{2}{5}$ **c** $1\frac{6}{7}+1\frac{5}{7}$ **d** $3\frac{7}{8}+1\frac{5}{8}$

3. **a** $3\frac{5}{9}-2\frac{2}{9}$ **b** $2\frac{3}{5}-1\frac{2}{5}$ **c** $1\frac{6}{7}-1\frac{5}{7}$ **d** $3\frac{7}{8}-1\frac{5}{8}$

4. **a** $\frac{2}{5}+\frac{3}{10}$ **b** $3\frac{1}{3}+2\frac{1}{6}$ **c** $6\frac{3}{4}+4\frac{1}{2}$ **d** $5\frac{1}{6}+1\frac{1}{2}$

5. **a** $\frac{2}{5}-\frac{3}{10}$ **b** $3\frac{1}{3}-2\frac{1}{6}$ **c** $6\frac{3}{4}-4\frac{1}{2}$ **d** $5\frac{1}{6}-1\frac{1}{2}$

You use common denominators in division too. Show your skills.

6. **a** $\frac{1}{3} \div \frac{1}{6}$ **b** $\frac{5}{8} \div \frac{1}{4}$ **c** $\frac{3}{4} \div \frac{1}{3}$ **d** $\frac{2}{5} \div \frac{1}{2}$

Renaming the product is the hardest part of multiplication.
But you have had lots of practice. Prove you can do it.
Multiply. Write the simplest name for each product.

7. **a** $2 \times \frac{1}{2}$ **b** $\frac{2}{3} \times 3$ **c** $\frac{1}{4} \times \frac{3}{5}$ **d** $\frac{1}{3} \times \frac{3}{4}$

8. **a** $1\frac{1}{2} \times \frac{1}{4}$ **b** $\frac{4}{5} \times 1\frac{1}{4}$ **c** $2\frac{1}{5} \times 1\frac{1}{2}$ **d** $1\frac{2}{3} \times \frac{3}{5}$

Operating with decimals is a joy.
Place value is the only thing you have to watch out for.

Add these.

9. **a** $\begin{array}{r} 2.26 \\ +1.82 \\ \hline \end{array}$ **b** $\begin{array}{r} 62.7 \\ +\ 1.3 \\ \hline \end{array}$ **c** $\begin{array}{r} 27.608 \\ 3.521 \\ +681.671 \\ \hline \end{array}$ **d** $9 + 0.01 + 6.7$

Now subtract.

10. **a** $\begin{array}{r} 6.70 \\ -3.21 \\ \hline \end{array}$ **b** $\begin{array}{r} 5.02 \\ -4.13 \\ \hline \end{array}$ **c** $21.7 - 9.18$ **d** $20 - 0.85$

Multiply.

11. **a** $\begin{array}{r} 6.01 \\ \times\ 0.2 \\ \hline \end{array}$ **b** $\begin{array}{r} 5.1 \\ \times\ 35 \\ \hline \end{array}$ **c** $\begin{array}{r} 0.04 \\ \times\ 0.2 \\ \hline \end{array}$ **d** $\begin{array}{r} 0.15 \\ \times\ 0.15 \\ \hline \end{array}$

And now divide.

12. **a** $5\overline{)6.25}$ **b** $0.7\overline{)3.57}$ **c** $0.10\overline{)500}$ **d** $8.2\overline{)21.32}$

Part IV-How well can you use what you know?

Provincial tax on fuel varies.
Complete the following tax tables.

1.

litres	tax
1	$0.05
2	**a**
5	**b**
10	**c**
15	**d**

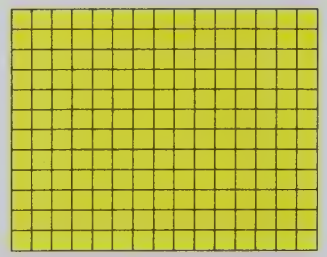
2.

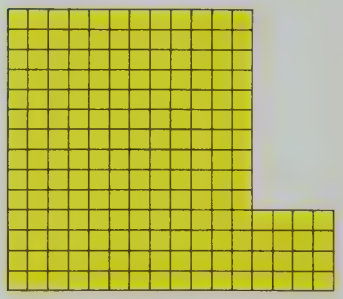
litres	tax
1	$0.0658
2	**a**
5	**b**
10	**c**
15	**d**

3.

litres	tax
1	$0.085
2	**a**
5	**b**
10	**c**
15	**d**

4. Find the perimeter of each figure. Then find the area. Let each square represent $1m^2$.



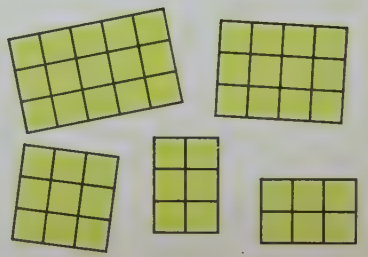


5. Suppose the figures above are really scale models of large offices.
Each of the floors needs to be covered with carpet.
The cost of carpet is $\$10.50/m^2$.
What is the total cost if you cover both offices?

6. Suppose the shapes on the right are in a paper bag.

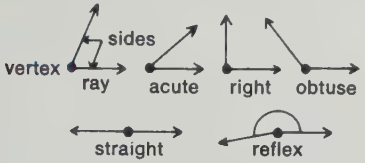
You have this one.

Someone is to pick one of the shapes out of the bag for you. What is the probability that you would get a shape congruent to the one you already have?



GLOSSARY

angle Two rays with a common endpoint



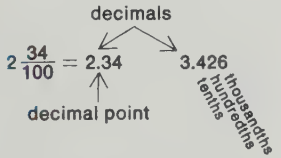
area The amount of surface inside a plane figure

Area is 8 square units.

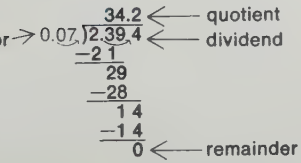
congruent Two figures are congruent if they have the same size and shape.

curve A figure drawn without lifting the pencil

decimal Another name for a fraction



division



exponent

$1000 = 10 \times 10 \times 10 = 10^3$ ← exponent—read "10 to the third power"

factoring Writing a number as a product of factors

$28 = 4 \times 7$ $\quad$ $24 = 2 \times 3 \times 4$

fraction A number that tells how much

$\frac{6}{7}$ ← numerator, ← denominator $\quad$ $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

equivalent fractions

$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$ $\quad$ $\frac{1}{5} = \frac{2}{10}$ ← common denominators

simplest name $\quad$ $\frac{7}{10} = \frac{7}{10}$

greatest common factor

factors of 24: 1, 2, 3, 4, ⑥, 8, 12, 24
factors of 18: 1, 2, 3; ⑥, 9, 18
6 is the greatest common factor of 18 and 24.

integer Any of the whole numbers 0, +1, +2, +3, and so on, or the negative numbers −1, −2, −3, −4, and so on

math sentence

true: $3 + 5 = 8$ $\quad$ open: $3 + \square = 7$
false: $4 < 3 \times 0$ $\quad$ open: $9 - a = b$

mixed number Another name for some fractions

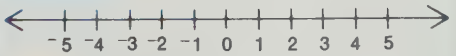
$2.3 = 2\frac{3}{10}$ $\quad$ $6\frac{7}{6} = 7\frac{1}{6}$; $2\frac{3}{5} = \frac{13}{5}$

mixed numbers

multiplication

0.6 ← factor
× 0.7 ← factor
0.42 ← product

number line



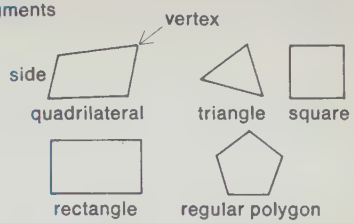
percent Parts of one hundred

$0.25 = \frac{25}{100} = 25\%$—read "25 percent"

perimeter The distance around a polygon or other simple closed curve

Perimeter is 15 cm.

polygon A simple closed curve made with line segments



All sides and angles are congruent in a regular polygon.

prime number A whole number greater than 1 whose only pair of factors is 1 and itself. 2, 3, 5, 7, 11 are some prime numbers.

probability A number from 0 to 1 that tells how likely something is to happen

The outcome of a spin is A or B.
Select the *event* A.

The probability of A is $\frac{1}{4}$.

The *odds* in favor of A are 1 to 3.

reflection

rotation The turning of a figure

Figure has rotated 120° clockwise.

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TABLES OF MEASURES

METRIC

LENGTH

10 millimetres (mm) = 1 centimetre
10 centimetres (cm) = 1 decimetre
10 decimetres (dm) = 1 metre
10 metres (m) = 1 decametre
10 decametres (dam) = 1 hectometre
10 hectometres (hm) = 1 kilometre (km)

AREA

100 square millimetres (mm^2) = 1 square centimetre
10 000 square centimetres (cm^2) = 1 square metre
10 000 square metres (m^2) = 1 hectare
100 hectares (ha) = 1 square kilometre (km^2)

MASS

10 milligrams (mg) = 1 centigram
10 centigrams (cg) = 1 decigram
10 decigrams (dg) = 1 gram
10 grams (g) = 1 decagram
10 decagrams (dag) = 1 hectogram
10 hectograms (hg) = 1 kilogram
1000 kilograms (kg) = 1 tonne (t)

VOLUME

1000 cubic millimetres (mm^3) = 1 cubic centimetre
1000 cubic centimetres (cm^3) = 1 cubic decimetre
1000 cubic decimetres (dm^3) = 1 cubic metre (m^3)

CAPACITY

Liquid and Dry

10 millilitres (ml) = 1 centilitre
10 centilitres (cl) = 1 decilitre
10 decilitres (dl) = 1 litre
10 litres (ℓ) = 1 decalitre
10 decalitres (dal) = 1 hectolitre
10 hectolitres (hl) = 1 kilolitre (kl)

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